

FINAL JEE-MAIN EXAMINATION – FEBRUARY, 2021

(Held On Thursday 25th February, 2021) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS

SECTION-A

1. Let A be a 3×3 matrix with $\det(A) = 4$. Let R_i denote the i^{th} row of A. If a matrix B is obtained by performing the operation $R_2 \rightarrow 2R_2 + 5R_3$ on 2A, then $\det(B)$ is equal to :
 (1) 16 (2) 80 (3) 128 (4) 64

Official Ans. by NTA (4)

2. The integral $\int \frac{e^{3 \log_e 2x} + 5e^{2 \log_e 2x}}{e^{4 \log_e x} + 5e^{3 \log_e x} - 7e^{2 \log_e x}} dx, x > 0$, is equal to :
 (where c is a constant of integration)

- (1) $\log_e |x^2 + 5x - 7| + c$
 (2) $4 \log_e |x^2 + 5x - 7| + c$
 (3) $\frac{1}{4} \log_e |x^2 + 5x - 7| + c$
 (4) $\log_e \sqrt{x^2 + 5x - 7} + c$

Official Ans. by NTA (2)

3. The shortest distance between the line $x - y = 1$ and the curve $x^2 = 2y$ is :

- (1) $\frac{1}{2}$ (2) $\frac{1}{2\sqrt{2}}$ (3) $\frac{1}{\sqrt{2}}$ (4) 0

Official Ans. by NTA (2)

4. If $\alpha, \beta \in \mathbb{R}$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to :

- (1) -3 (2) -7 (3) 7 (4) 3

Official Ans. by NTA (2)

5. A hyperbola passes through the foci of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities is one, then the equation of the hyperbola is :

- (1) $\frac{x^2}{9} - \frac{y^2}{25} = 1$ (2) $\frac{x^2}{9} - \frac{y^2}{16} = 1$
 (3) $x^2 - y^2 = 9$ (4) $\frac{x^2}{9} - \frac{y^2}{4} = 1$

Official Ans. by NTA (2)

TEST PAPER WITH ANSWER

6. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to :

- (1) $\frac{1}{2}$ (2) $\frac{1 + \sqrt{3}}{2}$
 (3) $\frac{\sqrt{3}}{2}$ (4) $\frac{1 - \sqrt{3}}{2}$

Official Ans. by NTA (2)

7. A plane passes through the points A(1, 2, 3), B(2, 3, 1) and C(2, 4, 2). If O is the origin and P is (2, -1, 1), then the projection of \overline{OP} on this plane is of length :

- (1) $\sqrt{\frac{2}{7}}$ (2) $\sqrt{\frac{2}{3}}$ (3) $\sqrt{\frac{2}{11}}$ (4) $\sqrt{\frac{2}{5}}$

Official Ans. by NTA (3)

8. In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is :

- (1) $\frac{7}{45}$ (2) $\frac{14}{45}$ (3) $\frac{28}{45}$ (4) $\frac{8}{45}$

Official Ans. by NTA (3)

9. $\operatorname{cosec} \left[2 \cot^{-1}(5) + \cos^{-1} \left(\frac{4}{5} \right) \right]$ is equal to :

- (1) $\frac{56}{33}$ (2) $\frac{65}{56}$
 (3) $\frac{65}{33}$ (4) $\frac{75}{56}$

Official Ans. by NTA (2)

10. If the curve $x^2 + 2y^2 = 2$ intersects the line $x + y = 1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is :

- (1) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{3}\right)$ (2) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{3}\right)$
 (3) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$ (4) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

Official Ans. by NTA (4)

11. The contrapositive of the statement "If you will work, you will earn money" is :

- (1) You will earn money, if you will not work
 (2) If you will earn money, you will work
 (3) If you will not earn money, you will not work
 (4) To earn money, you need to work

Official Ans. by NTA (3)

12. A function $f(x)$ is given by $f(x) = \frac{5^x}{5^x + 5}$,

then the sum of the series

$f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + \dots + f\left(\frac{39}{20}\right)$ is equal to :

- (1) $\frac{19}{2}$ (2) $\frac{49}{2}$ (3) $\frac{29}{2}$ (4) $\frac{39}{2}$

Official Ans. by NTA (4)

13. If for the matrix, $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$, $AA^T = I_2$, then

the value of $\alpha^4 + \beta^4$ is :

- (1) 4 (2) 2 (3) 3 (4) 1

Official Ans. by NTA (4)

