FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Tuesday 16th March, 2021) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS — TEST PAPER WITH ANSWER

SECTION-A

1. The maximum value of

$$f(x)=\begin{vmatrix} \sin^2 x & 1+\cos^2 x & \cos 2x \\ 1+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & \sin 2x \end{vmatrix}, x \in R \text{ is:}$$

(1) $\sqrt{7}$ (2) $\frac{3}{4}$ (3) $\sqrt{5}$ (4) 5

Official Ans by NTA (3)

2. Let A denote the event that a 6-digit integer formed by 0, 1, 2, 3, 4, 5, 6 without repetitions, be divisible by 3. Then probability of event A is equal to :

(1) $\frac{9}{56}$ (2) $\frac{4}{9}$ (3) $\frac{3}{7}$ (4) $\frac{11}{27}$

Official Ans by NTA (2)

3. Let $\alpha \in R$ be such that the function

$$f(x)=\begin{cases} \dfrac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, & x \neq 0 \\ \alpha, & x=0 \end{cases}$$

is continuous at $x = 0$, where $\{x\} = x - [x]$, $[x]$ is the greatest integer less than or equal to x. Then :

(1) $\alpha=\frac{\pi}{\sqrt{2}}$ (2) $\alpha = 0$

(3) no such α exists (4) $\alpha=\frac{\pi}{4}$

Official Ans by NTA (3)

4. If (x, y, z) be an arbitrary point lying on a plane P which passes through the point (42, 0, 0), (0, 42, 0) and (0, 0, 42), then the value of expression

$$3+\frac{x-11}{(y-19)^2(z-12)^2}+\frac{y-19}{(x-11)^2(z-12)^2}+\frac{z-12}{(x-11)^2(y-19)^2}-\frac{x+y+z}{14(x-11)(y-19)(z-12)}$$

(1) 0 (2) 3 (3) 39 (4) –45

Official Ans by NTA (2)

5. Consider the integral

$$I=\int_0^{10}\frac{[x]e^{[x]}}{e^{x-1}}dx,$$

where [x] denotes the greatest integer less than or equal to x. Then the value of I is equal to:

(1) 9(e – 1) (2) 45(e + 1)
(3) 45(e – 1) (4) 9(e + 1)

Official Ans by NTA (3)

6. Let C be the locus of the mirror image of a point on the parabola $y^2 = 4x$ with respect to the line $y = x$. Then the equation of tangent to C at P(2,1) is :

(1) $x - y = 1$ (2) $2x + y = 5$
(3) $x + 3y = 5$ (4) $x + 2y = 4$

Official Ans by NTA (1)

7. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx}+(\tan x)y=\sin x,\ 0\le x\le\frac{\pi}{3}$, with $y(0) = 0$, then $y\left(\frac{\pi}{4}\right)$ equal to :

(1) $\frac{1}{4}\log_e 2$ (2) $\left(\frac{1}{2\sqrt{2}}\right)\log_e 2$

(3) $\log_e 2$ (4) $\frac{1}{2}\log_e 2$

Official Ans by NTA (2)

8. Let A = {2, 3, 4, 5, ... , 30} and '$\simeq$' be an equivalence relation on A × A, defined by (a, b) $\simeq$ (c, d), if and only if ad = bc. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair (4, 3) is equal to :

(1) 5 (2) 6 (3) 8 (4) 7

Official Ans by NTA (4)

9. Let the lengths of intercepts on x-axis and y-axis made by the circle $x^2 + y^2 + ax + 2ay + c = 0$, $(a < 0)$ be $2\sqrt{2}$ and $2\sqrt{5}$, respectively. Then the shortest distance from origin to a tangent to this circle which is perpendicular to the line $x + 2y = 0$, is euqal to :

(1) $\sqrt{11}$ (2) $\sqrt{7}$ (3) $\sqrt{6}$ (4) $\sqrt{10}$

Official Ans by NTA (3)

10. The least value of |z| where z is complex number which satisfies the inequality

$$\exp\left(\frac{(|z|+3)(|z|-1)}{||z|+1|}\log_e 2\right) \geq \log_{\sqrt{2}}\left|5\sqrt{7}+9i\right|,$$

$i = \sqrt{-1}$, is equal to :

(1) 3 (2) $\sqrt{5}$ (3) 2 (4) 8

Official Ans by NTA (1)

