Question: For a damped oscillator, damping constant is 20 gm/s, mass is 500g. Find time taken for the amplitude to become half the initial?

Options:
(a) 50
(b) ln 2
(c) 50 ln2
(d) \( \frac{25}{2} \ln 2 \)

Answer: (c)

Solution:

\[
A = A_0 e^{-\frac{bt}{2m}}
\]

\[
A_0 = A e^{-\frac{bt}{2m}}
\]

\[
2 = e^{\frac{bt}{2m}}
\]

\[
\ln 2 = \frac{bt}{2m}
\]

\[
t = \frac{2m}{b} \ln 2
\]

\[
t = 2 \times 500 \times \frac{ln 2}{20}
\]

\[
t = 50 \ln 2
\]

Question: A square loop of side d is moved with velocity \( \vec{v} \) in a non-uniform magnetic field \( \frac{B_0}{a} \vec{z} \). Then the emf induced in the loop shown is?

Options:
Answer: (d)
Solution:

\[ \text{e.m.f} = \frac{d\phi}{dt} \]

\[ \phi = BA \]

\[ \phi = \frac{B_0}{a} x d^3 \]

\[ \frac{d\phi}{dt} = \frac{d}{dt} \left( \frac{B_0 d^2}{a} x \right) \]

\[ \text{e.m.f} = \frac{B_0 d^2}{a} \frac{dx}{dt} \]

\[ \text{e.m.f.} = \frac{B_0 d^2}{a} v \]

\[ \left\{ v = \frac{dx}{dt} \right\} \]

Question: For the diagram shown, what is the type of transformer?
Options:
(a) Step-up
(b) Step-down
(c) Auxiliary
(d) Axial
Answer: (a)
Solution:
\( V_{\text{input}} = 220V \)
\( P_{\text{output}} = 60W \)
\( P_{\text{output}} = V_{\text{out}} I_{\text{input}} \)
\( 60 = V_{\text{out}} \times 0.11 \)
\( V_{\text{out}} = 545.45V \)
\( V_{\text{out}} > V_{\text{in}} \)
Therefore, it is step-up transformer.

Question: If the range of single transmission between sending and receiving antennas of equal heights in 45 km. Then find the height of the antennas.
Options:
(a) 30 m
(b) 39.5 m
(c) 45 m
(d) 64 m
Answer: (b)
Solution:
Range = \( \sqrt{2Rh_r} + \sqrt{2Rh_R} \)
\( R = 6.4 \times 10^6 m \) sss
\( h_r = h_R = h \)
Range = 45 km
\( 45 \times 10^3 = 2\sqrt{2 \times 6.4 \times 10^6 \times h} \)
\( \sqrt{h} = 6.2889 \)
\( h = 39.55 m. \)
**Question:** Find the resistance if it dissipates 10 mJ of energy per second when current of 1 mA passes through it.

**Options:**
(a) 1 kΩ  
(b) 100 kΩ  
(c) 10 kΩ  
(d) 100 kΩ

**Answer:** (c)

**Solution:**

Given \( P = 10 \text{ mJ/s} \)

\( P = 10 \text{ mW} \)

\( I = 1 \text{ mA} \)

\( P = I^2R \)

\[ 10 \times 10^{-3} = (1 \times 10^{-3})^2 R \]

\[ 10 \times 10^{-3} = 1 \times 10^{-3} \times 10^{-3} \times R \]

\[ R = 10^4 \]

\[ R = 10 \text{ kΩ} \]

**Question:** The focal length of a lens whose refractive index is same as that of the outside medium is?

**Options:**
(a) Zero  
(b) Unity  
(c) Infinity  
(d) Can’t be found

**Answer:** (c)

**Solution:**

\[ \frac{1}{f} = \left( \frac{\mu_2}{\mu_1} - 1 \right) \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \]

Refractive index of lens is same as medium

So, \( \frac{\mu_2}{\mu_1} = 1 \)

\[ \frac{1}{f} = 0 \]

\[ \Rightarrow f = \infty \]

**Question:** The acceleration of a disc rolling (purely) down an inclined plane of inclination \( \theta \) is given as \( a = \frac{xg \sin \theta}{3} \) Find \( x \).
Answer: 2.00

Solution:
We know that for a body rolling down an inclined plane
\[ a = \frac{g \sin \theta}{1 + \frac{k^2}{R^2}} \]

For disc
\[ mk^2 = \frac{mR^2}{2} \]
\[ \Rightarrow k^2 = \frac{R^2}{2} \]
\[ a = \frac{g \sin \theta}{1 + \frac{R^2}{2R^2}} \]
\[ a = \frac{2}{3} g \sin \theta \]

So, \( x = 2 \)

Question: Find the equivalent capacitance for the given figure
\( A = 0.2m^2, d = 1m, k = 3.2 \)

