FINAL JEE–MAIN EXAMINATION – FEBRUARY, 2021

(Held On Friday 26th February, 2021) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS — TEST PAPER WITH ANSWER

SECTION-A

1. If vectors $\vec{a}_1 = x\hat{i} - \hat{j} + \hat{k}$ and $\vec{a}_2 = \hat{i} + y\hat{j} + z\hat{k}$ are collinear, then a possible unit vector parallel to the vector $x\hat{i} + y\hat{j} + z\hat{k}$ is

(1) $\frac{1}{\sqrt{2}}(-\hat{j}+\hat{k})$ (2) $\frac{1}{\sqrt{2}}(\hat{i}-\hat{j})$

(3) $\frac{1}{\sqrt{3}}(\hat{i}+\hat{j}-\hat{k})$ (4) $\frac{1}{\sqrt{3}}(\hat{i}-\hat{j}+\hat{k})$

Official Ans. by NTA (4)

2. Let A = {1, 2, 3, ..., 10} and $f : A \to A$ be defined as $f(k) = \begin{cases} k+1 & \text{if } k \text{ is odd} \\ k & \text{if } k \text{ is even} \end{cases}$

Then the number of possible functions $g : A \to A$ such that $gof = f$ is

(1) 10^5 (2) ${}^{10}C_5$ (3) 5^5 (4) 5!

Official Ans. by NTA (1)

3. Let $f : R \to R$ be defined as

$$f(x) = \begin{cases} 2\sin\left(-\frac{\pi x}{2}\right), & \text{if } x < -1 \\ |ax^2 + x + b|, & \text{if } -1 \le x \le 1 \\ \sin(\pi x), & \text{if } x > 1 \end{cases}$$

If $f(x)$ is continuous on R, then a + b equals:

(1) –3 (2) –1 (3) 3 (4) 1

Official Ans. by NTA (2)

4. For $x > 0$, if $f(x) = \int_1^x \frac{\log_e t}{(1+t)} dt$, then $f(e) + f\left(\frac{1}{e}\right)$ is equal to

(1) 1 (2) –1 (3) $\frac{1}{2}$ (4) 0

Official Ans. by NTA (3)

5. A natural number has prime factorization given by $n = 2^x 3^y 5^z$, where y and z are such that $y + z = 5$ and $y^{-1} + z^{-1} = \frac{5}{6}$, $y > z$. Then the number of odd divisors of n, including 1, is :

(1) 11 (2) 6 (3) 6x (4) 12

Official Ans. by NTA (4)

6. Let $f(x) = \sin^{-1}x$ and $g(x) = \frac{x^2 - x - 2}{2x^2 - x - 6}$. If $g(2) = \lim_{x \to 2} g(x)$, then the domain of the function fog is :

(1) $(-\infty, -2] \cup \left[-\frac{3}{2}, \infty\right)$

(2) $(-\infty, -2] \cup [-1, \infty)$

(3) $(-\infty, -2] \cup \left[-\frac{4}{3}, \infty\right)$

(4) $(-\infty, -1] \cup [2, \infty)$

Official Ans. by NTA (3)

7. The triangle of maximum area that can be inscribed in a given circle of radius 'r' is :

(1) An isosceles triangle with base equal to 2r.

(2) An equilateral triangle of height $\frac{2r}{3}$.

(3) An equilateral triangle having each of its side of length $\sqrt{3}r$.

(4) A right angle triangle having two of its sides of length 2r and r.

