F.E. SEM - I / CHOICE BASE / MAY 2019 / 10.05.2019

(3 Hours) **[Total marks : 80**

Note :-
1) Question number **1** is **compulsory.**
2) Attempt any **three** questions from the remaining **five** questions.
3) **Figures** to the **right** indicate **full** marks.

Q.1 a) If $u = \log\left(\frac{x}{y}\right) + \log\left(\frac{y}{x}\right)$, find $\frac{\partial u/\partial x}{\partial u/\partial y}$ 03

b) Find the value of $\tanh(\log x)$ if $x = \sqrt{3}$ 03

c) Evaluate $\lim\limits_{x\to 3}\left[\frac{1}{x-3} - \frac{1}{\log(x-2)}\right]$ 03

d) If $u = r^2\cos 2\theta,\ v = r^2\sin 2\theta$, find $\frac{\partial(u,v)}{\partial(r,\theta)}$ 03

e) Express the matrix $A = \begin{pmatrix} 2+3i & 2 & 3i \\ -2i & 0 & 1+2i \\ 4 & 2+5i & -i \end{pmatrix}$ as the sum of a Hermitian and a Skew-Hermitian matrix. 04

f) Expand $tan^{-1}x$ in powers of $\left(x - \frac{\pi}{4}\right)$ 04

Q.2 a) Expand $sin^7\theta$ in a series of sines of multiples of θ 06

b) If $y = sin^2 x cos^3 x$, find y_n 06

c) Find the stationary values of $x^3 + y^3 - 3axy,\ a > 0$ 08

Q. 3 a) Compute the real root of $x\, log_{10}x - 1.2 = 0$ correct to three places of decimals using Newton-Raphson method. 06

b) Show that the system of equations 06
$2x - 2y + z = \lambda x,\ 2x - 3y + 2z = \lambda y,\ \ -x + 2y = \lambda z$
can posses a non-trivial solution only if $\lambda = 1, \lambda = -3$. Obtain the general solution in each case.

c) If $\tan(\alpha + i\beta) = \cos\theta + i\sin\theta$, prove that 08
$\alpha = \frac{n\pi}{2} + \frac{\pi}{4}$ and $\beta = \frac{1}{2}\log\tan\left(\frac{\pi}{4} + \frac{\theta}{2}\right)$

Q. 4 a) Using the encoding matrix as $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, encode and decode the message MOVE 06

b) If $u = f(e^{x-y}, e^{y-z}, e^{z-x})$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ 06

c) If $y = a\cos(\log x) + b\sin(\log x)$, then show that $x^2 y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$ 08

Q. 5 a) If $1, \alpha, \alpha^2, \alpha^3, \alpha^4,$ are the roots of $x^5 - 1 = 0$, find them and show that $(1-\alpha)(1-\alpha^2)(1-\alpha^3)(1-\alpha^4) = 5$ 06

b) If $\theta = t^n e^{-r^2/(4t)}$, 06

Find n which will make $\frac{\partial \theta}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \theta}{\partial r}\right)$

c) Find the root (correct to three places of decimals) of $x^3 - 4x - 9 = 0$ lying between 2 and 3 by using Regula-Falsi method. 08

Q. 6 a) Find non-singular matrices P and Q such that 06

$A = \begin{pmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{pmatrix}$ is reduced to normal form. Also find its rank.

b) Find the principle value of $(1+i)^{1-i}$ 06

c) Solve the following equations by Gauss-Seidel method 08

$27x + 6y - z = 85$

$6x + 15y + 2z = 72$

$x + y + 54z = 110$

(Take three iterations)
