JEE-Main-25-02-2021-Shift-1 (Memory Based)

PHYSICS

Question: The distance of two points from the center of a loop on the axis is 0.05 cm and 0.20 cm and the ratio of the magnetic fields at these points is 8 : 1 respectively. Find the radius of the loop?

Options:

(a) 1 mm

(b) 0.1 mm

(c) 10 mm

(d) 0.01 mm

Answer: (a)

Solution:

Magnetic field due to circulation loop

$$B \propto \frac{1}{\left(r^2 + x^2\right)^{3/2}}$$

$$\Rightarrow \frac{B_1}{B_2} = \frac{\left((x_2)^2 + r^2\right)^{3/2}}{\left((x_1)^2 + r^2\right)^{3/2}}$$

$$\Rightarrow \frac{8}{1} = \frac{\left((.2)^2 + r^2\right)^{3/2}}{\left((.05)^2 + r^2\right)^{3/2}}$$

Solving we get

$$\Rightarrow \frac{4}{1} = \frac{\left((.2)^2 + r^2\right)}{\left((.05)^2 + r^2\right)}$$

$$\Rightarrow 4\left(\frac{25}{10000}\right) + 4r^2 = \frac{4}{100} + r^2$$

$$\Rightarrow 3r^2 = \frac{4-1}{100} = \frac{3}{100}$$

$$\Rightarrow r = \frac{1}{10} cm$$

$$= 1 mm$$

Question: Proton, deuteron and alpha particle have same momentum. They are projected in the same magnetic field. Then choose the correct ratio of forces and their speeds.

Options:

(a) $F_p : F_d : F_\alpha = 4:2:1; V_p : V_d : V_\alpha = 2:1:1$

(b) $F_p : F_d : F_\alpha = 2:1:1; V_p : V_d : V_\alpha = 4:2:1$

(c) $F_p : F_d : F_\alpha = 1:2:1; V_p : V_d : V_\alpha = 1:2:1$

(d) $F_p : F_d : F_\alpha = 1:1:1; V_p : V_d : V_\alpha = 1:1:1$
Answer: (b)
Solution:
We have same momentum for proton, deuteron and alpha particle.

$$F = qvB\sin\theta$$

$$F_p = ev_pB\sin\theta$$

$$F_d = ev_dB\sin\theta$$

$$F_\alpha = 2ev_\alpha B\sin\theta$$

$$F_p : F_d : F_\alpha = ev_pB\sin\theta : ev_dB\sin\theta : 2ev_\alpha B\sin\theta$$

Taking $\theta = 90^\circ$ & $m_p = m,\ m_d = 2m,\ m_\alpha = 4m$

$$V_p : V_d : V_\alpha = \frac{p}{m_p} : \frac{p}{m_d} : \frac{p}{m_a}$$

$$= \frac{1}{m_p} : \frac{1}{m_d} : \frac{1}{m_\alpha} = \frac{1}{1} : \frac{1}{2} : \frac{1}{4}$$

$$V_p : V_d : V_\alpha = 4:2:1$$

Now

$$F_p : F_d : F_\alpha = V_p : V_d : 2V_\alpha$$

$$= 4:2:2$$

$$= 2:1:1.$$

Question: STATEMENT 1: A free rod when heated experiences no thermal stress.
STATEMENT 2: The rod when heated increases in length.
Options:
(a) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of statement 1.
(b) Statement 1 is true, Statement 2 is true
(c) Statement 1 is true, Statement 2 is false
(d) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of statement 1.
Answer: (d)
Solution:
Thermal stress generates, when rod is clamped but in statement 1 rod is free so it will not experience any thermal stress. Hence statement 1 is correct.
On heating length of the rod increases. So statement 2 is also correct.
But statement 2 doesn't totally explain statement 1.
So correct option is D.

Question: STATEMENT 1: Two planets have same escape velocity & their masses are not equal.
STATEMENT 2: Ratio of mass to radius must be equal.
Options:
(a) Statement 1 is true, Statement 2 is true, Statement 2 is the correct explanation of statement 1.
(b) Statement 1 is true, Statement 2 is true

(c) Statement 1 is true, Statement 2 is false
(d) Statement 1 is true, Statement 2 is true, Statement 2 is not the correct explanation of statement 1.
Answer: (a)
Solution:
Escape velocity on a planet is given by

$$V_e = \sqrt{\frac{2GM}{R}}$$

Where M is mass of planet & R is the radius of planet.

Now taking escape velocities both planet equal. $(V_e)_{P_1} = (V_e)_{P_2}$

$$\sqrt{\frac{2GM_1}{R_1}} = \sqrt{\frac{2GM_2}{R_2}} = \sqrt{\frac{M_1}{R_1}} = \sqrt{\frac{M_2}{R_2}}$$

$$\frac{M_1}{R_1} = \frac{M_2}{R_2}$$

Question: The time period of a 2m long simple pendulum is 2 seconds. Find the value of ‘g’ at that place?
Options:

(a) $2\pi^2$

(b) π^2

(c) $4\pi^2$

(d) $\frac{\pi^2}{2}$

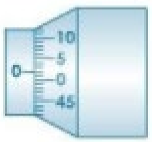
Answer: (a)
Solution:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$\Rightarrow 2 = 2\pi\sqrt{\frac{2}{g}}$$

$$\Rightarrow g = 2\pi^2$$

Question: The pitch of a micrometer screw gauge is 1 mm and the circular scale has 100 divisions. When there is nothing between the jaws, the zero of the circular scale is 8 divisions below the main scale. When a wire is put between the jaws, the 1st division of main scale is visible and 72nd division of circular scale coincides with main scale. The radius of wire is?



(b)
Positive zero error

Options:

(a) 1.8 mm
(b) 0.9 mm
(c) 1.64 mm
(d) 0.82 mm
Answer: (d)
Solution:

$LC = \frac{1}{100} = 0.01 mm$

Zero error $= +8$

Zero correction $= -8$ x LC

Main scale reading $= 1 mm$

Circular scale reading $= 72$

Diameter of wire $= 1 + (72 - 8) \times 0.01$

$= 1.64 mm$

Radius $= \frac{1.64}{2} = 0.82 mm$

Question: If a train engine crosses a signal with a velocity u and has constant acceleration and the last bogey of train crosses the signal with velocity v, then middle point of train crosses the signal with velocity?
Options:
(a) $\frac{v+u}{2}$
(b) $\sqrt{\frac{v^2+u^2}{2}}$
(c) $\sqrt{\frac{v^2-u^2}{2}}$
(d) $\frac{v-u}{2}$
Answer: (b)
Solution:

Let the length of the train be $= l$
Acc. to 3rd equation of motion

$v^2 - u^2 = 2as$

Where $s = l$
$v^2 - u^2 = 2al$

$a = \frac{v^2 - u^2}{2l}$

When mid point, $l' = \frac{l}{2}$

$$v_{last}^2 = u^2 + 2a\frac{l}{2}$$

$$= u^2 + al$$

$$= u^2 + \frac{v^2 - u^2}{2l}l$$

$$= \frac{2u^2 + v^2 - u^2}{2}$$

$$\boxed{v_{last} = \sqrt{\frac{u^2 + v^2}{2}}}$$

Question: Two satellites A & B revolve in $R_1 = 600$ km & $R_2 = 1600$ km. Find $T_2 : T_1$.

