1 All possible words with or without meaning were formed using all the letters of the word 'EXAMINATION'. The probability that 'M' appears at fourth position is :

(1) $\frac{2}{11}$ (2) $\frac{1}{11}$ (3) $\frac{4}{11}$ (4) $\frac{8}{11}$

Ans. (2)

Sol. EXAMINATION

$E \rightarrow 1$

$X \rightarrow 1$

$A \rightarrow 2$

$M \rightarrow 1$

$O \rightarrow 1$

$T \rightarrow 1$

$N \rightarrow 2$

$I \rightarrow 2$

$$n(S) = \frac{11!}{2!\,2!2!}$$

$$n(E) = \frac{10!}{2!\,2!2!}$$

$$P(E) = \frac{n(E)}{n(s)} = \frac{1}{11}$$

2 If a cricket team consist of 15 players have 6 batsmen , 7 Ballers and 2 wicket keepers then the number of ways in which cricket team formed with atleast 4 batsmen, 5 ballers and 1 wicket keeper

(1) 567 (2) 525 (3) 462 (4) 777

Ans. (4)

Sol. **Case-I :** Team consist 5 Batsman , 5 Bowlers and 1 wicket keeper then number of ways.

$= {}^6C_5 \times {}^7C_5 \times {}^2C_1 = 6 \times 21 \times 2 = 252$

Case - II 4 Batsmen, 6 blowers and 1 wicket keeper

$= {}^6C_4 \times {}^7C_6 \times {}^2C_1 = 15 \times 7 \times 2 = 210$

Case-III 4 Batsmen, 5 bowler and 2 wicket keepers

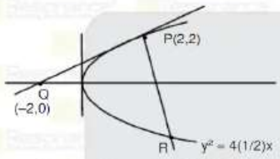
${}^6C_4 \times {}^7C_5 \times {}^2C_2 = 15 \times 21 \times 1 = 315$

3 Tangent drawn at a point P(2,2) to parabola $y^2 = 2x$ cuts x-axis at point Q and normal drawn at point P(2,2) to parabola cut parabola again at point R then area of ΔPQR is

(1) 25 (2) $\frac{25}{2}$ (3) $\frac{15}{2}$ (4) 50

Ans. **(2)**

Sol.



Equation of tangent at P(2,2) is T = 0

$2y = x + 2$

$y^2 = 4$ So, Q (–2,0)

$2at_1 = 1 \Rightarrow t_1 = 2$

$t_2 = -t_1 - \frac{2}{t_1} = -2 - \frac{2}{2} = -3$

$\therefore R\left(\frac{1}{2}(-3)^2, 2\left(\frac{1}{2}\right)(-3)\right) = \left(\frac{9}{2}, -3\right)$

Area of $\Delta PQR = \frac{1}{2}\begin{vmatrix} 2 & 2 & 1 \\ -2 & 0 & 1 \\ \frac{9}{2} & -3 & 1 \end{vmatrix}$

$= \frac{1}{2}[2(0+3) - 2(-2 - 9/2) + 1(6 - 0)] = \frac{1}{2}[6 + 4 + 9 + 6] = \frac{25}{2}$ sq. unit.

4. Coefficient of x^{256} in the expansion of $(1 - x)^{101}(x^2 + x + 1)^{100}$ is

(1) ${}^{100}C_{86}$ (2) ${}^{100}C_{85}$ (3) ${}^{100}C_{84}$ (4) ${}^{100}C_{83}$

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Ans. (2)

Sol. $\Rightarrow (1 - x)^{101}(x^2 + x + 1)^{100}$

$\Rightarrow (1 - x)^{100}(x^2 + x + 1)^{100}(1 - x)$

$\Rightarrow (1 - x^3)^{100}(1 - x)$

$\Rightarrow (1 - x)({}^{100}C_0 - {}^{100}C_1x^3 + {}^{100}C_2x^6 + ... + {}^{100}C_{84}x^{252} - {}^{100}C_{85}x^{255} + {}^{100}C_{86}x^{258} + ...)$

