CS Group (Computer Science)

C1. (i) Consider the two C programs given below.

C code (I)

```
#include<stdio.h>
int main() {
    int n=2, *ptr=&n; n*= 3;
    printf("%d", (*ptr**ptr)*(*ptr**ptr));
    }
```

C code (II)

```
#include<stdio.h>
int main() {
    int n=2, *ptr=&n; n*= 3;
    printf("%d", *ptr**ptr**ptr**ptr);
    }
```

Given C codes (I) and (II) above, decide which of the following statements is true.

(a) Output of (I) is 36 and (II) is 216
(b) Output of (I) is 216 and (II) is 1296
(c) Output of (I) is 1296 and (II) is 216
(d) Output of both (I) and (II) is 1296
(e) None of the above

(ii) What will be the output of the following C program? Justify your answer. Negative numbers are represented in 2's complement.

```
#include<stdio.h>
int main() {
    if(- ~ -1)   printf("COVID");
    if((~7 & 0x000f) == 8)   printf("19");
    printf("*");
    }
```

(iii) What does the following function compute for x≠0?

```
float isi1(float x, int y) {
    if (y == 0){ return 1; }
    else if (y > 0) {return isi1(x,-y);}
    else {return isi1(x,y+1)/x;}
    }
```

$[3 + 3 + 4 = 10]$

C2. An $n \times n$ binary matrix M is called a NICE matrix, if each row of M has exactly one non-zero element and each column also has exactly one non-zero element.

(i) Suggest a method of storing a NICE matrix in an $O(n)$ size array.

(ii) Design an $O(n)$ time algorithm (in pseudocode) for computing $R = PQ$, where P and Q are both NICE matrices each stored in an array of size $O(n)$ as in (i).

[$4 + 6 = 10$]

C3. You are given two sorted arrays $X[\ ]$ and $Y[\ ]$ of positive integers. The array sizes are not given. Accessing any index beyond the last element of the arrays returns -1. The elements in each array are distinct but the two arrays may have common elements. An intersection point between the two arrays is an element common to both the arrays, i.e., p ($p > 0$) be an intersection point if there exist i, j such that $X[i] = Y[j] = p$.

Given a positive integer q, design an algorithm (in pseudocode) to check if q is an intersection point between X and Y. No marks will be awarded if the time complexity of your algorithm is linear (or higher) in the maximum size of X and Y.

[10]

C4. Given a graph G and a vertex u in it, let $N(u)$ denote the set of neighbours of u in G. A graph G having n vertices is said to be k-*degenerate* if there is a linear ordering $v_1, v_2, \ldots, v_n$ of the vertices, in which each vertex has at most k neighbours after it. That is, for all $i \in \{1, 2, \ldots, n\}$, $|\{v_j \in N(v_i) \colon j > i\}| \leq k$.

(i) Calculate the maximum number of edges possible in a k-degenerate graph having n vertices.

(ii) Let H be any graph. Let H' be the graph obtained by removing a vertex of degree at most k from H. Prove that H is k-degenerate if and only if H' is k-degenerate.

[$5 + 5 = 10$]

C5. In a conference, the relation, `registered` (`participant`, `topic`) stores the names of participants and the topics registered by them. The primary key for this relation is (`participant`, `topic`). The relation, `fee` (`participant`, `amount`) stores the fees paid by participants and the primary key for this relation is `participant`. There are five categories of participants and fees paid under each category are Rs. 1000, Rs. 2000, Rs. 3000, Rs. 4000, and Rs. 5000. Assume that each category has equal number of participants.

Now consider the following query:
List all topics registered by participants who have paid more than a given amount x.
To execute the above query, there may be two strategies as follows.

Strategy I:
```
create table r1 as select * from fee where amount > x;
select distinct topic
from registered as R, r1 as T
where R.participant = T.participant;
```

Strategy II:
```
select distinct topic
from registered as R, fee as T
where R.participant = T.participant and amount > x;
```

Assume that there are no null values in the tables.

(a) Which strategy is faster for $x = 3000$? Justify your answer.

(b) Which strategy has less disk access time? Justify your answer.

[$7 + 3 = 10$]

C6. Let $L = \{s\overline{s_R} \mid s \in \{0,1\}^*\}$ be a language over alphabet $\{0,1\}$, where $\overline{s_R}$ describes the reverse complement of s. For an s, $\overline{s_R}$ is obtained by reversing the order of symbols in s and then replacing every 0 with 1 and every 1 with 0. As for example, if $s = 1101$, the reverse order of s is 1011 and the reverse complement $\overline{s_R}$ is 0100.

(i) Give a context-free grammar which generates L.

(ii) Draw a Pushdown Automata that recognizes L.

(iii) Is L a regular language? Justify your answer.

[$3 + 3 + 4 = 10$]

C7. Consider two hosts, **A** and **B**, which may belong to the same network or two different networks connected through a router. Recall the Internet protocol where a sender host sends packets directly to a destination if the sender finds the network ID of the destination host same as that of its own; otherwise, the sender sends packets to the router.

Let the IP addresses of **A** and **B** be 204.2.3.12 and 204.2.3.131, respectively. If the subnet mask of **A** is 255.255.255.128, which one of the following two options will **A** follow while communicating with **B**?

