PUMDET-2019

Subject : MATHEMATICS

(Booklet Number)

Duration: 90 Minutes | Full Marks: 100

INSTRUCTIONS

1. All questions are of objective type having four answer options for each. Only one option is correct. Correct answer will carry full marks 2. In case of incorrect answer or any combination of more than one answer, ½ mark will be deducted.

2. Questions must be answered on OMR sheet by darkening the appropriate bubble marked A, B, C, or D.

3. Use only **Black/Blue ball point pen** to mark the answer by complete filling up of the respective bubbles.

4. Mark the answers only in the space provided. Do not make any stray mark on the OMR.

5. Write question booklet number and your roll number carefully in the specified locations of the **OMR**. Also fill appropriate bubbles.

6. Write your name (in block letter), name of the examination centre and put your full signature in appropriate boxes in the OMR.

7. The OMR is liable to become invalid if there is any mistake in filling the correct bubbles for question booklet number/roll number or if there is any discrepancy in the name/ signature of the candidate, name of the examination centre. The OMR may also become invalid due to folding or putting stray marks on it or any damage to it. The consequence of such invalidation due to incorrect marking or careless handling by the candidate will be sole responsibility of candidate.

8. Candidates are not allowed to carry any written or printed material, calculator, pen, docu-pen, log table, wristwatch, any communication device like mobile phones etc. inside the examination hall. Any candidate found with such items will be **reported against** and his/her candidature will be summarily cancelled.

9. Rough work must be done on the question paper itself. Additional blank pages are given in the question paper for rough work.

10. Hand over the OMR to the invigilator before leaving the Examination Hall.

MATHEMATICS

1. Let $\Delta = \begin{vmatrix} 2 & 1 & 4 & 5 \\ 4 & 2 & 5 & 7 \\ 7 & 8 & 9 & 4 \\ 5 & 7 & 8 & 2 \end{vmatrix}$, then

(A) Δ is divisible by 11
(B) Δ is not divisible by 11
(C) Δ is not divisible by 2
(D) Δ represents a prime integer

2. Let $A = \begin{pmatrix} 2 & 3 & 0 \\ 6 & 5 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, then

(A) A is not diagonalisable
(B) A is diagonalisable
(C) Eigen values of A are not distinct
(D) Eigen vectors are not linearly independent.

3. Let (G,) be a group and for elements a, b $\in$ G, a ρ b holds iff there exists an element $x \in G$ such that $a = x.b.x^{-1}$, then

(A) ρ is only reflexive relation
(B) ρ is only symmetric relation
(C) ρ is only transitive relation
(D) ρ is equivalence relation

4. Let S be the set of all continuous functions $\mathbb{R} \rightarrow \mathbb{R}$. Let F: S $\rightarrow \mathbb{R}$ be defined by

$F(g) = \int_0^1 g(x)\, dx$. Then

(A) F is bijective
(B) F is one-one but not onto
(C) F is onto but not one-one
(D) F is neither one-one nor onto

5. Let $\alpha_1, \alpha_2, \cdots \alpha_6$ are roots of $x^6 + x^2 + 1 = 0$. Then the value of $(1-2\alpha_1)(1-2\alpha_2)\cdots(1-2\alpha_6)$ is

(A) 0 (B) 1

(C) 64 (D) 81

6. Let $(G, \cdot)$ be a group of order 35. Then

(A) G cannot be cyclic (B) G is cyclic

(C) G is only abelian (D) G is not abelian

7. Let $S = \left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in \mathbb{R} - \{0\}\right\}$ be a group with respect to matrix multiplication. Then 'Inverse' of $A = \begin{pmatrix} 3 & 3 \\ 3 & 3 \end{pmatrix}$ is

(A) $\begin{pmatrix} \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{pmatrix}$ (B) $\begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} \end{pmatrix}$

(C) $\begin{pmatrix} \frac{1}{12} & \frac{1}{12} \\ \frac{1}{12} & \frac{1}{12} \end{pmatrix}$ (D) $\begin{pmatrix} \frac{1}{24} & \frac{1}{24} \\ \frac{1}{24} & \frac{1}{24} \end{pmatrix}$

8. Let $(F, +, \cdot)$ be a field and let $M = \left\{\begin{pmatrix} a & b \\ 0 & 0 \end{pmatrix} : a, b \in F\right\}$. Then

(A) M is a right ideal in F.

(B) M is a left ideal in F.

(C) M is neither left ideal nor a right ideal in F.

(D) M is both sided ideal in F.

