FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Thursday 18th March, 2021) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS **TEST PAPER WITH ANSWER**

SECTION-A

1. Let y = y(x) be the solution of the differential equation $\frac{dy}{dx}=(y+1)\left((y+1)e^{x^2/2}-x\right), 0<x<2.1,$ with y(2) = 0. Then the value of $\frac{dy}{dx}$ at x = 1 is equal to :

(1) $\frac{-e^{3/2}}{(e^2+1)^2}$ (2) $-\frac{2e^2}{(1+e^2)^2}$

(3) $\frac{e^{5/2}}{(1+e^2)^2}$ (4) $\frac{5e^{1/2}}{(e^2+1)^2}$

Official Ans. by NTA (1)

2. In a triangle ABC, if $|\overrightarrow{BC}|=8$, $|\overrightarrow{CA}|=7$, $|\overrightarrow{AB}|=10$, then the projection of the vector $\overrightarrow{AB}$ on $\overrightarrow{AC}$ is equal to :

(1) $\frac{25}{4}$ (2) $\frac{85}{14}$ (3) $\frac{127}{20}$ (4) $\frac{115}{16}$

Official Ans. by NTA (2)

3. Let the system of linear equations

4x + λy + 2z = 0

2x – y + z = 0

μx + 2y + 3z = 0, λ, μ ∈ R.

has a non-trivial solution. Then which of the following is true ?

(1) μ = 6, λ ∈ R (2) λ = 2, μ ∈ R

(3) λ = 3, μ ∈ R (4) μ = –6, λ ∈ R

Official Ans. by NTA (1)

4. Let f : R – {3} → R – {1} be defined by $f(x)=\frac{x-2}{x-3}$. Let g : R → R be given as g(x) = 2x – 3. Then, the sum of all the values of x for which $f^{-1}(x) + g^{-1}(x) = \frac{13}{2}$ is equal to

(1) 7 (2) 2 (3) 5 (4) 3

Official Ans. by NTA (3)

5. Let the centroid of an equilateral triangle ABC be at the origin. Let one of the sides of the equilateral triangle be along the straight line x + y = 3. If R and r be the radius of circumcircle and incircle respectively of ΔABC, then (R + r) is equal to :

(1) $\frac{9}{\sqrt{2}}$ (2) $7\sqrt{2}$ (3) $2\sqrt{2}$ (4) $3\sqrt{2}$

Official Ans. by NTA (1)

6. Consider a hyperbola H : $x^2 - 2y^2 = 4$. Let the tangent at a point $P(4,\sqrt{6})$ meet the x-axis at Q and latus rectum at $R(x_1, y_1)$, $x_1 > 0$. If F is a focus of H which is nearer to the point P, then the area of ΔQFR is equal to

(1) $4\sqrt{6}$ (2) $\sqrt{6}-1$

(3) $\frac{7}{\sqrt{6}}-2$ (4) $4\sqrt{6}-1$

Official Ans. by NTA (3)

7. If P and Q are two statements, then which of the following compound statement is a tautology ?

(1) ((P ⇒ Q) ∧ ~ Q) ⇒ Q
(2) ((P ⇒ Q) ∧ ~ Q) ⇒ ~ P
(3) ((P ⇒ Q) ∧ ~ Q) ⇒ P
(4) ((P ⇒ Q) ∧ ~ Q) ⇒ (P ∧ Q)

Official Ans. by NTA (2)

8. Let $g(x)=\int_0^x f(t)\,dt$, where f is continuous function in [0, 3] such that $\frac{1}{3}\le f(t)\le 1$ for all $t\in[0,1]$ and $0\le f(t)\le\frac{1}{2}$ for all $t\in(1,3]$. The largest possible interval in which g(3) lies is :

(1) $\left[-1,-\frac{1}{2}\right]$ (2) $\left[-\frac{3}{2},-1\right]$

(3) $\left[\frac{1}{3},2\right]$ (4) [1, 3]

Official Ans. by NTA (3)