Answer: 4.35

Solution:

From Kepler's third law

$$T^2 \propto R^3$$

$$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3$$

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{1600}{600}\right)^3$$

$$\frac{T_2}{T_1} = \left(\frac{16}{6}\right)^{3/2} = 4.35$$

Question: A diatomic gas is heated at constant pressure. Find the ratio dU : dQ : dW (where symbol has usual meaning)

$$\left(\text{Given}: C_p = \frac{7}{2}R; C_v = \frac{5}{2}R\right)$$

Options:

(a) 5 : 7 : 1

(b) 5 : 7 : 2

(c) 2 : 7 : 5

(d) 1 : 1 : 1

Answer: (b)

Solution:

$$dQ = nC_p dT$$

$$dU = nC_v dT$$

$$dW = n\left(C_p - C_v\right)dT$$

$$dU : dQ : dW$$

$$C_u : C_p : \left(C_p - C_v\right)$$

$$\frac{5}{2}R : \frac{7}{2}R : R$$

$$5 : 7 : 2$$

Question: Match the following physical quantities with the correct dimensions?

1	2
h (planck's constant)	$[M^1 L^2 A^{-1} T^{-3}]$
KE (kinetic energy)	$[M^1 L^2 T^{-1}]$
V (voltage)	$[M^1 L^1 T^{-1}]$
P (momentum)	$[M^1 L^2 T^{-2}]$

Answer:
h ⟶ $[M^1 L^2 T^{-1}]$
KE ⟶ $[M^1 L^2 T^{-2}]$
V ⟶ $[M^1 L^2 A^{-1} T^{-3}]$
P ⟶ $[M^1 L^1 T^{-1}]$
Solution:

$$[KE]=[W]=[F][x]=MLT^{-2}\times L=ML^2T^{-2}$$

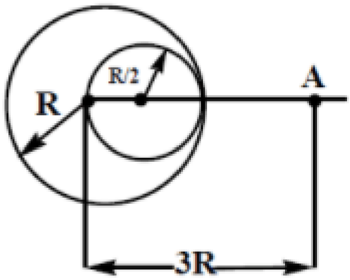
$$[P]=[m][v]=MLT^{-1}$$

$$V=\frac{W}{q}=\frac{W}{It}\Rightarrow[v]=\frac{[W]}{[I][t]}=\frac{ML^2T^{-2}}{AT}$$

$$=ML^2A^{-1}T^{-3}$$

$$[h]=ML^2T^{-1}$$

Question: A uniform solid sphere of mass M and ratio R applies an attractive gravitational force F_1 on a point mass m placed at a distance 3R from the center of sphere. Now a spherical mass of radius $\frac{R}{2}$ is removed from the sphere as shown. The force experienced by mass 'm' now is F_2. Find $\frac{F_1}{F_2}$?



Options:

(a) $\frac{F_1}{F_2}=\frac{50}{41}$

(b) $\frac{F_1}{F_2}=\frac{41}{50}$

(c) $\frac{F_1}{F_2} = \frac{41}{42}$

(d) None of these

Answer: (a)

Solution:

Let the particle of mass m be placed on A. Then

$$F_1 = \frac{GMm}{(3R)^2} = \frac{GMm}{9R^2}$$

After taking out $R/2$ radius sphere, mass of the remaining sphere,

$$= \left[\frac{4}{3}\pi R^3 - \frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right]d = \frac{4}{3}\pi R^3\left[\frac{7}{8}\right]d$$

$= \frac{7}{8}M$ (As $M = \frac{4}{3}\pi R^3 d$)

Now force on m placed at A,

$$F_2 = \frac{GMm}{9R^2} - \frac{GMm}{\theta\left(\frac{5}{2}R\right)^2} = \frac{GMm}{R^2}\left[\frac{1}{9} - \frac{1}{50}\right] = \frac{41}{450}\frac{GMm}{R^2}$$

$$\therefore \frac{F_1}{F_2} = \frac{\frac{GMm}{9R^2}}{\frac{41}{450}\frac{GMm}{R^2}} = \frac{50}{41}$$

Question: A thermodynamics process obeys the law p = KV^3 when the temperature is raised from 100°C to 300°C. Find the work done on one mole of gas? [R = 8.3]

Answer: (415)

Solution:

$P = Kv^3$

$n = 1$

$W = \int P.dv$

$W = \int K.v^3 dv$

$W = K\frac{v^4}{4}\Big|_{v_i}^{v_f}$

$W = \frac{k}{4}\left(v_f^4 - v_i^4\right) \quad ....(1)$

We have

$PV = nRT$

$\frac{nRT}{v} = kv^3$

$RT = kv^4$

$kv_f^4 = R\times(300+273)$

$kv_i^4 = R(100+273)$

$kv_f^4 - kv_i^4 = R(300-100)$

$= 200\,R$

$kv_f^4 - kv_i^4 = 200\times 8.3 \quad ...(2)$

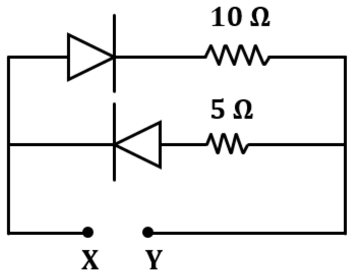
From (1) and (2)

$W = \dfrac{200\times 8.3}{4}$

$= 50\times 8.3$

$W = 415\,J$

Question: Find the current in ideal battery of 5v between X & Y such that X is at higher potential.



Options:
(a) 0.5 A
(b) 0.43 A
(c) 0.57 A
(d) 0.1 A
Answer: (b)
Solution:

Diode connected in series with 10Ω would be forward biased.
As some potential would drop to overcome barrier potential of diode,
So current would be less than $0.5A\left(=\dfrac{5V}{10\Omega}\right)$
And only option 0.43 A
Satisfies this condition.