Options:
(a) 0.1\( \varepsilon_0 \)
(b) 0.2\( \varepsilon_0 \)
(c) 0.3\( \varepsilon_0 \)
(d) 0.4\( \varepsilon_0 \)

Answer: (c)

Solution:
We can consider the shown capacitor as series combination of two capacitors.
\[ C_1 = \frac{2 \varepsilon_0 A}{d} \quad \text{and} \quad C_2 = \frac{2K \varepsilon_0 A}{d} \]
\[ C_{eq} = \frac{C_1C_2}{C_1 + C_2} \]
\[ \frac{2 \varepsilon_0 A \times 2K \varepsilon_0 A}{d} \]
\[ = \frac{2 \varepsilon_0 A}{d} \times \frac{2K}{1 + K} \]
\[ = \frac{2 \varepsilon_0 \times 3.2 \times 0.2}{(1 + 3.2)} \]
\[ C_{eq} = 0.304 \varepsilon_0 \approx 0.3 \varepsilon_0 \]

**Question:** This is equivalent to:

![Logic Gate Diagram]

**Options:**
(a) OR
(b) AND
(c) NOR
(d) NAND

**Answer:** (c)

**Solution:**

\[ Y = A.A.B.B \]
\[ = (A + A)(B + B) \]
\[ = AB \]
\[ = A + B \]

Hence it’s a NOR Gate.

**Question:** There are two species A & B with half-lives 54 & 18 minutes respectively. The time after which concentration of A is 16 times that of B will be -

**Options:**
(a) 27 min
(b) 54 min  
(c) 81 min  
(d) 108 min  

**Answer:** (d)  

**Solution:**  
\[ t_A = 54 \text{ min} \]  
\[ t_B = 18 \text{ min} \]  
\[ N_A = 16 N_B \]  
\[ N_0 e^{-\lambda t_A} = 16 N_0 e^{-\lambda t_B} \]  
\[ e^{(\lambda_A - \lambda_B)t} = 16 \]  
\[ e^{(t_A - t_B)/\ln 2} = 16 \]  
\[ 2^{(t_A - t_B) \ln 2} = 2^4 \]  
\[ t \left( \frac{1}{18} - \frac{1}{54} \right) = 4 \]  
\[ t = 108 \text{ min} \]  

**Question:** If half life of an element is 20 minutes. Find the time interval of 33.33% and 66.66% decay.  

**Options:**  
(a) 10 minutes  
(b) 20 minutes  
(c) 40 minutes  
(d) 80 minutes  

**Answer:** (b)  

**Solution:**  
The relation between decay constant \( \lambda \) and half-life \( T \) is:  
\[ \lambda = \frac{\log 2}{T} = \frac{0.693}{20} = \frac{0.693}{20} = 0.03465 \text{ per min} \]  
Time of decay, \( t = \frac{2.303}{\lambda} \log_{10} \frac{N_0}{N} \)  
Time of decay for 33.33% disintegration is:  
\[ t_1 = \frac{2.303}{0.03465} \log_{10} \frac{100}{66.66} = 11.71 \text{ min} \]  
Time of decay for 66.66% disintegration is:  
\[ t_2 = \frac{1.303}{0.03465} \log_{10} \frac{100}{33.33} = 31.71 \text{ min} \]  
Hence, difference of time is:  
\[ \Delta t = t_2 - t_1 = 31.71 - 11.71 = 20 \text{ min} \]
**Question:** Red and violet light have -
**Options:**
(a) Same frequency, different wavelength  
(b) Different frequency, same wavelength  
(c) Different frequency, different wavelength  
(d) Same frequency, same wavelength
**Answer:** (c)
**Solution:**
Violet light has a higher frequency and shorter wavelength than red light.

**Question:** A bimetallic strip consists of metals X and Y. it is mounted rigidly at the base as shown. The metal X has a higher coefficient of expansion compared to that for metal Y. when the bimetallic strip is placed in a cold bath:

![Bimetallic Strip Diagram]

**Options:**
(a) The combination will bend with X on convex side.  
(b) The combination will bend with Y on convex side  
(c) There will be no bending  
(d) Cannot be predicted
**Answer:** (b)
**Solution:**
As coefficient of thermal expansion of X is more. On cooling, it will shrink more. So the strip will bend with Y on convex side.

**Question:** An electron and a proton are accelerated by same voltage difference. Find the ratio of the de Broglie wavelength of electron: Proton. \( \frac{m_p}{m_e} = 1860 : 1 \)
**Options:**
(a) \( \frac{1860}{1} \)  
(b) \( \frac{41.4}{1} \)
Question: A charge ‘q’ moves by a distance ‘dl’ under the presence of magnetic field ‘B’. Find the work done by the field?