9. Let $\varphi : S_n \to (\mathbb{R} - \{0\}, \cdot)$ be defined by

$$\varphi(f) = \begin{cases} 1, & \text{if n is even} \\ -1, & \text{if n is odd} \end{cases} \text{ for all } f \in S_n. \text{ Then}$$

(A) φ is isomorphism
(B) φ is homomorphism
(C) φ is not homomorphism
(D) φ is monomorphism

10. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ map (1, 1) to (2, −3) & (1, −1) to (4, 7). Then the matrix representation of T relative to the standard basis of $\mathbb{R}^2$ is

(A) $\begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$
(B) $\begin{pmatrix} 3 & -1 \\ 2 & -5 \end{pmatrix}$
(C) $\begin{pmatrix} 1 & 2 \\ 3 & -5 \end{pmatrix}$
(D) $\begin{pmatrix} 2 & 1 \\ 3 & -5 \end{pmatrix}$

11. Consider the groups Z_{12} and Z_{18} and their direct product $Z_{12} \times Z_{18}$. Then the order of the element (8, 15) in $Z_{12} \times Z_{18}$ is

(A) 3
(B) 2
(C) 6
(D) 1

12. A basis of the subspace w of $\mathbb{R}^3$ where $w = \{(x, y, z) \in \mathbb{R}^3 : x + 2y + z = 0, 2x + y + 3z = 0\}$ is

(A) (5, −1, −3)
(B) (1, 1, 1)
(C) (1, 2, 0)
(D) (2, 1, 0)

13. Consider the equation $6^x + 8^x = 10^x$, $x \in \mathbb{R}$. Then

(A) it has no real root
(B) it has exactly one real root
(C) it has at least one real root
(D) it has infinitely many real roots

14. Let $\alpha > 0$, $\beta \geq 0$ and $f : \mathbb{R} \to \mathbb{R}$ be continuous at 0 with $f(0) = \beta$. If $g(x) = |x|^{\alpha} f(x)$ is differentiable at 0, then

(A) $\alpha = 1, \beta = 1$ (B) $0 < \alpha < 1, \beta = 0$

(C) $\alpha \geq 1, \beta = 0$ (D) $\alpha > 0, \beta > 0$

15. Let $f''(x)$ exist in [a, b], $f(a) = f(b) = 0$ and for some $c \in (a, b)$, $f(c) > 0$. Then

(A) there exists at least one point $\xi \in (a, b)$ such that $f''(\xi) < 0$.

(B) there exists no point in (a, b) at which $f''(x)$ is negative

(C) $f''(x) > 0$ for all x in (a, b)

(D) $f'(x)$ does not vanish anywhere in (a, b)

16. Let $C^1(\mathbb{R})$ denote the set of all continuously differentiable real valued functions defined on the real line. Define

$$A = \left\{ f \in C^1(\mathbb{R}) \;\middle|\; f(0) = 0, f(1) = 1, |f'(x)| \leq \frac{1}{2} \text{ for all } x \in \mathbb{R} \right\}. \text{ Then}$$

(A) A is an empty set (B) A is finite & non-empty set

(C) A is infinite set (D) A is countable set

17. Let $f(x) = \begin{cases} 0, & -1 \leq x \leq 0 \\ 1, & 0 < x \leq 1 \end{cases}$. Then

(A) there exists a function F such that $F'(x) = f(x)$ in [–1, 1]

(B) there does not exist any function F such that $F'(x) = f(x)$ in [–1, 1]

(C) there exists two different unique functions F and G such that $F'(x) = f(x)$ in [–1, 0] and $G'(x) = f(x)$ in [0, 1]

(D) Rolle's theorem is applicable to f in [–1, 1]

18. Let $f : \mathbb{R} \to \mathbb{R}$ be differentiable for all x and let $f(0) = 0$. Then

(A) $f(x)$ is a polynomial in x (B) $f(x) < x$ for all $x > 0$

(C) $f(x) = e^x - 1$ for all x (D) $f(x) = \sinh x$ for all x

19. For $R > 0$, let $I(R) = \int_0^{\pi/2} e^{-R \sin x}\, dx$, $J(R) = \frac{\pi}{2R}(1 - e^{-R})$. Then

(A) $I(R) > J(R)$ (B) $I(R) < J(R)$

(C) $I(R) = J(R)$ (D) $I(R) + J(R) = 0$

20. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function. Let $\int_0^{2x} f(3t)\, dt = \frac{x}{\pi} \sin(\pi x)$. Then $f\left(\frac{1}{2}\right)$ is