Question: An alpha particle and a proton, are accelerated from rest by a potential difference of 200 V. Find the ratio of their de broglie wavelengths?
Options:
(a) $\dfrac{\lambda_p}{\lambda_a} = \dfrac{\sqrt{8}}{1}$
(b) $\dfrac{\lambda_p}{\lambda_a} = \dfrac{1}{\sqrt{8}}$

(c) $\frac{\lambda_p}{\lambda_a} = \frac{1}{2}$

(d) $\frac{\lambda_p}{\lambda_a} = \frac{2}{1}$

Answer: (a)

Solution:

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}}$$

$$\lambda_a \propto \frac{1}{\sqrt{m_\alpha q_\alpha}}$$ (V is same for both)

$$\lambda_p \propto \frac{1}{\sqrt{m_p q_p}}$$

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{\sqrt{m_\alpha q_\alpha}}{\sqrt{m_p q_p}} = \frac{\sqrt{4m_p \times 2q_p}}{\sqrt{m_p q_p}} = \frac{\sqrt{8}}{1}$$

Question: Two radioactive samples X and Y have number of nuclei N_{10} and N_{20} respectively. The half life of X is half of that of Y. It is observed that after the time equal to three half life of Y, the number of nuclei of X is equal to that of Y. Find the ratio of initial number of nuclei of X ?

Options:

(a) $\frac{N_{20}}{N_{10}} = \frac{1}{8}$

(b) $\frac{N_{20}}{N_{10}} = 8$

(c) $\frac{N_{20}}{N_{10}} = \frac{1}{2}$

(d) $\frac{N_{20}}{N_{10}} = 2$

Answer: (a)

Solution:

$$N = N_0 \left(\frac{1}{2}\right)^n$$

$$N_X = N_{10} \left(\frac{1}{2}\right)^{3\times 2} \quad \ldots\text{(i)}$$

$$N_Y = N_{20} \left(\frac{1}{2}\right)^{3} \quad \ldots\text{(ii)}$$

According to question

$N_X = N_Y$

$N_{10} \cdot \left(\frac{1}{2}\right)^6 = N_{20}\left(\frac{1}{2}\right)^3$

$\Rightarrow \frac{N_{20}}{N_{10}} = \frac{(1/2)^6}{(1/2)^3} = \frac{1}{8}$

Question: Two coherent sources have intensity in the ratio of 2x. Find the value of

$\left[\frac{I_{max} - I_{min}}{I_{max} + I_{min}}\right]$

Options:

(a) $\frac{2\sqrt{x}}{x+1}$

(b) $\frac{2\sqrt{2x}}{x+1}$

(c) $\frac{2\sqrt{2x}}{2x+1}$

(d) $2x$

Answer: (c)

Solution:

$\frac{I_1}{I_2} = 2x$

$I_1 = 2xI_2$

$I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$

$I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

$I_{max} - I_{min} = 4\sqrt{I_1 I_2}$

$I_{max} + I_{min} = 2(I_1 + I_2)$

$\left(\frac{I_{max} - I_{min}}{I_{max} + I_{min}}\right) = \frac{2\sqrt{I_1 I_2}}{I_1 + I_2}$

$= \frac{2\sqrt{2xI_2^2}}{2xI_2 + I_2}$

$= \frac{2\sqrt{2x}}{2x+1}$

Question: 512 drops each of potential 2V are coalesced to form a big drop. The potential of the big drop (in V).

Answer: (128)

Solution:

For small drop

$\frac{Kq}{r} = 2$ ...(i)

When 512 drops coalesced

$$\left(\frac{4}{3}\pi R^3\right) = 512\left(\frac{4}{3}\pi r^3\right)$$

$\Rightarrow R = 8r$

Potential of large drop

$$\Rightarrow \frac{KQ}{R} = \frac{K\,512\,q}{8r} = 64\left(\frac{Kq}{r}\right)$$

Using eq^n (i)

$\Rightarrow 64(2) = 128\underline{V}$

Question: A circular coil of wire consisting of 100 turns each of radius 8.0 cm carries a current of 0.40 A. The magnitude of B at the centre of the coil is $n \times 10^{-5}\,T$, where n is closest to the integer:

Answer: (31)

Solution:

$$B_C = \frac{\mu_0 In}{2r} = \frac{4\pi \times 10^{-7} \times 0.4 \times 100}{2 \times 8 \times 10^{-2}}$$

$= 0.31 \times 10^{-3}$

$= 31 \times 10^{-5}\,T$

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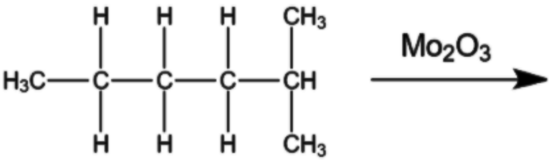
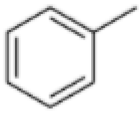
CHEMISTRY

Question:



Options:

(a)



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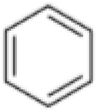
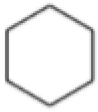


(c)



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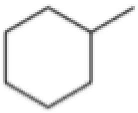
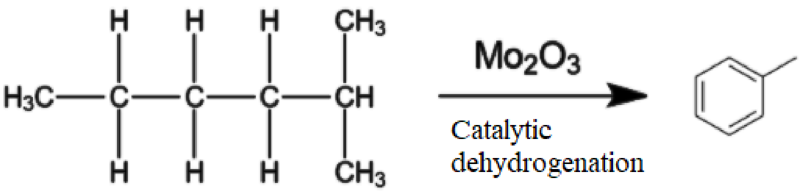


Answer: (a)

Solution:



Question:

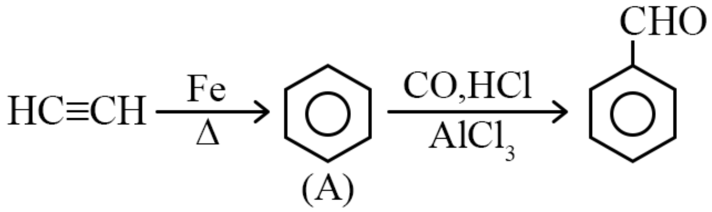
$$HC\equiv CH \xrightarrow[\Delta]{Fe} (A) \xrightarrow[AlCl_3]{CO,HCl} (B)$$

Number of sp^2 hybridised carbon in B

Options:

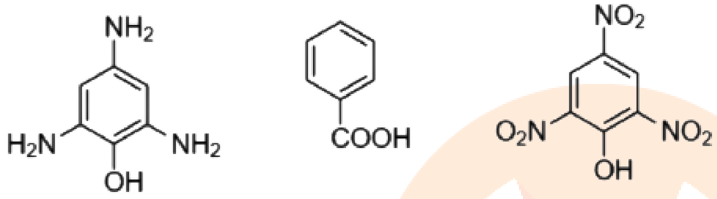
(a) 7

(b) 5
(c) 6
(d) 1
Answer: (a)
Solution:



{Total 7 sp^2 carbon}

Question: Which will liberate CO_2 with reaction with $NaHCO_3$



(a) (b) (c)

Options:
(a) Only (b)
(b) (a) only
(c) (c) and (a)
(d) (b) and (c)
Answer: (d)
Solution: $NaHCO_3$ being basic in nature, reacts with benzoic acid (b) and liberates CO_2. Picric acid (c) contains 3 NO_2 groups which are electron withdrawing and increase the acidity of phenolic hydrogen whereas (a) contains 3 amine groups which are electron releasing and decrease the acidity of phenolic hydrogen.
Hence, (b) and (c) liberates CO_2 on reaction with $NaHCO_3$ but (a) does not.