Options:
(a) \( q\overline{B}\overline{dl} \)
(b) \( q^2 \overline{B}\overline{dl} \)
(c) \( \infty \)
(d) Zero

Answer: (d)

Solution:
The magnetic force acts in such a way that the direction of the magnetic force and velocity and always perpendicular to each other. If force and velocity are perpendicular, then force \( F \) displacement will also be perpendicular
So, \( W = F.d \cos \theta \)
If \( \theta = 90^\circ \)
\[ W = 0 \]

Question: A block of mass = 5.99 kg hangs from string. A small mass \( m = 10 \) grams strikes it with velocity \( v \). if the height to which system rises is 9.8 cm, then find \( v \). Assume perfectly inelastic collision and \( g = 10 \) \( m/s^2 \).
Options:
(a) 800 m/s
(b) 840 m/s
(c) 900 m/s
(d) 1000 m/s
Answer: (b)
Solution:
By law of momentum conservation
\[ mV + 0 = (m + M)V' \]
\[ 0.01 \times V = (0.01 + 5.99)V' \]
\[ V' = \frac{0.01V}{6} = \frac{V}{600} \text{ m/s} \quad \text{(1)} \]
By energy conservation,
\[ \frac{1}{2}(m + M)V'^2 = (m + M)gh \]
From eq. (1)
\[ \frac{1}{2}(6)\left(\frac{V}{600}\right)^2 = (6)\times10\times\frac{98\times10^{-2}}{10} \]
\[ V^2 = 705600 \]
\[ V = 840 \text{ m/s} \]

Question: 500 Joules of heat is dissipated when 1.5 Amperes of current is passed through a resistor for 20 seconds. If current is changed to 3 A, then how much heat will be dissipated in same time.

Options:
(a) 500 Joules
(b) 125 Joules
(c) 2000 Joules
(d) 1000 Joules
Answer: (c)
Solution:
\[ H_i = i^2Rt \]
\[ H_1 = (1.5)^2 \times R \times 2 = 500 \]
\[ R = \frac{500}{20 \times (1.5)^2} = \frac{500}{20 \times 2.25} \quad \text{...(1)} \]

So for, \( i = 3 \text{Amp} \)

\[ H_2 = i_2^2 R t \]

From eq (1) \( H_2 = (3)^2 \times \left( \frac{500}{20 \times 2.25} \right) \times 20 \)

\[ H_2 = 2000 \text{ Joules} \]
JEE-Main-16-03-2021-Shift-2 (Memory Based)

CHEMISTRY

Question:

\[
\begin{align*}
&\text{C}_6\text{H}_5\text{MgBr} \ (1 \text{ eq.)} \\
&\text{H}_3\text{O}^+ \\
\end{align*}
\]

Major product

Options:

(a)

(b)

(c)

(d)
Wood laminates are made up of:

Options:

(a) Polystyrene
(b) PVC
(c) Urea-formaldehyde resins
(d) Bakelite

Answer: (c)

Solution:

Urea-formaldehyde resins
I) Urea, II) Formaldehyde

\[
\left(\text{NH} \longrightarrow \text{CO} \longrightarrow \text{NH} \longrightarrow \text{CH}_2\right)_n
\]

For making unbreakable cups and laminated sheets

Question: The number of orbitals that can be formed with \( n = 5, l = 4, m_l = +2 \)

Options:
(a) 1
(b) 2
(c) 3
(d) 4

Answer: (a)

Solution:

\[ n = 5, \ l = 4 \]

\[ 5g \]

\[ -4, -3, -2, -1, 0, +1, +2, +3, \]

**Question:** The constituents of greenhouse gases:

I. CO$_2$, II. H$_2$O
III. CH$_4$, IV. O$_3$

**Options:**

(a) Only I
(b) I and II
(c) I, II, III
(d) All of these

Answer: (d)

Solution: The main constituents of greenhouse gases are methane, Carbon dioxide, ozone, nitrous oxide and water vapours.

**Question:** Which of the following cannot be reduced by coke?

**Options:**

(a) FeO
(b) Al$_2$O$_3$
(c) CaO
(d) Cu$_2$O

Answer: (c)

Solution: Oxides of strong electropositive metals such as K, Ca, Na, Al and Mg are very stable

It is difficult to reduce them into metallic state by carbon reduction
Question:

\[
\begin{array}{c}
\text{Reagent} = ?
\end{array}
\]

Options:
(a) Zn-Hg
(b) HCl, Anhydrous AlCl₃
(c) Cl₂ (dark), HCl
(d) Cl₂, FeCl₃

Answer: (c)

Solution:

\[
\begin{array}{c}
\text{Cl}_2 \text{ (dark), HCl}
\end{array}
\]

Free radical substitution

Question: Ozonolysis of X gives A which is an aldehyde. A on heating with silver oxide gives beautiful silver mirror lining. X is ?

Options:
(a) 
(b) 

\[
\begin{array}{c}
\text{H}_3\text{C} \equiv \text{C} \equiv \text{C} \equiv \text{C} \equiv \text{CH}_3
\end{array}
\]
Answer: (b)

Solution:

\[ \text{HC} \equiv \text{C} - \text{CH}_3 \xrightarrow{\text{O}_2/\text{Zn}} \text{H}_3\text{C} - \text{C} - \text{CHO} \]

**Question:** S1: Sodium hydride can be used as an oxidizing agent.

S2: Pyridine is base because of lone pair.