11. Consider a rectangle ABCD having 5, 7, 6, 9 points in the interior of the line segments AB, CD, BC, DA respectively. Let α be the number of triangles having these points from different sides as vertices and β be the number of quadrilaterals having these points from different sides as vertices. Then $(\beta - \alpha)$ is equal to :

(1) 795 (2) 1173 (3) 1890 (4) 717

Official Ans by NTA (4)

12. If the point of intersections of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the circle $x^2 + y^2 = 4b$, $b > 4$ lie on the curve $y^2 = 3x^2$, then b is equal to:

(1) 12 (2) 5 (3) 6 (4) 10

Official Ans by NTA (1)

13. Given that the inverse trigonometric functions take principal values only. Then, the number of real values of x which satisfy

$$\sin^{-1}\left(\frac{3x}{5}\right) + \sin^{-1}\left(\frac{4x}{5}\right) = \sin^{-1}x$$ is equal to:

(1) 2 (2) 1 (3) 3 (4) 0

Official Ans by NTA (3)

14. Let A(–1, 1), B(3, 4) and C(2, 0) be given three points. A line y = mx, m > 0, intersects lines AC and BC at point P and Q respectively. Let A_1 and A_2 be the areas of ΔABC and ΔPQC respectively, such that $A_1 = 3A_2$, then the value of m is equal to :

(1) $\frac{4}{15}$ (2) 1 (3) 2 (4) 3

Official Ans by NTA (2)

15. Let f be a real valued function, defined on R – {–1, 1} and given by

$$f(x) = 3\log_e\left|\frac{x-1}{x+1}\right| - \frac{2}{x-1}.$$

Then in which of the following intervals, function f(x) is increasing?

(1) $(-\infty,-1)\cup\left(\left[\frac{1}{2},\infty\right)-\{1\}\right)$

(2) $(-\infty, \infty) - \{-1, 1\}$

(3) $\left(-1, \frac{1}{2}\right]$

(4) $\left(-\infty, \frac{1}{2}\right] - \{-1\}$

Official Ans by NTA (1)

16. Let f : S → S where S = (0, ∞) be a twice differentiable function such that f(x + 1) = xf(x). If g : S → R be defined as $g(x) = \log_e f(x)$, then the value of |g"(5) – g"(1)| is equal to :

(1) $\frac{205}{144}$ (2) $\frac{197}{144}$ (3) $\frac{187}{144}$ (4) 1

Official Ans by NTA (1)

17. Let $P(x) = x^2 + bx + c$ be a quadratic polynomial with real coefficients such that $\int_0^1 P(x)dx = 1$ and $P(x)$ leaves remainder 5 when it is divided by $(x - 2)$. Then the value of $9(b + c)$ is equal to:

(1) 9 (2) 15 (3) 7 (4) 11

Official Ans by NTA (3)

18. If the foot of the perpendicular from point (4, 3, 8) on the line $L_1 : \frac{x-a}{l} = \frac{y-2}{3} = \frac{z-b}{4}$, $l \neq 0$ is (3, 5, 7), then the shortest distance between the line L_1 and line $L_2 : \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is equal to :

(1) $\frac{1}{2}$ (2) $\frac{1}{\sqrt{6}}$ (3) $\sqrt{\frac{2}{3}}$ (4) $\frac{1}{\sqrt{3}}$

Official Ans by NTA (2)

19. Let C_1 be the curve obtained by the solution of differential equation $2xy\frac{dy}{dx} = y^2 - x^2$, $x > 0$. Let the curve C_2 be the solution of $\frac{2xy}{x^2 - y^2} = \frac{dy}{dx}$. If both the curves pass through (1,1), then the area enclosed by the curves C_1 and C_2 is equal to :

(1) $\pi - 1$ (2) $\frac{\pi}{2} - 1$ (3) $\pi + 1$ (4) $\frac{\pi}{4} + 1$

Official Ans by NTA (2)

20. Let $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = 2\hat{i} - 3\hat{j} + 5\hat{k}$. If $\vec{r} \times \vec{a} = \vec{b} \times \vec{r}$, $\vec{r}.\left(\alpha\hat{i} + 2\hat{j} + \hat{k}\right) = 3$ and $\vec{r}.\left(2\hat{i} + 5\hat{j} - \alpha\hat{k}\right) = -1$, $\alpha \in R$, then the value of $\alpha + \left|\vec{r}\right|^2$ is equal to :

(1) 9 (2) 15 (3) 13 (4) 11

Official Ans by NTA (2)

SECTION-B

1. If the distance of the point (1, –2, 3) from the plane $x + 2y - 3z + 10 = 0$ measured parallel to the line, $\frac{x-1}{3} = \frac{2-y}{m} = \frac{z+3}{1}$ is $\sqrt{\frac{7}{2}}$, then the value of $|m|$ is equal to ______.