Question: Which of the following is correct?
Options:
(a) Buna-S is a thermosetting and synthetic polymer
(b) Buna-N is a natural polymer
(c) Neoprene is used to manufacture buckets
(d) Nascent oxygen is used in the formation of Buna-S
Answer: (d)
Solution:
Buna – S is a synthetic polymer and a thermoplastic.
Buna – N is a synthetic polymer.
Neoprene is used to manufacture conveyor belts, gaskets and hoses.

Buna – S is formed in presence of peroxide as catalyst (nascent oxygen.)

Question: In the Freundlich isotherm at moderate pressure x/m is directly proportional to p^x, where x is:

Options:

(a) 1/n

(b) 0

(c) 1

(d) None of these

Answer: (a)

Solution: $\frac{x}{m} = \left(KP^{\frac{1}{n}} \right)$

Freundlich formula

Question: Which of the following have same number of electrons in outermost shell?

Options:

(a) Cr^{3+} and Fe^{+}

(b) Sc^{+} and V^{+}

(c) Mn^{2+} and Cr^{+}

(d) Sc^{+} and Ti^{+}

Answer: (c)

Solution: Both have $3d^5$ electronic configuration.

Question: Which of the following linkage is present in Lactose?

Options:

(a) C_1 - C_4, β-D galactose and β-D glucose

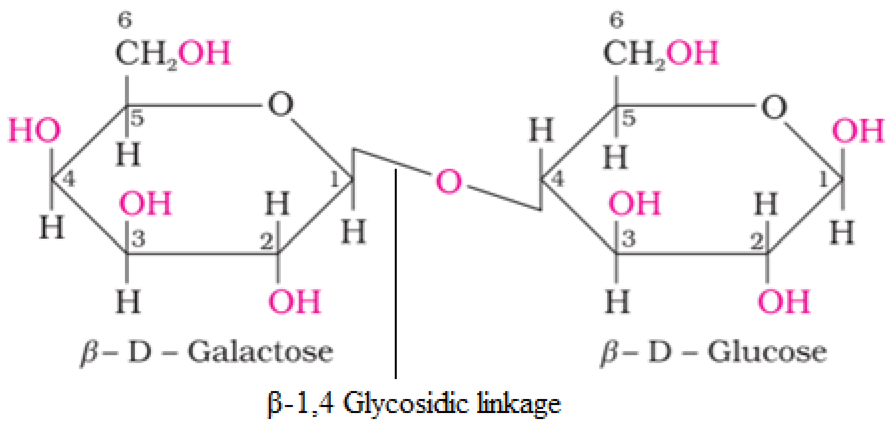
(b) C_1 - C_2, β-D galactose and β-D glucose

(c) C_1 - C_2, β-D glucose and β-D galactose

(d) C_1 – C_3, β-D glucose and β-D galactose

Answer: (a)

Solution: Lactose: It is more commonly known as milk sugar since this disaccharide is found in milk. It is composed of β-D galactose and β-D glucose. The linkage between C1 of galactose and C4 of glucose. Free aldehydes group may be produced at C-1 of glucose unit, hence it is also a reducing sugar.



Lactose

Question: Which of the following does not exist as per MOT?

Options:

(a) Be_2

(b) He_2

(c) He_2^+

(d) None of these

Answer: (b)

Solution:

$$He_2 = \sigma_{1s^2} \sigma^*_{1s^2}$$

$$B.O = \frac{1}{2}\left[e^-_{BMO} - e^-_{ABMO}\right]$$

$$B.O = \frac{1}{2}[2-2] = 0$$

According to MOT, If any molecule has zero bond order then it does not exist.

Question: Ellingham diagram is the plot between?

Options:

(a) ΔG vs T

(b) ΔH vs T

(c) ΔS vs T

(d) None of these

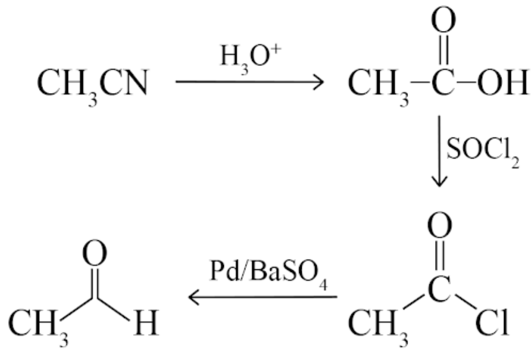
Answer: (a)

Solution: Ellingham diagram shows a graph between Gibbs energy charge (ΔG) and temperature.

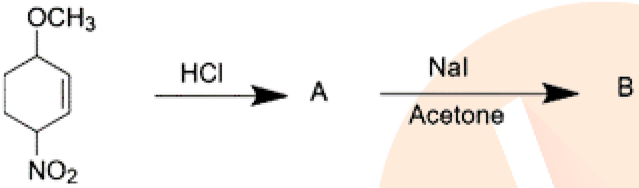
Question: $CH_3CN \xrightarrow[(3)Pd/BaSO_4]{(1)H_3O^+ \; (2)SOCl_2}$

Options:

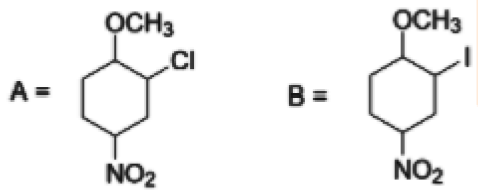
(a) CH_3COOH
(b) CH_3COCl
(c) CH_3CHO
(d) CH_3CH_2OH
Answer: (c)
Solution:



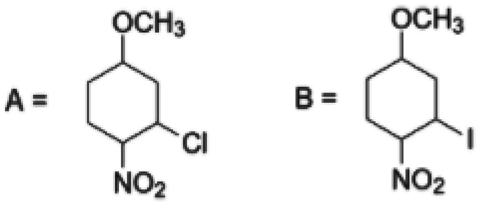
Question:



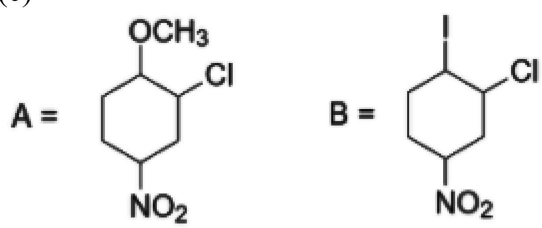
Find ‘A’ and ‘B’
Options:
(a)