**Options:**

(a) S1 is correct

(b) S2 is incorrect

(c) Both are correct

(d) Both are not correct

**Answer:** (b)

**Solution:**

1) NaH is reducing agent

2)
Pyridine is base

**Question:** The secondary group proteins have which of the following forces

**Options:**
(a) Vander waals forces  
(b) Hydrogen bond  
(c) Covalent bond  
(d) Ionic bond

**Answer:** (b)

**Solution:**

Secondary groups of proteins are produced maintained by H-bonding. Two types of secondary structure

i.e., α – Helix and β -pleated sheet

**Question:** Role of NaOH in ammonolysis of halide?

**Options:**
(a) Stabilizes the transition state
(b) Consumes the leaving group
(c) Both a and b
(d) None of these
Answer: (c)
Solution:

\[
Pb(NO_3)_2 + Cr_2(SO_4)_7 \rightarrow PbSO_4
\]

\[Pb(NO_3)_2 = 35 \text{ ml, } 0.15 \text{ M}\]

\[Cr_2(SO_4)_7 = 20 \text{ ml, } 0.12 \text{ M}\]

Find the moles of PbSO_4

Options:

(a) \(5.25 \times 10^{-3}\) moles
(b) \(3.25 \times 10^{-3}\) moles
(c) \(1.25 \times 10^{-3}\) moles
(d) \(2 \times 10^{-3}\) moles

Answer: (a)
Solution:
\[3\text{Pb(NO}_3\text{)}_2 + 3\text{Cr}_2\text{(SO}_4\text{)}_3 \rightarrow 3\text{PbSO}_4 + 2\text{Cr(NO}_3\text{)}_3\]

Millimole = 5.25 2.4

Here L.R is Pb(NO\(_3\))\(_2\)

Moles of PbSO\(_4\) formed = 5.25 millimoles = \(5.25 \times 10^{-3}\) moles

**Question:** Which of the following is incorrect statement regarding \(\text{H}_2\text{O}_2\)?

**Options:**

(a) It is used both as oxidising agent and reducing agent
(b) It is used in effluents
(c) Both hydroxyl groups are present in the same plane
(d) Its shape is open book type structure

**Answer:** (c)

**Solution:** Hydrogen peroxide is an important chemical used in pollution control treatment of domestic and industrial effluents.

Hydrogen peroxide has a non-planar structure. The molecular dimensions in the gas phase and solid phase are shown in figure.

**Question:** The volume of 1 M NaOH required for complete neutralization of 100 ml of 1 M of \(\text{H}_3\text{PO}_3\) and 100 ml of 2 M \(\text{H}_3\text{PO}_4\) is:

**Options:**

(a) 200 ml, 200 ml
(b) 200 ml, 400 ml
(c) 200 ml, 600 ml
(d) 200 ml, 800 ml

**Answer:** (c)

**Solution:**

\[\text{Eq of NaOH} = \text{eq of H}_3\text{PO}_3\]
\[ \frac{0.1 \times 1 \times 2}{V \times 1} = 0.2 \]
\[ V = 0.2 \text{ litre} = 200 \text{ ml} \]
Eq of NaOH = eq of H₃PO₄
\[ \frac{0.1 \times 2 \times 3}{V \times 1} = 0.6 \text{ litre} \]
\[ V = 600 \text{ ml} \]

**Question:** Which halogen cannot form FeX₃ and FeX₂?

**Options:**
(a) I
(b) Br
(c) F
(d) Cl

**Answer:** (a)

**Solution:** FeX₃ and FeX₂ is unstable

FeI₃ does not exist because Fe³⁺ oxidises I⁻ to I₂

**Question:** Atomic number of X, Y and Z are 33, 53, an 83 respectively, then:

**Options:**
(a) X and Z are non-metals and Y is metal
(b) X is metalloid, Y is non-metal and Z is metal
(c) X and Z are metals, Y is non-metal
(d) None of these

**Answer:** (b)

**Solution:**
X = Arsenic
Y = Iodine
Z = Bismuth

**Question:** If half-life of an element is 20 minutes. Find the time interval of 33% decay and 67% decay
Options:
(a) 13.05
(b) 23.45
(c) 33.25
(d) 41.15
Answer: (a)
Solution:
\[ t_{\frac{1}{2}} = 20 \text{ min} \]
\[ K = \frac{0.613}{20} = 0.03 \]
\[ t_{67\%} = \frac{2.303}{0.03} \log \left( \frac{100}{33} \right) \]
\[ = \frac{2.303}{0.03} \log (3.03) \]
\[ = \frac{2.303}{0.03} \times 0.48 = 36.84 \]
\[ t_{33\%} = \frac{2.303}{0.03} \log \left( \frac{100}{67} \right) \]
\[ = \frac{2.303}{0.03} \log (1.5) \]
\[ = \frac{2.303}{0.03} \times 0.17 = 13.05 \]

Question: The conversion is carried out

Options:
(a) NaBH₄
(b) KMnO₄/H⁺
(c) LiAlH₄  
(d) H₂O/H⁺  

**Answer:** (b)  

**Solution:**

![Chemical reaction diagram]

Question: Match the following.