Official Ans. by NTA (3)

8. Let L be a line obtained from the intersection of two planes $x + 2y + z = 6$ and $y + 2z = 4$. If point $P(\alpha, \beta, \gamma)$ is the foot of perpendicular from (3, 2, 1) on L, then the value of $21(\alpha + \beta + \gamma)$ equals :

(1) 142 (2) 68 (3) 136 (4) 102

Official Ans. by NTA (4)

9. Let $F_1(A,B,C) = (A \wedge \sim B) \vee [\sim C \wedge (A \vee B)] \vee \sim A$ and $F_2(A, B) = (A \vee B) \vee (B \to \sim A)$ be two logical expressions. Then :

(1) F_1 and F_2 both are tautologies

(2) F_1 is a tautology but F_2 is not a tautology

(3) F_1 is not tautology but F_2 is a tautology

(4) Both F_1 and F_2 are not tautologies

Official Ans. by NTA (3)

10. Let slope of the tangent line to a curve at any point P(x, y) be given by $\frac{xy^2 + y}{x}$. If the curve intersects the line $x + 2y = 4$ at $x = -2$, then the value of y, for which the point (3, y) lies on the curve, is :

(1) $\frac{18}{35}$ (2) $-\frac{4}{3}$ (3) $-\frac{18}{19}$ (4) $-\frac{18}{11}$

Official Ans. by NTA (3)

11. If the locus of the mid-point of the line segment from the point (3, 2) to a point on the circle, $x^2 + y^2 = 1$ is a circle of radius r, then r is equal to :

(1) 1 (2) $\frac{1}{2}$ (3) $\frac{1}{3}$ (4) $\frac{1}{4}$

Official Ans. by NTA (2)

12. Consider the following system of equations :

$x + 2y - 3z = a$

$2x + 6y - 11z = b$

$x - 2y + 7z = c$,

where a, b and c are real constants. Then the system of equations :

(1) has a unique solution when $5a = 2b + c$

(2) has infinite number of solutions when $5a = 2b + c$

(3) has no solution for all a, b and c

(4) has a unique solution for all a, b and c

Official Ans. by NTA (2)

13. If $0 < a, b < 1$, and $\tan^{-1}a + \tan^{-1}b = \frac{\pi}{4}$, then the value of

$$(a+b) - \left(\frac{a^2+b^2}{2}\right) + \left(\frac{a^3+b^3}{3}\right) - \left(\frac{a^4+b^4}{4}\right) + \ldots$$

is :

(1) $\log_e 2$ (2) $e^2 - 1$

(3) e (4) $\log_e\left(\frac{e}{2}\right)$

Official Ans. by NTA (1)

14. The sum of the series $\sum_{n=1}^{\infty} \frac{n^2 + 6n + 10}{(2n+1)!}$ is equal to :

(1) $\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

(2) $\frac{41}{8}e - \frac{19}{8}e^{-1} - 10$

(3) $\frac{41}{8}e + \frac{19}{8}e^{-1} + 10$

(4) $-\frac{41}{8}e + \frac{19}{8}e^{-1} - 10$

Official Ans. by NTA (2)

15. Let $f(x)$ be a differentiable function at $x = a$ with $f'(a) = 2$ and $f(a) = 4$. Then $\lim_{x \to a} \frac{x f(a) - a f(x)}{x - a}$ equals :

(1) $2a + 4$ (2) $4 - 2a$

(3) $2a - 4$ (4) $a + 4$

Official Ans. by NTA (2)

16. Let A(1, 4) and B(1, –5) be two points. Let P be a point on the circle $(x - 1)^2 + (y - 1)^2 = 1$ such that $(PA)^2 + (PB)^2$ have maximum value, then the points P, A and B lie on :

(1) a straight line (2) a hyperbola

(3) an ellipse (4) a parabola

Official Ans. by NTA (1)

17. If the mirror image of the point (1, 3, 5) with respect to the plane $4x - 5y + 2z = 8$ is (α, β, γ), then $5(\alpha + \beta + \gamma)$ equals :

(1) 47 (2) 43 (3) 39 (4) 41

Official Ans. by NTA (1)

18. Let $f(x) = \int_0^x e^t f(t)\,dt + e^x$ be a differentiable function for all $x \in R$. Then $f(x)$ equals :

(1) $2e^{(e^x-1)} - 1$ (2) $e^{e^x} - 1$

(3) $2e^{e^x} - 1$ (4) $e^{(e^x-1)}$

Official Ans. by NTA (1)

19. Let A_1 be the area of the region bounded by the curves $y = \sin x$, $y = \cos x$ and y-axis in the first quadrant. Also, let A_2 be the area of the region bounded by the curves $y = \sin x$, $y = \cos x$, x-axis and $x = \frac{\pi}{2}$ in the first quadrant.