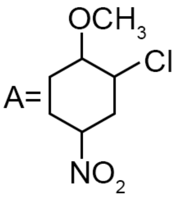
(b)



(c)

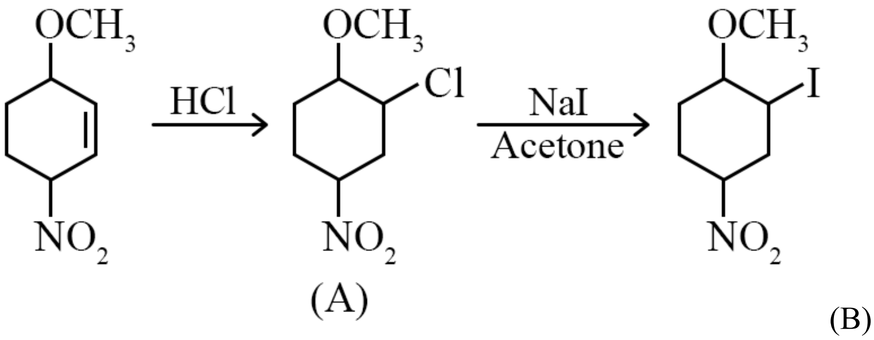


(d)



Answer: (a)

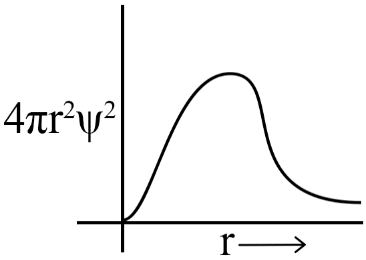
Solution:



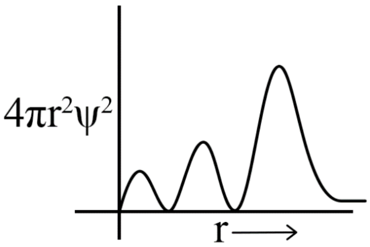
Question: Which of the following is the correct radial probability distribution curve for 3s orbital?

Options:

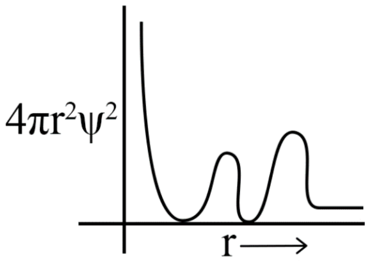
(a)



(b)



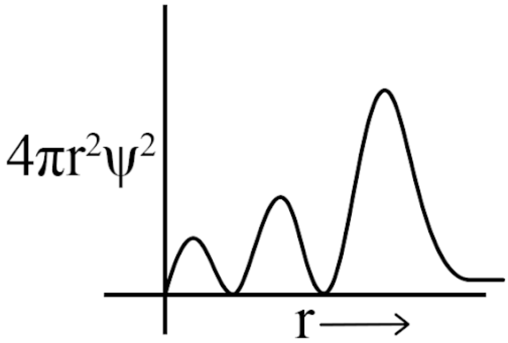
(c)



(d) None of these

Answer: (b)

Solution:



Radial node = $n - \ell - 1$
= 3 - 0 - 1
= 2

Question: Hybridization of $[Mn(CN)_6]^{4-}$ and magnetic nature of $[Fe(CN)_6]^{3-}$
Options:
(a) d^2sp^3 and diamagnetic
(b) sp^3d^2 and diamagnetic
(c) d^2sp^3 and paramagnetic
(d) sp^3d^2 and paramagnetic
Answer: (c)
Solution:
$[Mn(CN)_6]^{4-}$
CN^- is a strong field ligand and Mn is in +2 oxidation state ($3d^5$ configuration)
Hence, it forms inner sphere orbital complex and $\left[Mn(CN)_6\right]^{4-}$ is d^2sp^3 hybridised
$\left[Fe(CN)_6\right]^{3-}$ Fe is in $3d^5$ configuration.
CN^- is strong filed ligand

Fe^{3+} =>	⇅	⇅	1		

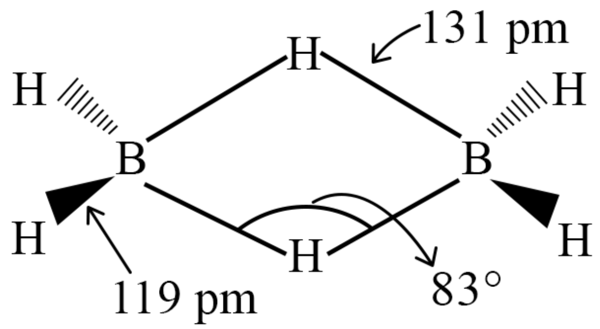
$\mu = \sqrt{n(n+2)} = \sqrt{1(1+2)} = \sqrt{3}$
$= 1.73\, BM$

Question: Find the boiling point of the aqueous solution of A_2B_3 considering 60 % dissociation. (given: K_b (H_2O) = 0.52. Molality = 1 molal
Answer: 101.768
Solution:
$A_2B_3 \rightleftharpoons 2A^{3+} + 3B^{2-}$
i = 1 – α + nα (for dissociation)
= 1 – 0.6 + 5 × 0.6
= 3.4
$\Delta T_b = i \times K_b \times m$

= 3.4 × 0.52 × 1
= 1.768
Boiling point = 101.768 ºC

Question: Statement 1: CeO_2 is used for the oxidation of aldehyde.
Statement 2: Aqueous solution of $EuSO_4$ acts as strong reducing agent.
Options:
(a) Both are true
(b) S_1 is true and S_2 is false
(c) S_1 is false and S_2 is true
(d) Both S_1 and S_2 are false
Answer: (a)
Solution: CeO_2 is mild oxidizing agent and used in oxidation of aldehyde into corresponding acid.
Eu is a lanthanide having electronic configuration $[Xe]4f^7 5d^1 6s^2$. Therefore, Eu^{+2} oxidises readily to give more stable Eu^{+3} and acts as a strong reducing agent.

Question: Correct statement about B_2H_6
Options:
(a) BH_3 is Lewis acid
(b) Terminal H has more p character than bridge H
(c) All B-H bond are of same length
(d) Bond angle B-H-B is 120º
Answer: (a)
Solution: BH_3 is Lewis acid because boron has 6 valence electron
∴ It can accept 2 electrons to complete its octet.



JEE-Main-25-02-2021-Shift-1 (Memory Based)

MATHEMATICS

Question: If $\frac{x^2}{a}+\frac{y^2}{b}=1$ and $\frac{x^2}{c}+\frac{y^2}{d}=1$ are orthogonal then relation between a, b, c, d is:

Options:

(a) $a-b=c-d$

(b) $ab=\frac{b+d}{c+d}$

(c)

(d)

Answer: (a)

Solution:

Let common point of curves be (p, q).