Official Ans by NTA (2)

2. Consider the statistics of two sets of observations as follows :

	Size	Mean	Variance
Observation I	10	2	2
Observation II	n	3	1

If the variance of the combined set of these two observations is $\frac{17}{9}$, then the value of n is equal to __________.

Official Ans by NTA (5)

3. Let $A = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ be two 2 × 1 matrices with real entries such that A = XB, where $X = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & -1 \\ 1 & k \end{bmatrix}$, and $k \in R$. If $a_1^2 + a_2^2 = \frac{2}{3}\left(b_1^2 + b_2^2\right)$ and $(k^2 + 1)b_2^2 \neq -2b_1b_2$, then the value of k is _______.

Official Ans by NTA (1)

4. For real numbers α, β, γ and δ, if

$$\int \frac{(x^2 - 1) + \tan^{-1}\left(\frac{x^2+1}{x}\right)}{(x^4 + 3x^2 + 1)\tan^{-1}\left(\frac{x^2+1}{x}\right)}dx$$

$$= \alpha \log_e\left(\tan^{-1}\left(\frac{x^2+1}{x}\right)\right)$$

$$+\beta\tan^{-1}\left(\frac{\gamma(x^2-1)}{x}\right) + \delta\tan^{-1}\left(\frac{x^2+1}{x}\right) + C$$

where C is an arbitrary constant, then the value of $10(\alpha + \beta\gamma + \delta)$ is equal to __________.

Official Ans by NTA (6)

5. Let $f : R \to R$ and $g : R \to R$ be defined as

$$f(x)=\begin{cases} x+a, & x<0 \\ |x-1|, & x\geq 0 \end{cases} \text{ and}$$

$$g(x)=\begin{cases} x+1, & x<0 \\ (x-1)^2+b, & x\geq 0 \end{cases}$$

where a, b are non-negative real numbers. If (gof)(x) is continuous for all $x \in R$, then a + b is equal to ___________.

Official Ans by NTA (1)

6. Let $\frac{1}{16}$, a and b be in G.P. and $\frac{1}{a},\frac{1}{b},6$ be in A.P., where a, b > 0. Then 72(a + b) is equal to ________.

Official Ans by NTA (14)

7. In ΔABC, the lengths of sides AC and AB are 12 cm and 5 cm, respectively. If the area of ΔABC is 30 cm² and R and r are respectively the radii of circumcircle and incircle of ΔABC, then the value of 2R + r (in cm) is equal to _______.

Official Ans by NTA (15)

8. Let n be a positive integer. Let

$$A=\sum_{k=0}^{n}(-1)^k\, {}^nC_k\left[\left(\frac{1}{2}\right)^k+\left(\frac{3}{4}\right)^k+\left(\frac{7}{8}\right)^k+\left(\frac{15}{16}\right)^k+\left(\frac{31}{32}\right)^k\right]$$

If $63A = 1-\frac{1}{2^{30}}$, then n is equal to _______.

Official Ans by NTA (6)

9. Let $\vec{c}$ be a vector perpendicular to the vectors $\vec{a}=\hat{i}+\hat{j}-\hat{k}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$.

If $\vec{c}.\left(\hat{i}+\hat{j}+3\hat{k}\right)=8$ then the value of $\vec{c}.\left(\vec{a}\times\vec{b}\right)$ is equal to _________.

Official Ans by NTA (28)

10. Let

$$S_n(x)=\log_{a^{1/2}}x+\log_{a^{1/3}}x+\log_{a^{1/6}}x$$
$$+\log_{a^{1/11}}x+\log_{a^{1/18}}x+\log_{a^{1/27}}x+.....$$

up to n-terms, where a > 1. If $S_{24}(x) = 1093$ and $S_{12}(2x) = 265$, then value of a is equal to _______.

Official Ans by NTA (16)