FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Wednesday 17th March, 2021) TIME : 3 : 00 PM to 6 : 00 PM

MATHEMATICS — TEST PAPER WITH ANSWER

SECTION-A

1. Let $f : R \to R$ be defined as $f(x) = e^{-x}\sin x$. If $F : [0, 1] \to R$ is a differentiable function such that $F(x)=\int_0^x f(t)\,dt$, then the value of $\int_0^1 (F'(x)+f(x))e^x dx$ lies in the interval

(1) $\left[\frac{327}{360},\frac{329}{360}\right]$ (2) $\left[\frac{330}{360},\frac{331}{360}\right]$

(3) $\left[\frac{331}{360},\frac{334}{360}\right]$ (4) $\left[\frac{335}{360},\frac{336}{360}\right]$

Official Ans. by NTA (2)

2. If the integral $\int_0^{10}\frac{[\sin 2\pi x]}{e^{x-[x]}}dx=\alpha e^{-1}+\beta e^{-\frac{1}{2}}+\gamma$, where α, β, γ are integers and $[x]$ denotes the greatest integer less than or equal to x, then the value of $\alpha + \beta + \gamma$ is equal to :

(1) 0 (2) 20 (3) 25 (4) 10

Official Ans. by NTA (1)

3. Let $y = y(x)$ be the solution of the differential equation

$\cos x\,(3\sin x + \cos x + 3)dy =$

$(1 + y\sin x\,(3\sin x + \cos x + 3))dx,$

$0 \le x \le \frac{\pi}{2}$, $y(0) = 0$. Then , $y\left(\frac{\pi}{3}\right)$ is equal to:

(1) $2\log_e\left(\frac{2\sqrt{3}+9}{6}\right)$ (2) $2\log_e\left(\frac{2\sqrt{3}+10}{11}\right)$

(3) $2\log_e\left(\frac{\sqrt{3}+7}{2}\right)$ (4) $2\log_e\left(\frac{3\sqrt{3}-8}{4}\right)$

Official Ans. by NTA (2)

4. The value of $\sum_{r=0}^{6}\left({}^6C_r\cdot{}^6C_{6-r}\right)$ is equal to :

(1) 1124 (2) 1324 (3) 1024 (4) 924

Official Ans. by NTA (4)

5. The value of $\lim\limits_{n\to\infty}\frac{[r]+[2r]+.....+[nr]}{n^2}$, where r is non-zero real number and [r] denotes the greatest integer less than or equal to r, is equal to :

(1) $\frac{r}{2}$ (2) r (3) 2r (4) 0

Official Ans. by NTA (1)

6. The number of solutions of the equation

$\sin^{-1}\left[x^2+\frac{1}{3}\right]+\cos^{-1}\left[x^2-\frac{2}{3}\right]=x^2$,

for $x \in [-1, 1]$, and $[x]$ denotes the greatest integer less than or equal to x, is :

(1) 2 (2) 0

(3) 4 (4) Infinite

Official Ans. by NTA (2)

7. Let a computer program generate only the digits 0 and 1 to form a string of binary numbers with probability of occurrence of 0 at even places be $\frac{1}{2}$ and probability of occurrence of 0 at the odd place be $\frac{1}{3}$. Then the probability that '10' is followed by '01' is equal to :

(1) $\frac{1}{18}$ (2) $\frac{1}{3}$ (3) $\frac{1}{6}$ (4) $\frac{1}{9}$

Official Ans. by NTA (4)

8. The number of solutions of the equation $x + 2\tan x = \frac{\pi}{2}$ in the interval $[0, 2\pi]$ is :

(1) 3 (2) 4 (3) 2 (4) 5

Official Ans. by NTA (1)

9. Let S_1, S_2 and S_3 be three sets defined as

$S_1 = \{z \in \mathbb{C} : |z-1| \le \sqrt{2}\}$

$S_2 = \{z \in \mathbb{C} : \text{Re}((1-i)z) \ge 1\}$

$S_3 = \{z \in \mathbb{C} : \text{Im}(z) \le 1\}$

Then the set $S_1 \cap S_2 \cap S_3$

(1) is a singleton
(2) has exactly two elements
(3) has infinitely many elements
(4) has exactly three elements

Official Ans. by NTA (3)

10. If the curve y = y(x) is the solution of the differential equation

$2(x^2 + x^{5/4})dy - y(x + x^{1/4})dx = 2x^{9/4}\,dx$, x > 0

which passes through the point $\left(1, 1-\frac{4}{3}\log_e 2\right)$, then the value of y(16) is equal to :