For $\frac{x^2}{a}+\frac{y^2}{b}=1$

Differentiating, we get

$$\frac{2x}{a}+\frac{2y}{b}\frac{dy}{dx}=0$$

$$\Rightarrow \frac{dy}{dx}=-\frac{bx}{ay}$$

So, slope for first curve at (p, q)

$$(=m_1)=-\frac{bp}{aq}$$

Similarly slope for second curve at (p, q)

$$(=m_2)=-\frac{dp}{cq}$$

Now, as both curves are orthogonal

$$\Rightarrow m_1m_2=-1$$

$$\frac{bd}{ac}\left(\frac{p^2}{q^2}\right)=-1 \qquad ....(1)$$

Now, (p, q) lies on both the curves

So, $\frac{p^2}{a}+\frac{q^2}{b}=1$ and

$$\frac{p^2}{c}+\frac{q^2}{a}=1$$

Subtracting these

$$p^2\left(\frac{1}{a}-\frac{1}{c}\right)+q^2\left(\frac{1}{b}-\frac{1}{d}\right)=0$$

$$\Rightarrow \frac{p^2}{q^2}=\frac{\left(\frac{1}{d}-\frac{1}{b}\right)}{\left(\frac{1}{a}-\frac{1}{c}\right)}=\frac{(b-d)ac}{(c-a)bd}$$

Putting this value in (1), we get

$$\frac{bd}{ac}\cdot\frac{(b-d)ac}{(c-a)bd}=-1$$

$$\Rightarrow b-d=a-c$$

$$\Rightarrow a-b=c-d$$

Question: The image of the point(3, 5) in line $x-y+1=0$, lies on

Options:

(a) $(x,2)^2+(y-4)^2=8$

(b) $(x+4)^2+(y-6)^2=16$

(c)

(d)

Answer: ()

Solution:

Image of point (3, 5) in $x-y+1=0$ is

$$\frac{x-3}{1}=\frac{y-5}{-1}=\frac{-2(3-5+1)}{1^2+1^2}$$

$$\Rightarrow \frac{x-3}{1}=\frac{y-5}{-1}=1$$

$$\Rightarrow x=4, y=4$$

From the given options

Question: $\lim_{x\to\infty}\left[1+\frac{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\ldots\frac{1}{n}}{n^2}\right]$

Options:

(a) 0

(b) $\frac{1}{e}$

(c) $\frac{1}{2}$

(d) 1

Answer: (d)

Solution:

Given, $\lim_{n\to\infty}\left[1+\frac{\frac{1}{2}+\frac{1}{2}+...+\frac{1}{n}}{n^2}\right]^n$

Its 1^∞ form

$\Rightarrow e^{\lim_{n\to\infty} n\left(\frac{\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+...+\frac{1}{n}}{n^2}\right)}$

$\Rightarrow e^{\lim_{n\to\infty}\left(\frac{1}{n}+\frac{1}{2n}+\frac{1}{3n}+....+\frac{1}{n^2}\right)}$

$\Rightarrow e^0 = 1$

Question: $A \to (B \to A)$ equal to

Options:

(a) $A \to (A \vee B)$

(b) $A \to (A \wedge B)$

(c) $A \to (A \to B)$

(d) $A \to (A \leftrightarrow B)$

Answer: (a)

Solution:

$A \to (B \to A)$

$\equiv A \to (\sim B \vee A)$

$\equiv \sim A \vee (\sim B \vee A)$

$\equiv \sim B \vee (\sim A \vee A)$ (Associative law)

$\equiv \sim B \vee t$

$\equiv t$

So, given statement is a tautology

Now option (A)

$A \to (A \vee B)$

$\equiv \sim A \vee (A \vee B)$

$\equiv (\sim A \vee A) \vee B$

$= t \vee B$

$\equiv t$

So, option (A) is correct

Question: $\frac{dy}{dx} = \frac{x^2 - 4x + 8 + y}{x - 1}$, if curve passes through origin, then it also passes through

Options:
(a) (5, 5)
(b) (4, 5)
(c) (5, 4)
(d) (4, 4)

Answer: (a)

Solution:

$\frac{dy}{dx} = \frac{x^2 - 4x + 8 + y}{x - 2}$

$\frac{dy}{dx} = \frac{(x^2 - 4x + 4)(y + 4)}{(x - 2)}$

$\frac{dy}{dx} = \frac{y + 4}{(x - 2)} + (x - 2)$

Let $x - 2 = X$

$y + 4 = Y$

$\frac{dY}{dX} = \frac{Y}{X} + X$

$\frac{dY}{dX} - \frac{Y}{X} = X$

Integrating factor $= e^{\int -\frac{1}{x} dx}$

$= e^{-\log x}$

$= \frac{1}{X}$

$\Rightarrow \frac{Y}{X} = \int 1 dx$

$\Rightarrow Y = X^2 + cX$

$\Rightarrow y + 4 = (x-2)^2 + c(x-2)$

It passes through origin (0, 0), so

$4 = 4 - 2c \Rightarrow c = 0$

$\Rightarrow (y+4) = (x-2)^2$

$(5, 5)$ satisfies it

Question: The coefficients a, b, c of quadratic equation $ax^2 + bx + c = 0$ are obtained by throwing a dice thrice. The probability that it has equal roots is

Options:

(a) $\frac{1}{36}$

(b) $\frac{1}{72}$

(c) $\frac{1}{54}$

(d) $\frac{5}{216}$

Answer: (d)

Solution:

For Equal roots D = 0

$\Rightarrow b^2 = 4ac$

For $b = 1 \Rightarrow ac = \frac{1}{4} \Rightarrow$ Not possible

For $b = 2 \Rightarrow ac = 1 \Rightarrow a = 1, c = 1$

For $b = 3 \Rightarrow ac = \frac{9}{4} \Rightarrow$ Not possible

For $b = 4 \Rightarrow ac = 4 \Rightarrow (1,4),(4,1),(2,2)$

For $b = 5 \Rightarrow ac = \frac{25}{4} \Rightarrow$ Not possible

For $b = 6 \Rightarrow ac = 9 \Rightarrow (3,3)$

So, cases with equal roots are

(1, 2, 1), (1, 4, 4), (4, 4, 1), (2, 4, 2), (3, 6, 3)

Total number of ways = $6 \times 6 \times 6 = 216$

Required probability = $\frac{5}{216}$

Question: $\int_{-1}^{1} x^2 e^{[x^3]} dx$

Options:

(a) $\frac{1}{3e}$

(b) $\frac{e+1}{3e}$

(c) $\frac{3e+1}{3e}$

(d) $\frac{1}{e}$

Answer: (b)