(A) 1 (B) $\frac{6 + \pi\sqrt{3}}{24\pi}$

(C) $\frac{3 + 2\sqrt{3\pi}}{12}$ (D) $\frac{2 + \pi\sqrt{3}}{6}$

21. Let f be monotone function in $\left[0, \frac{\pi}{2}\right]$, positive valued. Let $I_n = \int_0^{\pi/2} f(x) \sin nx\, dx$. Then

(A) $\lim_{n \to \infty} I_n$ does not exist (B) $\lim_{n \to \infty} I_n = 0$

(C) $\lim_{n \to \infty} I_n$ is oscillatory (D) $\lim_{n \to \infty} I_n$ cannot be determined

22. Let $p(x) = a_k x^k + a_{k-1} x^{k-1} + ... + a_0$ be a polynomial. Then $\lim_{n \to \infty} n \int_0^1 x^n p(x)\, dx$ equals

(A) $p(1)$ (B) $p(0)$

(C) $p(1) - p(0)$ (D) ∞

23. Let f (x, y) and g (x, y) be homogeneous functions of degree 0, having continuous first order partial derivatives. Let $J = \begin{vmatrix} f_x & f_y \\ g_x & g_y \end{vmatrix}$. Then for all (x, y),

(A) $J > 0$

(B) $J < 0$

(C) $J = 0$

(D) J cannot be determined

24. If $f(x, y) = x^7 + 100x^5y^2 + 200xy^6 + 10y^7$, then $x^2\frac{\partial^2 f}{\partial x^2} + 2xy\frac{\partial^2 f}{\partial x \partial y} + y^2\frac{\partial^2 f}{\partial y^2}$ is =

(A) $42x^7 + 4200x^5y^2 + 8400xy^6 + 420y^7$

(B) $42x^7 + 500x^5y^2 + 200xy^6 + 420y^7$

(C) $42x^7 + 1000x^5y^2 + 1400xy^6 + 10y^7$

(D) $7x^7 + 700x^5y^2 + 1400xy^6 + 70y^7$

25. Choose the correct one :

(A) An unbounded sequence in $\mathbb{R}$ has no convergent subsequence.

(B) An unbounded sequence in $\mathbb{R}$ may have convergent subsequence.

(C) If any two sub-sequences of a bounded sequence in $\mathbb{R}$ converge to same limit, then the sequence is convergent.

(D) A convergent sequence in $\mathbb{R}$ may have distinct subsequences converging to different limits.

26. The radius of convergence of the power series $\sum_{n} \{3 + (-1)^n\}^n \sin\frac{1}{n} x^n$ is

(A) 1

(B) $\frac{1}{2}$

(C) ∞

(D) $\frac{1}{4}$

27. Let $f(x) = \log(1+x)$, $I(x) = x - \frac{x^2}{2}$, $J(x) = x - \frac{2x^2}{9}$, $0 \le x \le \frac{1}{2}$, then

(A) $J(x) < f(x) < I(x)$ in $\left[0, \frac{1}{2}\right]$

(B) $I(x) < f(x) < J(x)$ in $\left[0, \frac{1}{2}\right]$

(C) $I(x) < f(x)$, $f(x) > J(x)$ in $\left[0, \frac{1}{2}\right]$

(D) $f(x) < I(x)$, $f(x) < J(x)$ in $\left[0, \frac{1}{2}\right]$

28. Let $f'(x)$ exist and be monotone on an open interval (a, b). Then

(A) f′ may not be continuous on (a, b)

(B) f′, at best, be piecewise continuous on (a, b)

(C) f′ is continuous on (a, b)

(D) f′ has infinite discontinuity on (a, b)

29. The area of the region bounded by the curve $y = x^2 - 4x$ and $x + y = 0$ is

(A) 9 sq. unit

(B) $\frac{9}{2}$ sq. unit

(C) 2 sq. unit

(D) 18 sq. unit

30. Let $f : [0,1] \to \mathbb{R}$ be bounded in [0, 1]. Consider the series $\sum_{n=0}^{\infty} n\, x^n f(x)$. Then the series

(A) is not even pointwise convergent in any sub-interval of [0, 1]

(B) is only pointwise convergent in every sub-interval of [0, 1]