<table>
<thead>
<tr>
<th>Tests/ reagents (Column I)</th>
<th>Tests/ reagents (Column II)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A) Lassaigne’s test</td>
<td>(i) Carbon</td>
</tr>
<tr>
<td>(B) CuO</td>
<td>(ii) N, P, S and halogen</td>
</tr>
<tr>
<td>(C) Silver nitrate</td>
<td>(iii) Halogen only</td>
</tr>
<tr>
<td>(D) Sodium Nitroprusside</td>
<td>(iv) Sulphur</td>
</tr>
</tbody>
</table>

Options:  
(a) (A) → (ii); (B) → (i); (C) → (iii); (D) → (iv)  
(b) (A) → (iii); (B) → (ii); (C) → (i); (D) → (iv)  
(c) (A) → (iv); (B) → (iii); (C) → (ii); (D) → (i)  
(d) (A) → (i); (B) → (iii); (C) → (iv); (D) → (ii)  

**Answer:** (a)  

**Solution:**  
Lassaigne’s test ⇒ N, P, S and halogen  
CuO ⇒ Carbon  
Silver nitrate ⇒ Halogen only  
Sodium Nitroprusside ⇒ Sulphur
JEE–Main–16–03–2021–Shift–2 (Memory Based)

MATHEMATICS

Question: \( F(x+1) = xF(x) \) and \( g(x) = \ln F(x) \) Find \( |g''(5) - g''(1)| \)

Options:
(a)
(b)
(c)
(d)

Answer: ()

Solution:
\[ f(x+1) = x f(x) \]
\[ f(x+N) = (x+N-1)f(x+N-1) \]
\[ = (x+N-1)(x+N-2)f(x+N-2) \ldots \]
\[ f(x+N) = (x+N-1)(x+N-2) \ldots (x-1)f(x) \]
\[ g(x+N) = \ln f(x+N) = \ln(x+N-1) + \ln(x+N-2) + \ldots \ln f(x) \]
\[ \therefore g'(x+N) = \frac{1}{x+N-1} + \frac{1}{x+N-2} + \ldots + g'(x) \]
\[ g''(x+N) - g''(x) = \frac{-1}{(x+N-1)^2} - \frac{1}{(x+N-2)^2} - \ldots - \frac{1}{x^2} \]

Put \( x = 1 \) and \( N = 4 \)
\[ g''(5) - g''(1) = \left[ \frac{1}{4^2} + \frac{1}{3^2} + \frac{1}{2^2} + \frac{1}{1^2} \right] \]
\[ |g''(5) - g''(1)| = 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} = \frac{205}{144} \]

Question: \( C_1 \) and \( C_2 \) are two curves intersecting at \((1,1)\) \( C_1 \) satisfy \( \frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \) and \( C_2 \) satisfy \( \frac{dy}{dx} = \frac{2xy}{x^2 - y^2} \) Then area bounded by these two curves is

Options:
(a)
(b)
(c)
(d)

**Answer:**

**Solution:**

\[ C_1: \frac{dy}{dx} = \frac{y^2 - x^2}{2xy} \]

Put \( y = vx \) \( \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx} \)

\[ v + x \frac{dv}{dx} = \frac{v^2 - 1}{2v} \Rightarrow x \frac{dv}{dx} = \frac{(1 + v^2)}{2v} \]

\[ \int \left( \frac{2v}{1 + v^2} \right) dv = -\int \frac{dx}{x} \Rightarrow \ln (1 + v^2) x = c \]

\[ C_1: \left( \frac{x^2 + y^2}{x} \right) = c = 2 \Rightarrow C_1: x^2 + y^2 = 2x \]

\[ C_2: \frac{dy}{dx} = \frac{2xy}{x^2 - y^2} \]

Put \( y = vx \) \( \Rightarrow \frac{dy}{dx} = v + x \frac{dv}{dx} \)

\[ v + x \frac{dv}{dx} = \frac{2v}{1 - v^2} \Rightarrow x \frac{dv}{dx} = \frac{v + v^3}{1 - v^2} \]

\[ \frac{1 - v^2}{v(1 + v^2)} \frac{dx}{dv} = \int \left( \frac{1}{v} - \frac{2v}{1 + v^2} \right) dv = \int \frac{dx}{x} \]

\[ \Rightarrow \ln \left( \frac{v}{1 + v^2} \right) = \ln x + c \Rightarrow \frac{y}{x^2 + y^2} = \frac{1}{2} \]

\[ C_2: x^2 + y^2 = 2y \]

\[ \therefore \text{Area bounded between } (x - 1)^2 + y^2 = 1 \text{ and } x^2 + (y - 1)^2 = 1 \text{ is} \]
Area = \int_0^1 (\sqrt{2x-x^2} - \sqrt{1-x^2} - 1) \, dx

