1 If element of matrix A is defined as $A = [a_{ij}]_{3\times 3}$ where $A = \begin{cases} (-1)^{j-i} & i < j \\ 2 & i = j \\ (-1)^{i+j} & i > j \end{cases}$, then the value of $|3\text{Adj}(2A^{-1})|$ is:

(1) 72 (2) 36 (3) 108 (4) 48

Ans. (3)

Sol. $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

So, $|A| = \begin{vmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix}$

$= 2(4-1) + 1(-2+1) + 1(1-2)$

$= 2(3) + 1(-1) + 1(-1)$

$= 4$

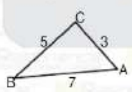
$|3\text{Adj}(2A^{-1})| = 3^3 |\text{Adj}(2A^{-1})| = 3^3 \times |2A^{-1}|^2$

$= 3^3 \times 2^6 \times |A^{-1}|^2 = 3^3 \times 2^6 \times \frac{1}{|A|^2} = 108$

2. In a ΔABC, if $|\overrightarrow{AB}| = 7$, $|\overrightarrow{BC}| = 5$, and $|\overrightarrow{CA}| = 3$. If projection of $\overline{BC}$ on $\overline{CA}$ is $\left(\frac{n}{2}\right)$, then the value of n is:

Ans. 05.00

Sol. $|\overrightarrow{AB}| = 7$, $|\overrightarrow{BC}| = 5$, $|\overrightarrow{CA}| = 3$,



Projection of $\overrightarrow{BC}$ on $\overrightarrow{CA}$ is $= |\overrightarrow{BC}| \cos\angle BCA$

$5\left|\left(\frac{3^2+5^2-7^2}{2.3.5}\right)\right| = = 5\left|\frac{-15}{30}\right| = \frac{5}{2}$

3. The value of $\tan(2\tan^{-1}(3/5) + \sin^{-1}(5/13))$ is :

(1) $\frac{220}{21}$ (2) $\frac{110}{21}$ (3) $\frac{55}{21}$ (4) $\frac{20}{11}$

Ans. (1)

Sol. $\tan\left(\tan^{-1}\frac{\frac{6}{5}}{1-\frac{9}{25}} + \tan^{-1}\frac{5}{12}\right)$

$\tan\left(\tan^{-1}\left(\frac{15}{8}\right) + \tan^{-1}\left(\frac{5}{12}\right)\right) = \frac{\frac{15}{8}+\frac{5}{12}}{1-\frac{15}{8}\cdot\frac{5}{12}} = \frac{220}{21}$

4. Mean of 6 observations is 10 and their variance is $\frac{20}{3}$. If observations are 15, 11, 10, 7, a, b then $|a - b|$ is equal to :

(1) 2 (2) 1 (3) 3 (4) 4

Ans. (2)

Sol. Mean = 10

$\frac{7+10+11+15+a+b}{6} = 10$

$a + b = 17$ ....(1)

Variance = $\frac{20}{3}$

$\frac{49+100+121+225+a^2+b^2}{6} - 100 = \frac{20}{3}$

$a^2 + b^2 = 145$ ....(2)

$(a + b)^2 = 289$

$ab = 72$

$(a - b)^2 = (a + b)^2 - 4ab$

$(a - b)^2 = 289 - 288 = 1$

$|a - b| = 1$

5. If $f(x) = x + 1$, then find $\lim_{n\to\infty}\frac{1}{n}\left[f(0)+f\left(\frac{5}{n}\right)+f\left(\frac{10}{n}\right)+..........+f\left(\frac{5(n-1)}{n}\right)\right]$

(1) $\frac{7}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{1}{2}$

Ans. (1)

6. Sum of 21 terms of series $\log_{9^{1/2}} x + \log_{9^{1/3}} x + \log_{9^{1/4}} x + \ldots\ldots$ is 252, then the value of x is :

(1) 7 (2) 243 (3) 9 (4) 81

Ans. (3)

Sol. $2\log_9 x + 3\log_9 x + 4\log_9 x \ldots\ldots 21$ terms

$= (2+3+4+5+\ldots\ldots+22)\log_9 x = \frac{21}{2}(2+22)\log_9 x$

$= 21 \times 12 \log_9 x$

$= 252 \log_9 x$

Given sum $= 252 \Rightarrow \log_9 x = 1$

$\Rightarrow x = 9$

7. $\int_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} ([x] - [\sin x])\,dx = ?$ (Where [·] represents G.I.F.)