(i) **A** sends packet to **B** directly,

(ii) **A** sends packet to **B** through the router.

Explain your answer (no marks will be awarded if justification is not given).

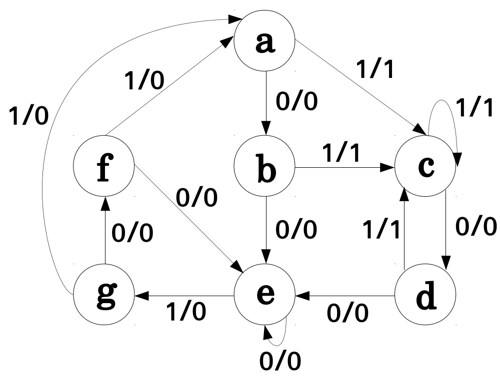
[Use: $(204)_{10} = (11001100)_2$; $(131)_{10} = (10000011)_2$]

[10]

C8. (i) Simplify the following Boolean function in product-of-sums form:

$$F(A, B, C, D) = \sum(0, 1, 2, 5, 8, 9, 10).$$

(ii) Consider the following state diagram of a sequential circuit, where each of **a**, **b**, **c**, **d**, **e**, **f** and **g** represents a state. Represent the



state diagram with minimum number of states without altering the input-output relationships. Justify your answer.

[$4 + (4 + 2) = 10$]

C9. Consider a RISC machine where page tables of the virtual memory system may be swapped to disk from memory and neither machine instructions nor page-table entries nor data words can cross a page boundary. Now recall the load word (`LW`) instruction of a RISC computer: `LW  Reg-A`, `Reg-B`, `Imme`. The instruction loads value from memory into `Reg-A` where memory address is formed by adding `Imme` with contents of `Reg-B`, i.e.,
`Reg-A` ← **Mem**[`Reg-B` + `Imme`].

Calculate how many page faults, in the worst case, could be generated as a result of the `fetch`, `decode`, and `execution` of an `LW` instruction in the above RISC machine? Explain your answer (no marks will be awarded if justification is not given).

[10]

C10. Suppose instead of a decoder with n input bits (n is even) to access a memory of size 2^n, one uses two decoders of input sizes k bits and $(n-k)$ bits. Explain how these two decoders can be used to access the memory of size 2^n.

Determine the value of k that achieves minimum address decoding time. Justify your answer. Assume that the time complexity of the decoder is measured by the number of output lines of that decoder.

$[6 + 4 = 10]$

Non-CS Group (Mathematics)

NC1. Let $\epsilon > 0$. Prove that there exists $n_0 \in \mathbb{N}$ such that

$$n \geq n_0 \Rightarrow 2 - \epsilon < \frac{2n+1}{n+2} < 2 + \epsilon.$$

[6]

NC2. Show that the sequence $\{x_n\}$, $n > 0$, defined by

$$x_n = \int_1^n \frac{cos(t)}{t^2} dt$$

is convergent.

[8]

NC3. Suppose A is an $(n \times n)$ matrix over $\mathbb{R}$ such that $A^p = 0$ for some positive integer p.

(a) Prove that $I + A$ is an invertible matrix, where I is the $(n \times n)$ identity matrix.

(b) Find the characteristic polynomial of A.

[$5 + 7 = 12$]

NC4. Let c be a positive real number for which the equation

$$x^4 - x^3 + x^2 - (c+1)x - (c^2 + c) = 0$$

has a real root α. Prove that $c = \alpha^2 - \alpha$.

[10]

NC5. (i) Prove that if N and K are normal subgroups of a group G such that $N \bigcap K = \{e_G\}$, then $xy = yx$, $\forall\, x \in N, \forall\, y \in K$.

(ii) Deduce that if N, H, K are normal subgroups of a group G such that

$$N \bigcap H = N \bigcap K = H \bigcap K = \{e_G\}$$

and $G = HK$, then N is an Abelian group.

[$5 + 9 = 14$]

NC6. Find all possible integers n for which $n^2 + 20n + 15$ is a perfect square.

[10]

NC7. Let $A = \{1, 2, 3, \cdots, 50\}$. In how many ways can three distinct numbers $x < y < z$ be chosen from A such that the product xyz is divisible by 125?

[10]

NC8. Prove that the function defined by

$$f(x) = \sum_{n=0}^{\infty} \left(\frac{x^n}{n!} \right)^2$$

is continuous on $\mathbb{R}$, for any real number x.

[10]

NC9. Let $X_i \sim (i.\ i.\ d.)$ Bernoulli $(\frac{\lambda}{n})$, $n \geq \lambda \geq 0$.

$Y_i \sim (i.\ i.\ d.)$ Poisson $(\frac{\lambda}{n})$, $\{X_i\}$ and $\{Y_i\}$ are independent.

Let $\sum_{i=1}^{n^2} X_i = T_n$ and $\sum_{i=1}^{n^2} Y_i = S_n$ (say).

Find the limiting distribution of $\frac{T_n}{S_n}$ as $n \to \infty$.

[10]

NC10. Prove that if $T_1, T_2, \ldots, T_k$ are pairwise-intersecting subtrees of a tree T, then T has a vertex that belongs to all of $T_1, T_2, \ldots, T_k$.

[10]