= \left[ \frac{(x-1)}{2} \sqrt{2x-x^2} - \frac{1}{2} \sin^{-1}(x-1) - \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x - x \right]_0

= \left( \frac{\pi}{4} - 1 \right) - \left( \frac{-\pi}{4} \right) = \frac{\pi}{2} - 1

**Question:** A six digit number is formed by the numbers 0, 1, 2, 3, 4, 5, 6 without repetition. Then the probability that the number is divisible by 3 is

**Options:**
(a) \frac{11}{24}
(b) \frac{3}{7}
(c) \frac{4}{9}
(d) \frac{9}{56}

**Answer:** (c)

**Solution:**
Given numbers are 0, 1, 2, 3, 4, 5, 6

Total number of 6-digit number = 6 \times 6! = 720 \times 6

6-digit number divisible by 3
(a) when ‘0’ is excluded = 6! = 720
(b) when ‘0’ is included = 2 \times 5 \times 5! = 1200

\therefore \text{ Required probability} = \frac{1920}{4320} = \frac{4}{9}
**Question:** Let ‘c’ be the locus of the mirror image of a point on the parabola \( y^2 = 4x \) with respect to the line \( y = x \). Then the equation of tangent to ‘c’ at \( p(2,1) \) is:

**Options:**
(a) \( x + 3y = 5 \)
(b) \( x + 2y = 4 \)
(c) \( x - y = 1 \)
(d) \( 2x + y = 5 \)

**Answer:** (c)

**Solution:**
Any point on parabola \( y^2 = 4x \) is \( (t^2, 2t) \)

Mirror image of \( (t^2, 2t) \) w.r.t \( y = x \) is \( (2t, t^2) \)

\[ \therefore \text{locus of 'C' is } x^2 = 4y \]

\[ \therefore \text{Equation of tangent to ‘C’ is at (2, 1) is } 2x = 2(y+1) \text{ or } x = y+1 \]

**Question:** The maximum value of \( f(x) = \begin{vmatrix} \sin^2 x & 1 + \cos^2 x & \cos 2x \\ 1 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x \in \mathbb{R} \)

**Options**
(a) \( 5 \)
(b) \( 3 \)
(c) \( \frac{3}{4} \)
(d) \( \sqrt{7} \)

**Answer:** (a)

**Solution:**
\[
f(x) = \begin{vmatrix} \sin^2 x & 1 + \cos^2 x & \cos 2x \\ 1 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix} = \sin^2 x \begin{vmatrix} 1 & 1 \cos^2 x & \cos 2x \\ 0 & -1 & \sin 2x - \cos 2x \end{vmatrix}
\]
\[ f(x) = -\cos 2x + (\sin 2x - \cos 2x)(-\sin^2 x - 1 - \cos^2 x) \]

\[ f(x) = \cos 2x - 2\sin 2x \]

\[ \therefore \text{Maximum value of } f(x) = \sqrt{1^2 + 2^2} = \sqrt{5} \]

**Question:** If the points of intersections of the ellipse \( \frac{x^2}{16} + \frac{y^2}{b^2} = 1 \) and the circle \( x^2 + y^2 = 4b \), \( b > 4 \) lie on the curve \( y^2 = 3x^2 \) then ‘b’ is equal to

Options
(a) 12  
(b) 6  
(c) 10  
(d) 5

**Answer:** (a)

**Solution:**
\[ \frac{x^2}{16} + \frac{y^2}{b^2} = 1 \]
\[ x^2 + y^2 = 4b \]
\[ y^2 = 3x^2 \]
\[ \Rightarrow b = x^2 \]
\[ y^2 = 3b \]

\[ \therefore \frac{b}{16} + \frac{3}{b} = 1 \]
\[ \Rightarrow b^2 - 16b + 48 = 0 \]
\[ \Rightarrow b = 4, 12 \]
\[ \therefore b > 4 \Rightarrow b = 12 \]

**Question:** \( f(x) = \begin{cases} \frac{\cos^{-1}(1-x^2) \sin^{-1}(1-x)}{x(1-x)(1+x)} & x \neq 0 \\ \alpha & x = 0 \end{cases} \), Find \( \alpha \) if \( f(x) \) is continuous
Options:
(a)
(b)
(c)
(d)

Answer: ()