Solution:

$\left[x^3\right] = 0$ for $0 \le x < 1$

And $\left[x^3\right] = -1$ for $-1 < x < 0$

So, $\int_{-1}^{1} x^2 e^{[x^3]} dx = \int_{-1}^{0} x^2 e^{[x^3]} dx + \int_{0}^{1} x^2 e^{[x^3]} dx$

$= \int_{-1}^{0} x^2 \cdot e^{-1} dx + \int_{0}^{1} x^2 dx$

$= \frac{1}{e} \times \frac{x^3}{3}\Big|_{-1}^{0} + \frac{x^3}{3}\Big|_{0}^{1}$

$= \frac{1}{3e}\left(0-(-1)\right) + \frac{1}{3}(1-0)$

$= \frac{1}{3e} + \frac{1}{3}$

Question: If $x = \sum_{n=0}^{\infty} \cos^{2n}\theta$, $y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and, $z = \sum_{n=0}^{\infty} \cos^{2n}\theta \sin^{2n}\phi$. Then which of the following is true

Options:

(a) $xy + z = (x+y)z$

(b) $xy - z = (x+y)z$

(c) $xyz = 4$
(d) $xy + yz + zx = z$
Answer: (a)
Solution:
$x = 1 + \cos^2\theta + \cos^4\theta + ...\infty$

$$x = \frac{1}{1-\cos^2 1} = \text{cosec}^2\theta \qquad ...(1)$$

$y = 1 + \sin^2\phi + \sin^4\phi + ...\infty$

$$y = \frac{1}{1-\sin^2\phi} = \sec^2\phi \qquad ...(ii)$$

$z = 1 + \cos^2\theta\sin^2\phi + \cos^4\theta\sin^4\phi + ...\infty$

$$z = \frac{1}{1-\cos^2\theta\sin^2\phi} \qquad ...(iii)$$

From (1), (ii) and (iii)

$$z = \frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)} = \frac{xy}{xy-(x-1)(y-1)}$$

$xz + yz - z = xy$

$= xy + z = (x+y)z$

Question: The polynomial $f(x) = x^3 - bx^2 + cx - 4$ satisfies the conditions of Rolle's theorem for $x \in [1, 2]$, $f\left(\frac{4}{3}\right)$ the order pair (b, c) is

Options:
(a) (5, 8)
(b) (-5, 8)
(c) (-5, -8)
(d) (5, -8)
Answer: (a)
Solution:
Since, $f(x) = x^3 - bx^2 + cx - 4$ satisfies Rolle's Theorem condition

$\therefore f(1) = f(2) = 0$

$\Rightarrow 1 - b + c - 4 = 0 \ \Rightarrow c - b = 3 \qquad ...(i)$

$\Rightarrow 8 - 4b + 26 - 4 = 0 \ \Rightarrow c - 2b = -2 \qquad ...(ii)$

On solving above equation (i) and (2)
$b = 5, c = 8$

Question: $\int \frac{\sin\theta.\sin 2\theta\left(\sin^6\alpha+\sin^4\theta+\sin^2\theta\right)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{1-\cos 2\theta}$

Answer: $\frac{1}{18}\left(2\sin^6\theta+3\sin^4\theta+6\sin^2\theta\right)^{\frac{3}{2}}+C$

Solution:

Given, $I=\int \frac{\sin\theta\cdot 2\sin\theta\cdot\cos\theta\cdot\sin^2\theta\left(\sin^4\theta+\sin^2\theta+1\right)\sqrt{2\sin^4\theta+3\sin^2\theta+6}}{2\sin^2\theta}dx$

$I=\int \sin\theta\cdot\cos\theta\left(\sin^4\theta+\sin^2\theta+1\right)\sqrt{2\sin^6\theta+3\sin^4\theta+6\sin^2\theta}\,dx$

Let $2\sin^6\theta+3\sin^4\theta+6\sin^2\theta=t$

$\Rightarrow 12\sin\theta\cdot\cos\theta\left(\sin^4\theta+\sin^2\theta+1\right)dx=dt$

$\therefore\ I=\int\sqrt{t}\cdot\frac{dt}{12}$

$=\frac{1}{12}\left(\frac{t^{3/2}}{\frac{3}{2}}\right)+C=\frac{1}{18}\left(2\sin^6\theta+3\sin^4\theta+6\sin^2\theta\right)+C$

Question: $xyz=24,\ x,\ y,\ z\in N.$ Find number of ordered pairs $(x,\ y,\ z)$

Answer: 30.00

Solution:

$xyz=24$

$xyz=2^3.3$

Let $x=2^{a_1}3^{b_1}$

$y=2^{a_2}3^{b_2}$

$z=2^{a_3}3^{b_3}$

So, $2^{a_1+a_2+a_3}.3^{b_1+b_2+b_3}=2^3.3^1$

So, $a_1+a_2+a_3=3\quad ...(1)$

$b_1+b_2+b_3=1\qquad ...(2)$

Number of solutions of (1) are ${}^5C_2=10$

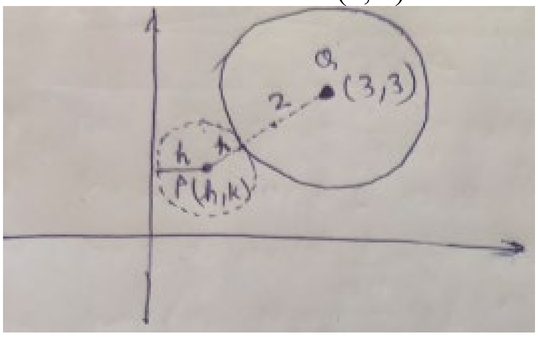
Number of solutions of (2) are ${}^3C_2=10$

Total number $=10\times 3=30$

Question: Calculate the locus of centre of circle which touches externally $x^2+y^2-6x-6y+14=0$ and also touches the y-axis

Answer: 0.00

Solution:
Given circle has centre at (3, 3) and radius is 2 units



So $PQ = 2 + h$

$PQ^2 = (2+h)^2$

$(h-3)^2 + (k-3)^2 = (2+h)^2$

$h^2 + 9 - 6h + k^2 + 9 - 6k = h^2 + 4 + 4 + h$

$k^2 - 6k - 10h + 14 = 0$

So, locus is $y^2 - 6y - 10x + 14 = 0$

Question: The number of points where $f(x) = |2x+1| - 3|x+2| + |x^2 + x - 2|,\ x \in R$ is not differentiable.

Answer: 2.00

Solution:

As $f(x)$ involves sum and difference of mod functions with polynomials inside them

So, $f(x)$ is a continuous function and may or may not be differentiable ay points where mod values become 0.