9. Let S_1 be the sum of first 2n terms of an arithmetic progression. Let S_2 be the sum of first 4n terms of the same arithmetic progression. If $(S_2 - S_1)$ is 1000, then the sum of the first 6n terms of the arithmetic progression is equal to:

(1) 1000 (2) 7000 (3) 5000 (4) 3000

Official Ans. by NTA (4)

10. Let a complex number be $w = 1-\sqrt{3}i$. Let another complex number z be such that $|zw| = 1$ and $\arg(z)-\arg(w)=\frac{\pi}{2}$. Then the area of the triangle with vertices origin, z and w is equal to :

(1) 4 (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) 2

Official Ans. by NTA (2)

11. Let in a series of 2n observations, half of them are equal to a and remaining half are equal to –a. Also by adding a constant b in each of these observations, the mean and standard deviation of new set become 5 and 20, respectively. Then the value of $a^2 + b^2$ is equal to :

(1) 425 (2) 650 (3) 250 (4) 925

Official Ans. by NTA (1)

12. Let $S_1 : x^2 + y^2 = 9$ and $S_2 : (x-2)^2 + y^2 = 1$. Then the locus of center of a variable circle S which touches S_1 internally and S_2 externally always passes through the points :

(1) $\left(0,\pm\sqrt{3}\right)$ (2) $\left(\frac{1}{2},\pm\frac{\sqrt{5}}{2}\right)$

(3) $\left(2,\pm\frac{3}{2}\right)$ (4) $(1,\pm 2)$

Official Ans. by NTA (3)

13. Let $\vec{a}$ and $\vec{b}$ be two non-zero vectors perpendicular to each other and $|\vec{a}|=|\vec{b}|$. If $|\vec{a}\times\vec{b}|=|\vec{a}|$, then the angle between the vectors $\left(\vec{a}+\vec{b}+\left(\vec{a}\times\vec{b}\right)\right)$ and $\vec{a}$ is equal to :

(1) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (2) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

(3) $\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (4) $\sin^{-1}\left(\frac{1}{\sqrt{6}}\right)$

Official Ans. by NTA (2)

14. Let in a Binomial distribution, consisting of 5 independent trials, probabilities of exactly 1 and 2 successes be 0.4096 and 0.2048 respectively. Then the probability of getting exactly 3 successes is equal to :

(1) $\frac{32}{625}$ (2) $\frac{80}{243}$ (3) $\frac{40}{243}$ (4) $\frac{128}{625}$

Official Ans. by NTA (1)

15. Let a tangent be drawn to the ellipse $\frac{x^2}{27}+y^2=1$ at $\left(3\sqrt{3}\cos\theta,\sin\theta\right)$ where $\theta\in\left(0,\frac{\pi}{2}\right)$. Then the value of θ such that the sum of intercepts on axes made by this tangent is minimum is equal to :

(1) $\frac{\pi}{8}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{3}$

Official Ans. by NTA (3)

16. Define a relation R over a class of n × n real matrices A and B as "ARB iff there exists a non-singular matrix P such that $PAP^{-1} = B$". Then which of the following is true ?

(1) R is symmetric, transitive but not reflexive,
(2) R is reflexive, symmetric but not transitive
(3) R is an equivalence relation
(4) R is reflexive, transitive but not symmetric

Official Ans. by NTA (3)

17. A pole stands vertically inside a triangular park ABC. Let the angle of elevation of the top of the pole from each corner of the park be $\frac{\pi}{3}$. If the radius of the circumcircle ot ΔABC is 2, then the height of the pole is equal to :

(1) $\frac{2\sqrt{3}}{3}$ (2) $2\sqrt{3}$ (3) $\sqrt{3}$ (4) $\frac{1}{\sqrt{3}}$

Official Ans. by NTA (2)

18. If $15\sin^4\alpha + 10\cos^4\alpha = 6$, for some $\alpha \in R$, then the value of $27\sec^6\alpha + 8\operatorname{cosec}^6\alpha$ is equal to :

(1) 350 (2) 500 (3) 400 (4) 250

Official Ans. by NTA (4)