Solution:
\[ f(x) \text{ is continuous} \]
\[ \therefore f(0) = \lim_{x \to 0} f(x) \]
\[ \alpha = \lim_{x \to 0} \frac{\cos^{-1}\left(1 - \{x\}^2\right) \cdot \sin^{-1}\left(1 - \{x\}\right)}{\{x\}(1-\{x\})(1+\{x\})} \]
\[ = \frac{\pi}{2} \lim_{x \to 0} \frac{\cos^{-1}\left(1 - x^2\right)}{x} \]
\[ = \frac{\pi}{2} \lim_{x \to 0} \frac{-1(-2x)}{\sqrt{1-(1-x^2)^2}} \]
\[ = \frac{\pi}{2} \lim_{x \to 0} \frac{2x}{x\sqrt{2-x^2}} = \frac{\pi}{\sqrt{2}} \]

Question: \( \int_{0}^{10} \left[ x e^{[x]} \right] e^{x-1} \, dx \)

Options:
(a) \( 9(e-1) \)
(b) \( 9(e+1) \)
(c) \( 45(e-1) \)
(d) \( 45(e+1) \)

Answer: (c)

Solution:
\[ I = e \int_{0}^{10} \left[ x e^{[x]} \right] e^{x} \, dx \]
\[
\begin{align*}
&= e \left[ e \int_1^2 e^{-e} \, dx + 2e^{\frac{3}{2}} \int_2^3 e^{-e} \, dx + \ldots + 9e^{\frac{10}{9}} \int_9^e e^{-e} \, dx \right] \\
&= -e \left[ e(\frac{e^2 - e^{-1}}{2}) + 2e^{\frac{3}{2}}(\frac{e^3 - e^{-2}}{3}) + 3e^{\frac{4}{3}}(\frac{e^4 - e^{-3}}{4}) + \ldots + 9e^{\frac{10}{9}}(\frac{e^{10} - e^{-9}}{10}) \right] \\
&= -e \left[ (e^3 - 1) + 2(e^4 - 1) + 3(e^5 - 1) + \ldots + 9(e^{10} - 1) \right] \\
&= -e \left[ 45e^{e^{-1}} - 45 \right] = 45[e - 1]
\end{align*}
\]

**Question:** ABCD is a rectangle with 5, 6, 7, 9 points on side AB, CD, BC and AD respectively. Let \( \alpha \) be the number of quadrilateral that can be formed using these points with vertices and different sides and let \( \beta \) be the number of triangles formed with vertices on different side. Then what is \( \alpha - \beta \)?

**Options:**
(a) 1890  
(b) 1173  
(c) 717  
(d) 819  

**Answer:** (c)
\[
\alpha = 5C_1 \times 6C_1 \times 7C_1 + 6C_1 \times 7C_1 \times 8C_1 \times 5C_1 + 8C_1 \times 5C_1 \times 6C_1 \\
= 210 + 378 + 315 + 270 = 1173 \\
\beta = 5C_1 \times 6C_1 \times 7C_1 \times 8C_1 = 1890 \\
\therefore \beta - \alpha = 717
\]

**Question:** \( x, y, z \) be a point on plane passing through (42, 0, 0), (0, 42, 0) and (0, 0, 42) then find the value of:

\[
\frac{x-11}{(y-19)^2 (z-12)^2} + \frac{y-19}{(x-11)^2 (z-12)^2} + \frac{z-12}{(x-11)^2 (y-19)^2} + \frac{x+y+z}{14(x-11)(y-19)(z-12)}
\]

**Answer:** 3.00

**Solution:**
Equation of plane is \( x + y + z = 42 \)
\[
\Rightarrow (x-11) + (y-19) + (z-12) = 0
\]
Let \( x-11 = u; y-19 = v; z-12 = w \)
i.e. \( u + v + w = 0 \)
\[
\therefore \frac{u}{v^2 \cdot w^2} + \frac{v}{u^2 \cdot w^2} + \frac{w}{u^2 \cdot v^2} - \frac{3}{u \cdot v \cdot w} + 3
\]
\[
= \left(\frac{u^3 + v^3 + w^3 - 3uvw}{u^2v^2 \cdot w^2}\right) + 3 = 3
\]
\[
\left[ \therefore u + v + w = 0 \implies u^3 + v^3 + w^3 = 3uvw \right]
\]

\[
\left\{ \begin{array}{l}
H^3 + H + 1 \\
H + 1
\end{array} \right\} \geq 2 \log_2 |5\sqrt{7} + 9i|
\]

Question: Find the minimum value of |z|.