(C) is uniformly convergent in every compact sub-interval [0, r], $0 < r < 1$

(D) is divergent throughout the interval

31. Let $u(x, y) = x^2 - y^2$, $v(x, y) = \frac{y}{x^2 + y^2}$ and $f(z) = u + iv$. Then

(A) f (z) is analytic function

(B) f (z) is not an analytic function

(C) CR equations do hold everywhere

(D) f (z) is differentiable everywhere

32. Let $f(z) = u(x, y) + iv(x, y)$ be analytic function. Let $v(x, y) = e^x(x \sin y + y \cos y)$. Then f (z) is

(A) $ze^z + c$, where c is suitable constant

(B) $e^z + c$, where c is suitable constant

(C) $z^2e^z + c$, where c is suitable constant

(D) $e^{z^2} + c$, where c is suitable constant

33. Let f (z) = Re z. Then

(A) f is discontinuous everywhere on $\mathbb{C}$

(B) f is differentiable everywhere on $\mathbb{C}$

(C) f is differentiable at no point of $\mathbb{C}$

(D) f is analytic on $\mathbb{C}$

34. Consider the series $\sum_{n\geq 1} n^{-z}$, then it

(A) is nowhere convergent

(B) is everywhere convergent on $\mathbb{C}$

(C) converges in $D = \{z \in \mathbb{C}, \text{Re } z > 1\}$

(D) converges in $E = \left\{ z \in \mathbb{C}, \text{Re } z < \frac{1}{2} \right\}$

35. Given that C [a, b] denote the set of all real valued continuous functions on [a, b] and a metric d on C [a, b] be defined by $d(f, g) = \sup\{|f(x) - g(x)| : a \leq x \leq b\}$. Then (C[a, b], d) is

(A) incomplete metric space

(B) complete metric space

(C) such that all Cauchy sequences do not have convergent sub sequences

(D) such that Cauchy sequences have at best only bounded sub sequences

36. Let X denote the set C [0, 1] of all real valued continuous functions on [0, 1] & the metric d on X be defined by $d(x, y) = \int_0^1 |x(t) - y(t)|\, dt$ for all $x, y \in X$. Let $x_n : [0, 1] \to \mathbb{R}$ be given by

$$x_n(t) = \begin{cases} n, \text{if } 0 \leq t \leq \frac{1}{n^2} \\ \frac{1}{\sqrt{t}}, \text{if } \frac{1}{n^2} \leq t \leq 1 \end{cases}. \text{ Then}$$

(A) $\{x_n\}_n$ is not Cauchy in X

(B) $\{x_n\}_n$ is Cauchy and convergent in X

(C) $\{x_n\}_n$ is Cauchy but not convergent in X

(D) $\{x_n\}_n$ is only bounded in X

37. Let $d : \mathbb{Z} \times \mathbb{Z} \to \mathbb{R}$ be defined by

$$d(x, y) = \begin{cases} 0, & \text{if } x = y \\ m^{-1}, & \text{where m is the positive integer such that } 10^{m-1} \text{ divide } (x - y) \\ & \text{but } 10^{m} \text{ does not divide } (x - y) \end{cases}$$

(A) d defines a metric on $\mathbb{R}$

(B) d does not obey the triangle property

(C) d is not well-defined

(D) Symmetry axiom is not tenable

38. Let ΔABC be a right-angled triangle with $\angle ABC = 90°$. Let P be the mid-point of BC and Q be a point on AB. Suppose that the length of BC is $2x$ (unit), $\angle ACQ = \alpha$, $\angle APQ = \beta$. Then the length of AQ is

(A) $\dfrac{3x}{2\cot\alpha - \cot\beta}$

(B) $\dfrac{2x}{3\cot\alpha - 2\cot\beta}$

(C) $\dfrac{3x}{\cot\alpha - 2\cot\beta}$

(D) $\dfrac{2x}{2\cot\alpha - 3\cot\beta}$

39. The vertex and the length of Latusrectum of the parabola

$x^2 - 2xy + y^2 - 2ax - 2ay + a^2 = 0$ $(a > 0)$ are given respectively by

(A) $\left(\frac{a}{2}, \frac{a}{2}\right)$, 2a unit

(B) $\left(\frac{a}{4}, \frac{a}{4}\right)$, $a\sqrt{2}$ unit

(C) (a, a), 4a unit

(D) (2a, 2a), 4a unit

40. The equation of the plane which touches the cone $x^2 + 4y^2 - 5z^2 + 4xy - 6yz + 2xz = 0$ along the generator whose direction vector is (1, 1, 1) is

(A) $4x + 3y - 7z = 0$

(B) $4x + 3y + 7z = 0$

(C) $3x - 4y - 8z = 0$

(D) $3x + 4y - 7z = 0$

41. A variable point P is such that its distance from the y-axis is always equal to its distance from the plane $x - z = 1$. The locus of P is