(1) –2 (2) 1 (3) 0 (4) –1

Ans. (3)

Sol. $I = \int_{-\frac{3\pi}{2}}^{\frac{3\pi}{2}} ([x] - [\sin x])\,dx$

using property $\int_{-a}^{a} f(x)\,dx = \int_0^a f(x)\,dx + \int_0^a f(-x)\,dx$

$I = \int_0^{\frac{3\pi}{2}} ([x] + [-x])\,dx - \int_0^{\frac{3\pi}{2}} ([\sin x] + [-\sin x])\,dx = 0$

8. If $\lim_{x \to 0} \frac{\alpha x e^x - \beta \ln(1+x) + \gamma x^2 e^{-x}}{x^3} = 10$, then the value of $\alpha + \beta + \gamma$ is :

Ans. (3)

$\alpha x\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\right) - \beta\left(x - \frac{x^2}{2} + \frac{x^3}{3} + \ldots\right) + \gamma x^2\left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots\right)$

9. The value of x satisfying the equation $\log_{(x+1)}(2x^2+7x+5)+\log_{(2x+5)}(x+1)^2=4$ is :

(1) –2 (2) 2 (3) –4 (4) 4

Ans. (2)

Sol. $\log_{(x+1)}((2x+5)(x+1))+\log_{(2x+5)}(x+1)^2=4$

$1+\log_{(x+1)}(2x+5)+2\log_{(2x+5)}(x+1)=4$

Put $\log_{(x+1)}(2x+5)=t$

$\therefore 1+t+\frac{2}{t}=4$

$t^2+t+2=4t \Rightarrow t^2-3t+2=0$

$t=1, t=2$

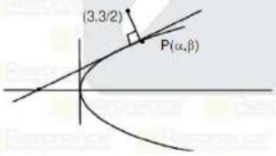
For $t=1$	For $t=2$
$2x+5=x+1$	$2x+5=(x+1)^2$
$\Rightarrow x=-4$ (rejected)	$x=2, x=-2$ (rejected)

10. If (α, β) is the point on $y^2=6x$, that is closest to $\left(3,\frac{3}{2}\right)$ then find $2(\alpha+\beta)$

(1) 6 (2) 9 (3) 7 (4) 5

Ans. (2)

Sol.



$y^2=6x$

$2yy'=6$

$\frac{dy}{dx}=\frac{3}{\beta}$

$-\frac{\beta}{3}=\frac{\beta-3/2}{\alpha-3}$

11. Two circles pass through (–1,4) and their centres lie on $x^2 + y^2 + 2x + 4y = 4$ r. If r_1 and r_2 are maximum 4 minimum radii and $\frac{r_1}{r_2} = a + b\sqrt{2}$ then the value of a + b is

Ans. 3

Sol. Given circle

$(x + 1)^2 + (y + 2)^2 = (3)^2$

any point on this circle is (3cosθ – 1, 3sinθ2) equation of circle having centre (3cosθ – 1, 3sinθ –2)

$\Rightarrow$ radius of circle is

$r = \sqrt{(3\cos\theta - 1 + 1)^2 + (3\sin\theta - 2 - 4)^2}$

$= \sqrt{9\cos^2\theta + 9\sin^2\theta + 36 - 36\sin\theta}$

$\sqrt{45 - 36\sin\theta}$

$\Rightarrow r_{max} = 9 = r_1$ and $r_{min} = 3 = r_2$

$\Rightarrow \frac{r_1}{r_2} = \frac{9}{3} = 3 \; 3 + 0\sqrt{2}$

$\Rightarrow a + b = 3$

12. If ΔABC is right angled triangle with sides a,b & c and smallest angle θ. If $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ are also the sides of right angled triangle then find sin θ

(1) $\sqrt{\frac{3-\sqrt{5}}{2}}$ (2) $\frac{3-\sqrt{5}}{2}$ (3) $\sqrt{\frac{3+\sqrt{5}}{2}}$ (4) $\frac{3+\sqrt{5}}{2}$

$\text{cosec}^2\theta - 1$

13. If Re $[(1+\cos\theta + 2i\sin\theta)^{-1}] = 4$ then value of θ is :

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $-\frac{\pi}{2}$ (4) π

Ans. (4)

Sol. $\frac{1}{1+\cos^2\theta+2i\sin\theta} \times \frac{1+\cos\theta-2i\sin\theta}{1+\cos\theta-2i\sin\theta}$

$= \frac{1+\cos\theta-2i\sin\theta}{(1+\cos\theta)^2+4\sin^2\theta}$

$\Rightarrow \frac{1+\cos\theta}{1+\cos^2\theta+2\cos\theta+4\sin^2\theta} = 4$

$\Rightarrow \frac{1+\cos\theta}{1+\cos^2\theta+2\cos\theta+4-4\cos^2\theta} = 4$

$\Rightarrow \frac{1+\cos\theta}{5+2\cos\theta-3\cos^2\theta} = 4$

$\Rightarrow 1 + \cos\theta = 20 + 8\cos\theta - 12\cos^2\theta$

$\Rightarrow 12\cos^2\theta - 7\cos\theta - 19 = 0$

$\Rightarrow 12\cos^2\theta - 19\cos\theta + 12\cos\theta - 19 = 0$

$\Rightarrow \dfrac{1+\cos\theta}{5+2\cos\theta-3\cos^2\theta} = 4$

$\Rightarrow 1 + \cos\theta = 20 + 8\cos\theta - 12\cos^2\theta$

$\Rightarrow 12\cos^2\theta - 7\cos\theta - 19 = 0$

$\Rightarrow 12\cos^2\theta - 19\cos\theta + 12\cos\theta - 19 = 0$

$\Rightarrow \cos\theta(12\cos\theta - 19) + 1\,(12\cos\theta - 19) = 0$

$\Rightarrow \cos\theta = -1$ or $\cos\theta = \dfrac{19}{12}$ (rejected)