$\Rightarrow {}^{100}C_{85}x^{256}$

so, the coefficient of x^{256} is ${}^{100}C_{85}$

5. The value of lim(2 – cos x√cos 2x) ... is

5. The value of $\lim\limits_{x \to 0}\left(2 - \cos x\sqrt{\cos 2x}\right)^{\frac{x^2+2}{x}}$ is

Ans. 1

Sol. $\lim\limits_{x \to 0}\left(2 - \cos\sqrt{\cos 2x}\right)^{\frac{x^2+2}{x}}$ $\quad (1^{\infty}$ form$)$

$$= e^{\lim\limits_{x \to 0} \frac{\left(1 - \cos x\sqrt{\cos 2x}\right)}{x}\left(x^2+2\right)}$$

$$= e^{\lim\limits_{x \to 0} \left\{\frac{1 - \cos^2 x(\cos 2x)}{x}\right\}(x^2+2)}$$

$$= e^{\lim\limits_{x \to 0} \left\{\frac{1 - \cos^2 x(\cos 2x)}{x}\right\}\left(\frac{x^2+2}{1+\cos x\sqrt{\cos 2x}}\right)}$$

$$= e^{\lim\limits_{x \to 0} \left\{\frac{1 - \cos^2\left(2\cos^2 x - 1\right)}{x}\right\}\left(\frac{x^2+2}{1+\cos x\sqrt{\cos 2x}}\right)}$$

$$= e^{\lim\limits_{x \to 0} \frac{\left(1 - 2\cos^4 x + \cos^2 x\right)}{x} \frac{x^2+2}{1+\cos x\sqrt{\cos 2x}}}$$

$$= e^{\lim\limits_{x \to 0} \frac{-\left(2\cos^4 x - \cos^2 x - 1\right)}{x} \frac{x^2+2}{1+\cos x\sqrt{\cos 2x}}}$$

$$= e^{\lim\limits_{x \to 0} \frac{(2\cos^2 x+1)(\cos^2 x-1)}{x} \frac{x^2+2}{1+\cos x\sqrt{\cos 2x}}}$$

$$= e^{\lim\limits_{x \to 0} \frac{+(2\cos^2 x+1)\sin^2 x}{x} \frac{x^2+2}{1+\cos x\sqrt{\cos 2x}}}$$

$$= e^{\lim\limits_{x \to 0} (2\cos^2 x+1)\frac{\sin x}{x}.\sin x \frac{x^2+2}{1+\cos x\sqrt{\cos 2x}}}$$

$$= e^0 = 1$$

$\sqrt{2}$

$\therefore\ 4\sqrt{2}(m+c)=\pm 34$

7. The mean of 6 numbers is 6.5 and its variance is 10.25 if 4 numbers are 2, 4, 5 and 7, then find the other two :

(1) 10 ,11 (2) 11, 12 (3) 9, 12 (4) 9, 11

Ans. (1)

Sol. Let two number x and y according to question

$18 + x + y = 39$

$x + y = 21$ ....(1)

$10.25 = \frac{\Sigma x_i^2}{n} - (\bar{x})^2$

$10.25 = \frac{x^2+y^2+4+16+25+49}{6} - (6.5)^2$

$10.25 = \frac{x^2+y^2+94}{6} - (6.5)^2$

$\Rightarrow x^2 + y^2 = 221$ ....(2)

solving (1) and (2)

So, x = 10 or y = 11

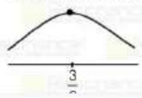
8. A continuous & differentiable function f(x) is increasing in $\left(-\infty,\frac{3}{2}\right)$ and decreasing in $\left(\frac{3}{2},\infty\right)$ then $x=\frac{3}{2}$ is:

(1) point of local maxima (2) point of local minima

(3) point of Inflection (4) None of these

Ans. (1)

Sol. Roughly graph of f(x) can be drawn as



9. If the roots of the quadratic equation $x^2 + 3^{\frac{1}{4}}x + 3^{\frac{1}{2}} = 0$ are α and β then the value of

$\alpha^{96}(\alpha^{12}-1) + \beta^{96}(\beta^{12}-1)$

(1) 50.3^{24} (2) 51.3^{24} (3) 52.3^{24} (4) 104.3^{24}

Ans. (3)