(A) $x^2 + z^2 + 2xz + 2x - 2z - 1 = 0$

(B) $x^2 + z^2 + 2xz + 2x + 2z + 1 = 0$

(C) $x^2 + z^2 + 2xz - 2x - 2z - 1 = 0$

(D) $x^2 + z^2 - 2xz - 2x - 2z - 1 = 0$

42. The equation of the cylinder with guiding curve $2y^2 + 3z^2 = 1, x = 0$ and generators parallel to $x = y = z$ is

(A) $5x^2 + 2y^2 + 3z^2 + 4xy + 6xz + 1 = 0$

(B) $5x^2 + 2y^2 - 3z^2 + 4xy + 6zx + 1 = 0$

(C) $5x^2 + 2y^2 + 3z^2 - 4xy + 6zx + 1 = 0$

(D) $5x^2 + 2y^2 + 3z^2 - 4xy - 6zx - 1 = 0$

43. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $r = |\vec{r}|$ and $\Phi(x, y, z) = (x^2 + y^2 + z^2)e^{-\sqrt{x^2+y^2+z^2}}$, then $\vec{\nabla}\Phi$ equals to

(A) $(2 - r)e^{-r}\vec{r}$

(B) $re^{-r}\vec{r}$

(C) r^2e^{-r}

(D) $e^{-r}\vec{r}$

44. For a vector $\vec{\alpha} = (2x - yz)\hat{i} + (2y - zx)\hat{j} + (2z - xy)\hat{k}$, a scalar function Φ, such that $\vec{\alpha} = \text{grad}\,\Phi$, is

(A) $x^2 + y^2 + z^2 + c$, where c is any constant

(B) $x^2 + y^2 + z^2 - xyz + c$, where c is any constant

(C) $x + y + z + c$, where c is any constant

(D) $xy + yz + zx + c$, where c is any constant

45. General solutions of $(D^2 - 3D + 2)y = e^{3x}$ is

(A) $y = Ae^{2x} + Be^{x} + \frac{1}{2}e^{3x}$, where A, B are arbitrary constants

(B) $y = Ae^{3x} + Be^{-x} + \frac{1}{2}e^{3x}$, where A, B are arbitrary constants

(C) $y = Ae^{2x} + Be^{x} + \frac{1}{3}e^{3x}$, where A, B are arbitrary constants

(D) $y = Ae^{2x} + Be^{x} + e^{3x}$, where A, B are arbitrary constants

46. One root of $e^x - 3x^2 = 0$ lies in (3, 4). The least number of iterations of the bisection method so that |error| is $\le 10^{-3}$

(A) 9

(B) 10

(C) 11

(D) 12

47. The values of the constants α, β, γ such that the formula for the first derivative $f'(x) = \alpha f(x - h) + \beta f(x) + \gamma f(x + h)$ gives exact values for all polynomials of maximum degree two are respectively

(A) $-\frac{1}{h}, 0, \frac{1}{2h}$

(B) $-\frac{1}{2h}, 0, \frac{1}{2h}$

(C) $-\frac{1}{h}, 0, \frac{1}{h}$

(D) $-\frac{1}{2h}, 0, \frac{1}{h}$

48. The iteration scheme $x_{n+1} = \frac{1}{2} x_n \left(1 + \frac{a}{x_n^2}\right)$ converges to $\sqrt{a}$. The order of convergence is

(A) 1 (B) 2

(C) greater than 2 (D) between 1 and 2

49. The random variable x takes the values 0, 1, 2, 3 and its mean is $\frac{32}{15}$. If $P(x = 0) = \frac{2}{15}$ and $P(x = 1) = \frac{1}{5}$, then $P(x \le 2)$ is

(A) $\frac{2}{5}$ (B) $\frac{1}{15}$

(C) $\frac{3}{5}$ (D) $\frac{2}{3}$

50. In a simple harmonic motion, the distance of a particle from the midpoint of its path at three consecutive seconds are x, y, z. The time period of a complete oscillation is

(A) $2\pi \cos^{-1}\left(\frac{x+z}{2y}\right)$ (B) $2\pi \sin^{-1}\left(\frac{x+z}{2y}\right)$

(C) $\frac{2\pi}{\cos^{-1}\left(\frac{x+z}{2y}\right)}$ (D) $\frac{2\pi}{\sin^{-1}\left(\frac{x+z}{2y}\right)}$