19. The area bounded by the curve $4y^2 = x^2(4 - x)(x - 2)$ is equal to :

(1) $\frac{\pi}{8}$ (2) $\frac{3\pi}{8}$ (3) $\frac{3\pi}{2}$ (4) $\frac{\pi}{16}$

Official Ans. by NTA (3)

20. Let $f : R \to R$ be a function defined as

$$f(x) = \begin{cases} \dfrac{\sin(a+1)x + \sin 2x}{2x} & , \text{if } x < 0 \\ b & , \text{if } x = 0 \\ \dfrac{\sqrt{x + bx^3} - \sqrt{x}}{bx^{5/2}} & , \text{if } x > 0 \end{cases}$$

If f is continuous at x = 0, then the value of a + b is equal to :

(1) $-\frac{5}{2}$ (2) -2 (3) -3 (4) $-\frac{3}{2}$

Official Ans. by NTA (4)

SECTION-B

1. If $f(x)$ and $g(x)$ are two polynomials such that the polynomial $P(x) = f(x^3) + xg(x^3)$ is divisible by $x^2 + x + 1$, then P(l) is equal to______.

Official Ans. by NTA (0)

2. Let I be an identity matrix of order 2 × 2 and $P = \begin{bmatrix} 2 & -1 \\ 5 & -3 \end{bmatrix}$. Then the value of $n \in N$ for which $P^n = 5I - 8P$ is equal to _______.

Official Ans. by NTA (6)

3. If $\sum_{r=1}^{10} r!\left(r^3 + 6r^2 + 2r + 5\right) = \alpha(11!)$, then the value of α is equal to _______.

Official Ans. by NTA (160)

4. The term independent of x in the expansion of $\left[\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right]^{10}$, $x \neq 1$, is equal to _______.

Official Ans. by NTA (210)

5. Let P(x) be a real polynomial of degree 3 which vanishes at x = –3. Let P(x) have local minima at x = 1, local maxima at x = –1 and $\int_{-1}^{1} P(x)dx = 18$, then the sum of all the coefficients of the polynomial P(x) is equal to _______.

Official Ans. by NTA (8)

6. Let the mirror image of the point (1, 3, a) with respect to the plane $\vec{r}.\left(2\hat{i} - \hat{j} + \hat{k}\right) - b = 0$ be (–3, 5, 2). Then the value of |a + b| is equal to _______.

Official Ans. by NTA (1)

7. Let f : R $\to$ R satisfy the equation $f(x + y) = f(x).f(y)$ for all x, y $\in$ R and f(x) $\neq$ 0 for any x $\in$ R. If Ihe function f is differentiable at x = 0 and $f'(0) = 3$, then

$\lim\limits_{h\to 0}\frac{1}{h}(f(h)-1)$ is equal to ______.

Official Ans. by NTA (3)

8. Let ${}^nC_r$ denote the binomial coefficient of x^r in the expansion of $(1 + x)^n$.

If $\sum\limits_{k=0}^{10}(2^2+3k)\,{}^nC_k = \alpha.3^{10}+\beta.2^{10}$, α, β $\in$ R,

then $\alpha + \beta$ is equal to ______.

Official Ans. by NTA (19)

Allen Answer (Bonus)

9. Let P be a plane containing the line $\frac{x-1}{3}=\frac{y+6}{4}=\frac{z+5}{2}$ and parallel to the line $\frac{x-3}{4}=\frac{y-2}{-3}=\frac{z+5}{7}$. If the point $(1, -1, \alpha)$ lies on the plane P, then the value of $|5\alpha|$ is equal to ______.

Official Ans. by NTA (38)

10. Let y = y(x) be the solution of the differential equation $xdy - ydx = \sqrt{(x^2-y^2)}\,dx$, $x \geq 1$, with $y(1) = 0$. If the area bounded by the line $x = 1$, $x = e^{\pi}$, $y = 0$ and $y = y(x)$ is $\alpha e^{2\pi} + \beta$, then the value of $10(\alpha + \beta)$ is equal to ______.

Official Ans. by NTA (4)