Duration – 3 Hours **Total Marks: 80**

N.B 1) Question **No. 1** is **Compulsory**.

2) **Answer** any **three** questions from remaining questions.

3) Figures to the right indicate full marks.

Q.1 a) Evaluate $\int_0^\infty y^4 e^{-y^6} dy$. **3**

b) Find the circumference of a circle of radius r by using parametric equations of the circle $x = r\cos\theta, y = r\sin\theta$. **3**

c) Solve $(D^2 + D - 6)y = e^{4x}$. **3**

d) Evaluate $\int_0^1 \int_{x^2}^{x} xy(x^2 + y^2)dydx$. **3**

e) $Solve\ (\tan y + x)dx + (x\sec^2 y - 3y)dy = 0$. **4**

f) Solve $\frac{dy}{dx} = 1 + xy$ with initial condition $x_0 = 0, y_0 = 0.2$ by Euler's method. Find the approximate value $of\ y\ at\ x = 0.4\ with\ h = 0.1$. **4**

Q.2 a) Solve $(D^2 - 4D + 3)y = e^x \cos 2x + x^2$. **6**

b) Show that $\int_0^\infty \frac{\tan^{-1} ax}{x(1+x^2)} dx = \frac{\pi}{2}\log(1 + a)$. **6**

c) Change the order of integration and evaluate $\int_0^2 \int_{\frac{x^2}{2}}^{4-x} xydydx$. **8**

Q.3 a) Evaluate $\iiint x^2yz\, dxdydz$ throughout the volume bounded by the planes $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$. **6**

b) Find the mass of lamina of a cardioid $r = a(1 + \cos\theta)$. If the density at any point varies as the square of its distance from its axis of symmetry. **6**

c) Solve $(3x + 2)^2 \frac{d^2y}{dx^2} + 5(3x + 2)\frac{dy}{dx} - 3y = x^2 + x + 1$. **8**

Q.4 a) Find by double integration the area common to the circles $r = 2cos\theta \ and \ r = 2sin\theta$. **6**

b) Solve $sin2x\frac{dy}{dx} = y + tanx$. **6**

c) Solve $\frac{dy}{dx} = 3x + y^2$ with initial conditions $y_0 = 1$, $x_0 = 0$ at at x=0.2 in steps of h=0.1 by Runge Kutta method of fourth order. **8**

Q.5 a) Evaluate $\int_0^1 x^5 sin^{-1}x \, dx$ and find the value of $\beta\left(\frac{7}{2}, \frac{1}{2}\right)$. **6**

b) The differential equation of a moving body opposed by a force per unit mass of value cx and resistance per unit mass of value bv^2 where x and v are the displacement and velocity of the particle at that time is given by $v\frac{dv}{dx} = -cx - bv^2$. Find the velocity of the particle in terms of x, if it starts from the rest. **6**

c) Evaluate $\int_0^6 \frac{dx}{1+4x}$ by using i) Trapezoidal ii) Simpsons (1/3)rd and iii) Simpsons (3/8)th rule. **8**

Q.6 a) Find the volume of the region that lies under the paraboloid $z = x^2 + y^2$ and above the triangle enclosed by the lines $y = x, x = 0 \ and \ x + y = 2$ in the xy plane. **6**

b) Change to polar coordinates and evaluate $\int\int y^2 dxdy$ Over the area outside $x^2 + y^2 - ax = 0$ and inside $x^2 + y^2 - 2ax = 0$. **6**

c) Solve by method of variation of parameters **8**

$$\frac{d^2y}{dx^2} + y = \frac{1}{1+sinx}.$$

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