Paper / Subject Code: 40001 / APPLIED MATHEMATICS - IV

SE/Sem IV/CBCGS/AUTO/MJ 2019/07-05-2019

Duration – 3 Hours Total Marks : 80

N.B.:- 1. Question no 1 is compulsory.
2. Attempt any THREE questions out of remaining FIVE questions.

Q.1 a) Write the dual of the given LPP (5)
Maximize $Z=4x_1+9x_2+2x_3$
Subject to: $2x_1+3x_2+2x_3\le 7$, $3x_1-2x_2+4x_3=5$, $x_1,x_2,x_3\ge 0$.

b) If X is a Random Variable with probability density function (5)

$$f(x)=\begin{cases} kx; 0\le x\le 2\\ 2k; 2\le x\le 4\\ 6k-kx; 4\le x\le 6\end{cases}$$

Find k, expectation and $P(1\le x\le 3)$.

c) A tyre company claims that the life of the tyres have mean 42,000 kms with standard deviation of 4,000 kms. A change in the production process is believed to a result in better product.A test sample of 81 new tyres has a mean life 42,500 kms.Test at 5% level of significance that the new product is significantly better than the old one. (5)

d) Find the minimal polynomial of $A=\begin{bmatrix}2&2&1\\1&3&1\\1&2&2\end{bmatrix}$. Is A derogatory? (5)

Q.2 a) Use Big-M method to solve the following LPP (6)
Minimize $Z=2x_1+x_2$
subject to $3x_1+x_2=3$,
$4x_1+3x_2\ge 6$,
$x_1+2x_2\le 3$, $x_1,x_2\ge 0$

b) Find e^A and 4^A if $A=\begin{bmatrix}3/2&1/2\\1/2&3/2\end{bmatrix}$. (6)

c) Verify Green's theorem for $\int_C (3x^2-8y^2)dx+(4y-6xy)dy$ where C is the closed curve given by $y=x^2, y=\sqrt{x}$. (8)

Q.3 a) Prove that $\overline{F}=2xyz^2 i+(x^2z^2+z\cos yz)j+(2x^2yz+y\cos yz)k$ is a conservative field. Find ϕ such that $\overline{F}=\nabla\phi$. Hence find the work done in moving an object in this field from $(0,0,1)$ to $(1,\frac{\pi}{4},2)$. (6)

b) The standard deviations calculated from two random samples of sizes 9 and 13 are 1.99 and 1.9. Can the samples be regard as drawn from the normal populations with same standard Deviations. (6)
(Given: $F(0.025)=3.51$ with d. f. 8 & 12 and $F(0.025)=4.20$ with d. f. 12 & 8.)

c) Find the index, rank, signature and class of the Quadratic Form $x_1^2 + 2x_2^2 + 3x_3^2 + 2x_1x_2 - 2x_1x_3 + 2x_2x_3$ by reducing it to canonical form using congruent transformation method. **(8)**

Q. 4 a) Evaluate $\iint_S \overline{F} \bullet d\overline{S}$ where $\overline{F} = (2xy+z)i + y^2 j - (x+3y)k$ and S is the closed surface bounded by $x=0, y=0, z=0, 2x+2y+z=6$. **(6)**

b) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 2 \\ 2 & 2 \end{bmatrix}$ and hence find $2A^4 - 5A^3 - 7A + 6I$. **(6)**

c) A sample of 400 students of under-graduate and 400 students of post-graduate classes was taken to know their opinion about autonomous colleges.290 of the under-graduate and 310 of the post-graduate students favoured the autonomous status.Use chi-square test and test that the opinion regarding autonomous status of colleges is independent of the level of classes of students. **(8)**

Q. 5 a) Prove that $\nabla \times \left[\frac{\overline{a} \times \overline{r}}{r^3} \right] = \frac{-\overline{a}}{r^3} + \frac{3(\overline{a} \bullet \overline{r})\overline{r}}{r^5}$ **(6)**

b) Show that the matrix $A = \begin{bmatrix} 3 & -1 & 1 \\ -1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ is diagonalizable and hence find the transforming matrix and diagonal matrix. **(6)**

c) Ten school boys were given a test in statistics and their scores were recorded. They were given a month special coaching and a second test was given to them in the same subject at the end of the coaching period .Test at 5% level of significance, if the marks given below give evidence to the fact that the students are benefited by coaching. **(8)**

Mark in test 1:	70	68	56	75	80	90	68	75	56	58
Mark in test 2:	68	70	52	73	75	78	80	92	54	55

Q. 6 a) In a sample of 1000 cases, the mean of a certain test is 14 and Standard Deviation is 2.5. Assuming the distribution to be normal, find **(6)**
1] how many students score between 12 & 15.
2] how many score above 18.

b) Evaluate by Stoke's theorem $\int_C xy\,dx + xy^2 dy$, where C is the square in the xy-plane with vertices (1, 0),(0,1),(-1,0),(0,-1). **(6)**

c) Using duality solve the following L.P.P. **(8)**

Minimise $z = 0.7x_1 + 0.5x_2$

subject to $x_1 \geq 4, x_2 \geq 6, x_1 + 2x_2 \geq 20,\ 2x_1 + x_2 \geq 18,$

$x_1, x_2, x_3 \geq 0$.