(1) $4\left(\frac{31}{3}+\frac{8}{3}\log_e 3\right)$ (2) $\left(\frac{31}{3}+\frac{8}{3}\log_e 3\right)$

(3) $4\left(\frac{31}{3}-\frac{8}{3}\log_e 3\right)$ (4) $\left(\frac{31}{3}-\frac{8}{3}\log_e 3\right)$

Official Ans. by NTA (3)

11. If the sides AB, BC and CA of a triangle ABC have 3, 5 and 6 interior points respectively, then the total number of triangles that can be constructed using these points as vertices, is equal to :

(1) 364 (2) 240 (3) 333 (4) 360

Official Ans. by NTA (3)

12. If x, y, z are in arithmetic progression with common difference d, x ≠ 3d, and the determinant of the matrix $\begin{bmatrix} 3 & 4\sqrt{2} & x \\ 4 & 5\sqrt{2} & y \\ 5 & k & z \end{bmatrix}$ is zero, then the value of k^2 is

(1) 72 (2) 12 (3) 36 (4) 6

Official Ans. by NTA (1)

13. Let O be the origin. Let $\overrightarrow{OP} = x\hat{i} + y\hat{j} - \hat{k}$ and $\overrightarrow{OQ} = -\hat{i} + 2\hat{j} + 3x\hat{k}$, x, y ∈ R, x > 0, be such that $|\overrightarrow{PQ}| = \sqrt{20}$ and the vector $\overrightarrow{OP}$ is perpendicular to $\overrightarrow{OQ}$. If $\overrightarrow{OR} = 3\hat{i} + z\hat{j} - 7\hat{k}$, z ∈ R, is coplanar with $\overrightarrow{OP}$ and $\overrightarrow{OQ}$, then the value of $x^2 + y^2 + z^2$ is equal to

(1) 7 (2) 9 (3) 2 (4) 1

Official Ans. by NTA (2)

14. Two tangents are drawn from a point P to the circle $x^2 + y^2 - 2x - 4y + 4 = 0$, such that the angle between these tangents is $\tan^{-1}\left(\frac{12}{5}\right)$, where $\tan^{-1}\left(\frac{12}{5}\right) \in (0, \pi)$. If the centre of the circle is denoted by C and these tangents touch the circle at points A and B, then the ratio of the areas of ΔPAB and ΔCAB is :

(1) 11 : 4 (2) 9 : 4 (3) 3 :1 (4) 2 : 1

Official Ans. by NTA (2)

15. Consider the function f : R → R defined by

$f(x) = \begin{cases} \left(2-\sin\left(\frac{1}{x}\right)\right)|x|, & x \ne 0 \\ 0, & x = 0 \end{cases}$. Then f is :

(1) monotonic on (–∞, 0) ∪ (0, ∞)
(2) not monotonic on (–∞, 0) and (0, ∞)
(3) monotonic on (0, ∞) only
(4) monotonic on (–∞, 0) only

Official Ans. by NTA (2)

16. Let L be a tangent line to the parabola $y^2 = 4x - 20$ at (6, 2). If L is also a tangent to the ellipse $\frac{x^2}{2}+\frac{y^2}{b}=1$, then the value of b is equal to :

(1) 11 (2) 14 (3) 16 (4) 20

Official Ans. by NTA (2)

17. The value of the limit $\lim_{\theta \to 0}\frac{\tan(\pi\cos^2\theta)}{\sin(2\pi\sin^2\theta)}$ is equal to :

(1) $-\frac{1}{2}$ (2) $-\frac{1}{4}$ (3) 0 (4) $\frac{1}{4}$

Official Ans. by NTA (1)

18. Let the tangent to the circle $x^2 + y^2 = 25$ at the point R(3, 4) meet x-axis and y-axis at point P and Q, respectively. If r is the radius of the circle passing through the origin O and having centre at the incentre of the triangle OPQ, then r^2 is equal to

(1) $\frac{529}{64}$ (2) $\frac{125}{72}$ (3) $\frac{625}{72}$ (4) $\frac{585}{66}$

Official Ans. by NTA (3)

19. If the Boolean expression $(p \wedge q) \circledast (p \otimes q)$ is a tautology, then $\circledast$ and $\otimes$ are respectively given by