Such points are $x = -\frac{1}{2}, 1, -2$

For $x < -2$,

$f(x) = -2x - 1 + 3x + 6 + x^2 + x - 2$

$= x^2 + 2x + 3$

For $-2 < x < -\frac{1}{2}$

$f(x) = -2x - 1 - 3x - 6 - x^2 - x + 2$

$= -x^2 - 6x - 5$

For $-\frac{1}{2} < x < 1$

$f(x) = 2x + 1 - 3x - 6 - x^2 - x + 2$

$= -x^2 - 2x - 3$

For $x > 1$

$f(x) = 2x + 1 - 3x - 6 + x^2 + x - 2$

$= x^2 - 7$

$$f'(x) = \begin{cases} 2x+2 & , \quad x < -2 \\ -2x-6 & , \quad -2 < x < -\frac{1}{2} \\ -2x-2 & , \quad -\frac{1}{2} < x < 1 \\ 2x & , \quad x > 1 \end{cases}$$

Now, $f'\left(-\frac{1}{2}^-\right) = -5,\ f'\left(-\frac{1}{2}^+\right) = -1 \Rightarrow$ Not differentiable at $x = -\frac{1}{2}$

$f'(-2^-) = -2,\ f'(-2^+) = -2 \Rightarrow$ Differentiable at $x = -2$

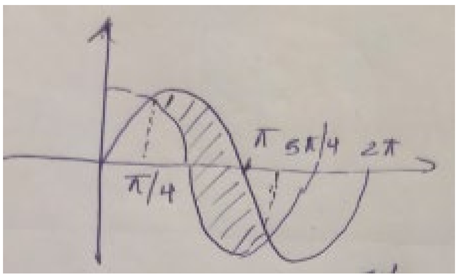
$f'(1^-) = -4,\ f'(1^+) = 2 \Rightarrow$ Not differentiable at $x = 1$

So, 2 points where $f(x)$ is not differentiable

Question: Sine and cosine graph intersect each other, a number of times. If the area of one cross section is A. Then A^4 = ?

Answer: 64.00

Solution:



Required area $= \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\, dx$

$= -\cos\Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}} - \sin x\Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}}$

$$=\left(-\left(-\frac{1}{\sqrt{2}}\right)-\left(-\frac{1}{\sqrt{2}}\right)\right)-\left(-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)$$

$$=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=2\sqrt{2}$$

So, $A^4=\left(2\sqrt{2}\right)^4=64$

Question: $f(x)$ is a polynomial of degree 6 with coefficient of x^6 equal to 1. If extreme values occur at $x=1$ and $x=-1$, $\lim\limits_{x\to 0}\frac{f(x)}{x^3}=1$, then $5f(2)=$

Answer: 144.00

Solution:

Let $f(x)=x^6+ax^5+bx^4+cx^3+dx^2+ex+f$

$$\because \lim_{x\to 0}\frac{f(x)}{x^3}=1$$

$$\therefore \lim_{x\to 0}\frac{x^6+ax^5+bx^4+cx^3+dx^2+ex+f}{x^3}=1$$

$$\therefore\ c=1,\ d=e=f=0$$

$$\therefore\ f(x)=x^6+ax^5+bx^4+x^3$$

So, $f'(x)=6x^5+5ax^4+4bx^3+3x^2$

$$\because\ f'(1)=f'(-1)=0$$

$$\therefore\ 6+5a+4b+3=0 \qquad ....(1)$$

$$-6+5a-4b+3=0 \qquad ....(2)$$

On adding (1) & (2)

$$a=-\frac{3}{5}$$

On subtracting (1) & (2)

$$b=-\frac{3}{2}$$

$$\therefore f(x)=x^6-\frac{3}{5}x^5-\frac{3}{2}x^4+x^3$$

$$\therefore 5f(2) = 5\left[64 - \frac{96}{5} - 24 + 8\right] = 144$$

Question: If $A = \begin{bmatrix} 0 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 0 \end{bmatrix}$, $(I+A)(I-A)^{-1} = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, find $13\left(a^2+b^2\right)$

Answer: 13.00

Solution:

$$A = \begin{bmatrix} 0 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 0 \end{bmatrix}$$

$$I + A = \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$I - A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$(1-A)^{-1} = \frac{1}{\left(1+\tan^2\frac{\theta}{2}\right)}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$$

So, $(I+A)(I-A)^{-1}$

$$= \frac{1}{\left(\sec^2\frac{\theta}{2}\right)}\begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$$

$$= \frac{1}{\left(\sec^2\frac{\theta}{2}\right)}\begin{bmatrix} \sec^2\frac{\theta}{2} & 0 \\ -0 & \sec^2\frac{\theta}{2} \end{bmatrix}$$

$$= \frac{1}{\left(\sec^2\frac{\theta}{2}\right)}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$=\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So, $a=1, h=0$

and $13\left(a^2+b^2\right)=13$

Question: A missile fires a target. The probability of its getting intercepted is $\frac{1}{3}$ and if it is not intercepted then probability of hitting the target is $\frac{3}{4}$. Three independent missiles are fired. Find the probability of all three hit.

Answer: $\frac{1}{8}$

Solution:

Probability of missile not getting intercepted = $\frac{2}{3}$

Probability of missile hitting the target = $\frac{3}{4}$

$\therefore$ Probability of all three missiles to hit target = $\left(\frac{2}{3}\times\frac{3}{4}\right)\times\left(\frac{2}{3}\times\frac{3}{4}\right)\times\left(\frac{2}{3}\times\frac{3}{4}\right)=\frac{1}{8}$

Question: $\sqrt{3}kx-yk+4\sqrt{3}=0$ and $\sqrt{3}x+y-4\sqrt{3}k=0$

The locus of point of intersection of these lines form a conic with eccentricity ……….

Answer: 2.00

Solution:

$\sqrt{3}kx+ky=4\sqrt{3}$ ...(i)

$\sqrt{3}kx-ky=4\sqrt{3}k^2$ ...(ii)

On adding (i) and (ii)

$2\sqrt{3}kx=4\sqrt{3}\left(k^2+1\right)$

$x=2\left(k+\frac{1}{k}\right)$ ...(iii)

On subtracting (i) and (ii)

$2ky=4\sqrt{3}\left(1-k^2\right)$

$y=2\sqrt{3}\left(\frac{1}{k}-k\right)$ ....(iv)

$$\therefore \left(\frac{x}{2}\right)^2 - \left(\frac{y}{2\sqrt{3}}\right)^2 = \left(k+\frac{1}{k}\right)^2 - \left(k-\frac{1}{k}\right)^2 = 4$$

$$\Rightarrow \frac{x^2}{16} - \frac{y^2}{48} = 1$$

$\therefore$ Eccentricity of Hyperbola

$$\Rightarrow e^2 = 1 + \frac{48}{16} = 4$$

$$\therefore e = 2$$