14. The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$, where $a, x \in \mathbb{R}$ and $a > 0$, is equal to :

- (1) $2a$ (2) $2\sqrt{a}$
 (3) $a + \frac{1}{a}$ (4) $a + 1$

Official Ans. by NTA (2)

15. If $I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x \, dx$, then :

- (1) $\frac{1}{I_2 + I_4}, \frac{1}{I_3 + I_5}, \frac{1}{I_4 + I_6}$ are in G.P.
 (2) $I_2 + I_4, I_3 + I_5, I_4 + I_6$ are in A.P.
 (3) $I_2 + I_4, (I_3 + I_5)^2, I_4 + I_6$ are in G.P.
 (4) $\frac{1}{I_2 + I_4}, \frac{1}{I_3 + I_5}, \frac{1}{I_4 + I_6}$ are in A.P.

Official Ans. by NTA (4)

16. $\lim_{n \rightarrow \infty} \left[\frac{1}{n} + \frac{n}{(n+1)^2} + \frac{n}{(n+2)^2} + \dots + \frac{n}{(2n-1)^2} \right]$

is equal to :

- (1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

Official Ans. by NTA (1)

17. Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is :

- (1) $\frac{2}{9}$ (2) $\frac{122}{297}$ (3) $\frac{97}{297}$ (4) $\frac{1}{5}$

Official Ans. by NTA (3)

18. Let α and β be the roots of $x^2 - 6x - 2 = 0$. If

$a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{3a_9}$

is :

- (1) 2 (2) 1 (3) 4 (4) 3

Official Ans. by NTA (1)

19. Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set $A \times B$. Then :

- (1) $y = 273x$ (2) $2y = 91x$
 (3) $y = 91x$ (4) $2y = 273x$

Official Ans. by NTA (2)

20. The following system of linear equations
 $2x + 3y + 2z = 9$
 $3x + 2y + 2z = 9$
 $x - y + 4z = 8$
 (1) has a solution (α, β, γ) satisfying
 $\alpha + \beta^2 + \gamma^3 = 12$
 (2) has infinitely many solutions
 (3) does not have any solution
 (4) has a unique solution
Official Ans. by NTA (4)

SECTION-B

1. The total number of two digit numbers 'n', such that $3^n + 7^n$ is a multiple of 10, is _____.
Official Ans. by NTA (45)
2. A function f is defined on $[-3, 3]$ as

$$f(x) = \begin{cases} \min\{|x|, 2 - x^2\}, & -2 \leq x \leq 2 \\ [x], & 2 < |x| \leq 3 \end{cases}$$
 where $[x]$ denotes the greatest integer $\leq x$. The number of points, where f is not differentiable in $(-3, 3)$ is _____.
Official Ans. by NTA (5)
3. Let $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors \vec{a} and \vec{b} is $8\sqrt{3}$ square units, then $\vec{a} \cdot \vec{b}$ is equal to _____.
Official Ans. by NTA (2)
4. If the remainder when x is divided by 4 is 3, then the remainder when $(2020 + x)^{2022}$ is divided by 8 is _____.
Official Ans. by NTA (1)
5. If the curves $x = y^4$ and $xy = k$ cut at right angles, then $(4k)^6$ is equal to _____.
Official Ans. by NTA (4)

6. A line is a common tangent to the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then $2(a + c)$ is equal to _____.
Official Ans. by NTA (9)
7. If $\lim_{x \rightarrow 0} \frac{ax - (e^{4x} - 1)}{ax(e^{4x} - 1)}$ exists and is equal to b, then the value of $a - 2b$ is _____.
Official Ans. by NTA (5)
8. If the curve, $y = y(x)$ represented by the solution of the differential equation $(2xy^2 - y)dx + xdy = 0$, passes through the intersection of the lines, $2x - 3y = 1$ and $3x + 2y = 8$, then $|y(1)|$ is equal to _____.
Official Ans. by NTA (1)
9. The value of $\int_{-2}^2 |3x^2 - 3x - 6| dx$ is _____.
Official Ans. by NTA (19)
10. A line 'l' passing through origin is perpendicular to the lines
 $l_1 : \vec{r} = (3 + t)\hat{i} + (-1 + 2t)\hat{j} + (4 + 2t)\hat{k}$
 $l_2 : \vec{r} = (3 + 2s)\hat{i} + (3 + 2s)\hat{j} + (2 + s)\hat{k}$
 If the co-ordinates of the point in the first octant on 'l' at a distance of $\sqrt{17}$ from the point of intersection of 'l' and 'l₁' are (a, b, c), then $18(a + b + c)$ is equal to _____.
Official Ans. by NTA (44)