(1) $\rightarrow, \rightarrow$ (2) $\wedge, \vee$ (3) $\vee, \rightarrow$ (4) $\wedge, \rightarrow$

Official Ans. by NTA (1)

20. If the equation of plane passing through the mirror image of a point (2, 3, 1) with respect to line $\frac{x+1}{2} = \frac{y-3}{1} = \frac{z+2}{-1}$ and containing the line $\frac{x-2}{3} = \frac{1-y}{2} = \frac{z+1}{1}$ is $\alpha x + \beta y + \gamma z = 24$, then $\alpha + \beta + \gamma$ is equal to :

(1) 20 (2) 19 (3) 18 (4) 21

Official Ans. by NTA (2)

SECTION-B

1. If 1, $\log_{10}(4^x - 2)$ and $\log_{10}\left(4^x + \frac{18}{5}\right)$ are in arithmetic progression for a real number x, then the value of the determinant

$$\begin{vmatrix} 2\left(x-\frac{1}{2}\right) & x-1 & x^2 \\ 1 & 0 & x \\ x & 1 & 0 \end{vmatrix}$$ is equal to :

Official Ans. by NTA (2)

2. Let $f : [-1, 1] \rightarrow R$ be defined as $f(x) = ax^2 + bx + c$ for all $x \in [-1, 1]$, where $a, b, c \in R$ such that $f(-1) = 2$, $f'(-1) = 1$ and for $x \in (-1, 1)$ the maximum value of $f''(x)$ is $\frac{1}{2}$. If $f(x) \le \alpha$, $x \in [-1, 1]$, then the least value of α is equal to ______.

Official Ans. by NTA (5)

3. Let $f : [-3, 1] \rightarrow R$ be given as

$$f(x) = \begin{cases} \min\{(x+6), x^2\}, & -3 \le x \le 0 \\ \max\{\sqrt{x}, x^2\}, & 0 \le x \le 1. \end{cases}$$

If the area bounded by $y = f(x)$ and x-axis is A, then the value of 6A is equal to ______.

Official Ans. by NTA (41)

4. Let $\tan\alpha$, $\tan\beta$ and $\tan\gamma$; $\alpha, \beta, \gamma \neq \frac{(2n-1)\pi}{2}$, $n \in N$ be the slopes of three line segments OA, OB and OC, respectively, where O is origin.If circumcentre of ΔABC coincides with origin and its orthocentre lies on y-axis, then the value of $\left(\frac{\cos 3\alpha + \cos 3\beta + \cos 3\gamma}{\cos\alpha \cos\beta \cos\gamma}\right)^2$ is equal to :

Official Ans. by NTA (144)

5. Consider a set of 3n numbers having variance 4. In this set, the mean of first 2n numbers is 6 and the mean of the remaining n numbers is 3. A new set is constructed by adding 1 into each of first 2n numbers, and subtracting 1 from each of the remaining n numbers. If the variance of the new set is k, then 9k is equal to ______.

Official Ans. by NTA (68)

6. Let the coefficients of third, fourth and fifth terms in the expansion of $\left(x + \frac{a}{x^2}\right)^n$, $x \neq 0$, be in the ratio 12 : 8 : 3. Then the term independent of x in the expansion, is equal to _______.

Official Ans. by NTA (4)

7. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and $B = \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ such that AB = B and a + d = 2021, then the value of ad – bc is equal to ________.

Official Ans. by NTA (2020)

8. Let $\vec{x}$ be a vector in the plane containing vectors $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - \hat{k}$. If the vector $\vec{x}$ is perpendicular to $(3\hat{i} + 2\hat{j} - \hat{k})$ and its projection on $\vec{a}$ is $\frac{17\sqrt{6}}{2}$, then the value of $|\vec{x}|^2$ is equal to ________.

Official Ans. by NTA (486)

9. Let $I_n = \int_1^e x^{19}(\log|x|)^n dx$, where $n \in N$. If $(20)I_{10} = \alpha I_9 + \beta I_8$, for natural numbers α and β, then $\alpha - \beta$ equal to _______.

Official Ans. by NTA (1)

10. Let P be an arbitrary point having sum of the squares of the distance from the planes $x + y + z = 0$, $lx - nz = 0$ and $x - 2y + z = 0$, equal to 9. If the locus of the point P is $x^2 + y^2 + z^2 = 9$, then the value of $l - n$ is equal to _______.

Official Ans. by NTA (0)