Answer: 3.00

Solution:
\[
\left\{ \begin{array}{l}
H^3 + H + 1 \\
H + 1
\end{array} \right\} \geq 2 \log_2 |5\sqrt{7} + 9i|
\]
\[
\therefore |5\sqrt{7} + 9i| = \sqrt{25 \times 7 + 81} = \sqrt{256} = 16
\]
\[
\therefore 2 \log_2 16 = 2 \log_2 2^4 = 8 = 2^3
\]
\[
\Rightarrow \left(\left|z\right| + 3\right)\left(\left|z\right| - 1\right) \geq 3 \implies |z|^2 + 2|z| - 3 \geq 3|z| + 3
\]
\[
|z|^2 - |z| - 6 \geq 0 \Rightarrow (|z| - 3)(|z| + 2) \geq 0
\]
\[
|z| \geq 3
\]
\[
|z|_{\text{min}} = 3
\]

Question: \( \frac{1}{16}, a, b \) are G.P., \( \frac{1}{a}, \frac{1}{b}, 6 \) are in A.P. Then find the value of \( 72(a + b) \)

Answer: 54.00

Solution:
\[
a^2 = \frac{b}{16}; \quad \frac{2}{b} = \frac{1}{a} + 6 \Rightarrow b = \frac{2a}{1 + 6a}
\]
\[
\therefore 16a^2 = \frac{2a}{1 + 6a} \Rightarrow 96a^2 + 16a - 2 = 0
\]
\[
48a^2 + 8a - 1 = 0 \Rightarrow 48a^2 + 12a - 4a - 1 = 0
\]
12a(4a + 1) − (4a + 1) = 0 \Rightarrow a = -\frac{1}{4}, \frac{1}{12} \Rightarrow b = 1, \frac{1}{9}

∴ 72(a + b) = 54 or 14

**Question:** Two sides of \( \triangle ABC \) are 5 and 12. Area of \( \triangle ABC \) is 30. Find \( 2R + r \), where \( R \) is circumradius and \( r \) is inradius.

**Answer:** 15.00

**Solution:**

Let \( a = 5, b = 12, \Delta = 30 \)

\[
\Delta = \frac{1}{2} \cdot a \cdot b \cdot \sin c = \frac{1}{2} \times 5 \times 12 \cdot \sin c = 30
\]

\[
\therefore \sin c = 1 \Rightarrow c = 90 \Rightarrow c = 13
\]

\[
\Rightarrow 2R = \frac{c}{\sin c} = 13; \ r = \frac{\Delta}{s} = \frac{30}{15} = 2
\]

\[
\therefore 2R + r = 13 + 2 = 15
\]

**Question:** \( A = XB \), \( A \) and \( B \) are \( 2 \times 1 \) matrices

\[
A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad X = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}, \quad A = XB, \quad a_1^2 + a_2^2 = \frac{2}{3}(b_1^2 + b_2^2).
\]

Find \( k \).

**Answer:** 1.00

**Solution:**

\[
A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad X = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}
\]

\[
A = XB
\]

\[
\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \frac{1}{\sqrt{3}} \begin{bmatrix} b_1 - b_2 \\ b_1 + kb_2 \end{bmatrix}
\]
\[ a_1 = \frac{b_1 - b_2}{\sqrt{3}}; \quad a_2 = \frac{b_1 + kb_2}{\sqrt{3}} \]

\[ a_1^2 + a_2^2 = \frac{b_1^2 + b_2^2 - 2b_1b_2 + b_1^2 + k^2b_2^2 + 2kb_1b_2}{3} = \frac{(b_1^2 + b_2^2)}{3} \]

\[ b_2^2 = kb_2^2 + 2b_1b_2(k-1) \]

\[ \Rightarrow k = 1 \]

**Question:** \( A = \{2, 3, 4, \ldots, 30\} \), \((a, b)\) and \((c, d)\) are equivalent of \( ad = bc \) then number of elements equivalent to \((4, 3)\)

**Answer:** 7.00

**Solution:**

\( A = \{2, 3, 4, \ldots, 30\} \); \( \frac{a}{b} = \frac{c}{d} \)

\( (4,3) = \frac{4}{3} = \frac{8}{6} = \frac{12}{9} = \frac{16}{12} = \frac{20}{15} = \frac{24}{18} = \frac{28}{21} \)

\( \therefore \) Total number of elements = 7

**Question:** \( \sum_{k=0}^{n} (-1)^k \cdot C_k \left[ \left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \ldots + \left(\frac{31}{32}\right)^k \right] \), \( 63A = 1 - \frac{1}{2^{30}} \) Find \( n \)

**Answer:** 6.00

**Solution:**

\( A = \sum_{k=0}^{n} (-1)^k \cdot C_k \left[ \left(\frac{1}{2}\right)^k + \left(\frac{3}{4}\right)^k + \left(\frac{7}{8}\right)^k + \ldots + \left(\frac{31}{32}\right)^k \right] \)

\[ = \left(\frac{1}{2}\right)^n + \left(\frac{1}{4}\right)^n + \left(\frac{1}{8}\right)^n + \left(\frac{1}{16}\right)^n + \left(\frac{1}{32}\right)^n \]

\[ = \left(\frac{1}{2}\right)^n \left[ 1 - \left(\frac{1}{2}\right)^5 n \right] \]

\[ A = \frac{\left(\frac{1}{2}\right)^n}{1 - \left(\frac{1}{2}\right)^n} \]

When \( n = 6 \Rightarrow 63A - 1 = \frac{1}{2^{30}} \Rightarrow n = 6 \)