Then,

(1) $A_1 : A_2 = 1 : \sqrt{2}$ and $A_1 + A_2 = 1$

(2) $A_1 = A_2$ and $A_1 + A_2 = \sqrt{2}$

(3) $2A_1 = A_2$ and $A_1 + A_2 = 1 + \sqrt{2}$

(4) $A_1 : A_2 = 1 : 2$ and $A_1 + A_2 = 1$

Official Ans. by NTA (1)

20. A seven digit number is formed using digits 3, 3, 4, 4, 4, 5, 5. The probability, that number so formed is divisible by 2, is :

(1) $\frac{6}{7}$ (2) $\frac{1}{7}$ (3) $\frac{3}{7}$ (4) $\frac{4}{7}$

Official Ans. by NTA (3)

SECTION B

1. Let z be those complex numbers which satisfy $|z + 5| \le 4$ and $z(1+i) + \bar{z}(1-i) \ge -10, i = \sqrt{-1}$.

If the maximum value of $|z + 1|^2$ is $\alpha + \beta\sqrt{2}$, then the value of $(\alpha + \beta)$ is _______.

Official Ans. by NTA (48)

2. Let the normals at all the points on a given curve pass through a fixed point (a, b). If the curve passes through (3, –3) and $\left(4, -2\sqrt{2}\right)$, and given that $a - 2\sqrt{2}\,b = 3$, then $(a^2 + b^2 + ab)$ is equal to ______.

Official Ans. by NTA (9)

3. Let α and β be two real numbers such that $\alpha + \beta = 1$ and $\alpha\beta = -1$. Let $p_n = (\alpha)^n + (\beta)^n$, $p_{n-1} = 11$ and $p_{n+1} = 29$ for some integer $n \ge 1$. Then, the value of p_n^2 is ______.

Official Ans. by NTA (324)

4. If $I_{m,n} = \int_0^1 x^{m-1}(1-x)^{n-1}\,dx$, for $m, n \ge 1$ and $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}}\,dx = \alpha I_{m,n}$, $\alpha \in R$, then α equals ____.

Official Ans. by NTA (1)

5. If the arithmetic mean and geometric mean of the p^{th} and q^{th} terms of the sequence –16, 8, –4, 2, ... satisfy the equation $4x^2 - 9x + 5 = 0$, then p + q is equal to ______.

Official Ans. by NTA (10)

6. The total number of 4-digit numbers whose greatest common divisor with 18 is 3, is ______.

Official Ans. by NTA (1000)

7. Let L be a common tangent line to the curves $4x^2 + 9y^2 = 36$ and $(2x)^2 + (2y)^2 = 31$. Then the square of the slope of the line L is ______.

Official Ans. by NTA (3)

8. Let a be an integer such that all the real roots of the polynomial $2x^5+5x^4 + 10x^3 + 10x^2 + 10x + 10$ lie in the interval (a, a + 1). Then, |a| is equal to ______.

Official Ans. by NTA (2)

9. Let $X_1, X_2, \ldots, X_{18}$ be eighteen observations such that $\sum_{i=1}^{18}(X_i - \alpha) = 36$ and $\sum_{i=1}^{18}(X_i - \beta)^2 = 90$, where α and β are distinct real numbers. If the standard deviation of these observations is 1, then the value of $|\alpha - \beta|$ is ______.

Official Ans. by NTA (4)

10. If the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 3 & 0 & -1 \end{bmatrix}$ satisfies the equation $A^{20} + \alpha A^{19} + \beta A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ for some real numbers α and β, then $\beta - \alpha$ is equal to ______.

Official Ans. by NTA (4)