$\Rightarrow \theta = \pi$

14. If $x = ay - 1 = z - 2$, and $x = 3y - 2 = bz - 2$ lie in same plane then the value of a, b, is

(1) $a = 2, b = 3$ (2) $a = 1, b = 1$ (3) $b = 1, a \in R - \{0\}$ (4) $a = 3, b = 2$

Ans. (3)

Sol. $\dfrac{x}{1} = \dfrac{y-\frac{1}{a}}{\frac{1}{a}} = \dfrac{z-2}{1}$, $\quad x = \dfrac{y-\frac{2}{3}}{\frac{1}{3}} = \dfrac{z-\frac{2}{b}}{\frac{1}{b}}$

$(\vec{a}_1 - \vec{a}_2)\cdot(\vec{b}_1 \times \vec{b}_2) = 0$

$$\begin{vmatrix} 0 & \frac{1}{a}-\frac{2}{3} & 2-\frac{2}{b} \\ 1 & \frac{1}{a} & 1 \\ 1 & \frac{1}{3} & \frac{1}{b} \end{vmatrix} = 0$$

$\Rightarrow \dfrac{1}{ab} - \dfrac{1}{a} = 0$

$b = 1,\ a \in R - \{0\}$

15. If $P(\bar{A} \cap B) + P(A \cap \bar{B}) = 1 - K$

$P(\bar{A} \cap C) + P(A \cap \bar{C}) = 1 - 2K$

15. If $P(\overline{A}\cap B)+P(A\cap\overline{B})=1-K$

$P(\overline{A}\cap C)+P(A\cap\overline{C})=1-2K$

$P(\overline{B}\cap C)+P(\overline{B}\cap\overline{C})=1-K$

$P(A\cap B\cap C)=K^2, K\in(0,1)$

Then the value of P(at least one of A,B,C) is:

(1) $>\frac{1}{2}$ (2) $\left[\frac{1}{8},\frac{1}{4}\right]$ (3) $<\frac{1}{4}$ (4) $\frac{1}{4}$

Ans. (1)

Sol. $P(A) + P(B) - 2P(A\cap B) = 1 - K$

$P(A) + P(C) - 2P(A\cap C) = 1 - 2K$

$P(B) + P(C) - 2P(B\cap C) = 1 - K$

$P(A\cup B\cup C) = P(A) + P(B) + P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C)$

$= \frac{3-4k}{2} + k^2 = \frac{2k^2-4k+3}{2}$

$\because$ The value of $2k^2 - 4k + 3$ is greater than 1

$\therefore\ P(A\cup B\cup C) > \frac{1}{2}$

16. If $f(x) = \dfrac{5x+3}{6x+a}$ and $f(f(x)) = x$ then the value of a is :

(1) –5 (2) 5 (3) 6 (4) –6

Ans. (1)

Sol. $f(f(x)) = \dfrac{5f(x)+3}{6f(x)+a} = x \Rightarrow 5f(x) + 3 = 6x\, f(x) + ax$

$\Rightarrow \dfrac{25x+15}{6x+a} + 3 = 6x\left(\dfrac{5x+3}{6x+a}\right) + ax$

$\Rightarrow 25x + 15 + 18x + 3a = 30x^2 + 18x + 6ax^2 + a^2x$

$\Rightarrow (30+6a)x^2 + (a^2 - 25)\,x - (3a+15) = 0$

$\Rightarrow 6(a+5)x^2 + (a-5)(a+5)x - 3(a+5) = 0, \quad \forall x$

$\Rightarrow a+5 = 0 \Rightarrow a = -5$

17. If $g(t) = \begin{cases} \max(t^3 - 6t^2 + 9t - 3, 0), & t \in [0,3] \\ 4 - t, & t \in (3,4] \end{cases}$ then the number of points at which g(t) is non differentiable is :

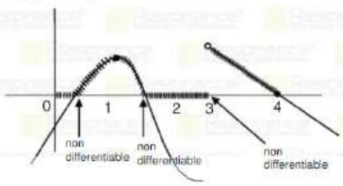
(1) 1 (2) 3 (3) 2 (4) 4

Ans. (2)

Sol. $y = t^3 - 6t^2 + 9t - 3$

$y' = 3t^2 - 12t + 9$

$= 3(t^2 - 4t + 3)$



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18. A : if 2 + 4 = 7, then 3+4 = 8

B : if 3 + 5 = 8, then earth is flat

C : if A and B are true, then 5+4 = 11

(1) A is true, B and C are false
(2) B is true, A and C are false
(3) C is true, A and B are false
(4) B is false, A and C are true

Ans. (4)

Sol. Truth table $p \to q$

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

A is true, B is false, C is true .

19. If $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B \sum_{r=1}^{2021} A^r$ then value of $|B|$ is

(1) 2021
(2) $(2021)^2$
(3) -2021
(4) 0

Ans. (2)

Sol. $A = I$, $B = I + I + \ldots\ldots$ 2021 times

$B = 2021 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$,

$|B| = (2021)^2$