Sol. $x^2 + \sqrt{3} = -3^{\frac{1}{4}}x$

$\Rightarrow x^4 + 2\sqrt{3}x^2 + 3 = \sqrt{3}x^2$

$\Rightarrow x^4 + \sqrt{3}x^2 + 3 = 0$

$\Rightarrow x^8 + 6x^4 + 9 = 3x^4$

$\Rightarrow x^8 + 3x^4 + 9 = 0$

$\Rightarrow \alpha^8 = -9 - 3\alpha^4$

$\Rightarrow \alpha^{12} = -9\alpha^4 - 3\alpha^8 = -9\alpha^4 - 3(-9-3\alpha^4) = 27$

Similarly $\beta^{12} = 27$

$\Rightarrow \alpha^{96}(\alpha^{12}-1) + \beta^{96}(\beta^{12}-1) = (27)^8.26 + (27)^8.26 = 52.(27)^8 = 52.3^{24}$

10. In a ΔABC, If AB = 5, $\angle B = \cos^{-1}(3/5)$ and radius of circumcircle of triangle is 5 then the area of ΔABC is :

(1) $6+8\sqrt{3}$ (2) $3+4\sqrt{3}$ (3) $3+8\sqrt{3}$ (4) $6+4\sqrt{3}$

Ans. (1)

Sol. $\cos B = \frac{3}{5} \Rightarrow \sin B = \frac{4}{5}$, $R = 5$

$\Rightarrow \frac{b}{2R} = \frac{4}{5} \Rightarrow b = 8$, $c = 5$

$\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{3}{5} \Rightarrow \frac{a^2+25-64}{2a(5)} = \frac{3}{5}$

$a^2 - 39 = 6a \Rightarrow a^2 - 6a - 39 = 0$

$\Rightarrow a = \frac{6+8\sqrt{3}}{2} \Rightarrow a = 3+4\sqrt{3}$

12. If the shortest distance between the lines $\vec{r}_1 = \alpha\hat{i} + 2\hat{j} + 2\hat{k} + \lambda(\hat{i} - 2\hat{j} + 2\hat{k}), \lambda \in R\ \alpha > 0$ and $\vec{r}_2 = -4\hat{i} - \hat{k} + \mu(3\hat{i} - 2\hat{j} - 2\hat{k}), \mu \in R$ is 9, then the value of α is

(1) 2 (2) 4 (3) 6 (4) $\sqrt{6}$

Ans. (3)

Sol. Shortest distance $= \left|\frac{(a_2 - a_1).(b_1 \times b_2)}{|b_1 \times b_2|}\right|$

$\Rightarrow 9 = \left|\frac{((\alpha+4)\hat{i} + 2\hat{j} + 3\hat{k}).(8\hat{i} + 8\hat{j} + 4\hat{k})}{\sqrt{64+64+16}}\right|$

$\Rightarrow \left|\frac{8(\alpha+4)+16+12}{12}\right| = 9$

$\therefore \alpha = 6$

13. If a, b, c are mutually $\perp$ unit vectors equally inclined to $\vec{a} + \vec{b} + \vec{c}$ at an angle θ, find $36\cos^2 2\theta$.

Ans. 4

Sol. $|\vec{a} + \vec{b} + \vec{c}|^2 = (\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c}) = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 = 3$

$\Rightarrow |\vec{a} + \vec{b} + \vec{c}| = \sqrt{3}$

Now $\vec{a}.(\vec{a} + \vec{b} + \vec{c}) = |\vec{a}||\vec{a} + \vec{b} + \vec{c}|\cos\theta$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}} \Rightarrow \cos 2\theta = 2\cos^2\theta - 1$

$\Rightarrow \cos 2\theta = -\frac{1}{3} \Rightarrow \cos^2 2\theta = \frac{1}{9} \Rightarrow 36\cos^2 2\theta = 4$

14. If z and ω are complex number such that $|z\omega| = 1$, $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$. Find the $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$.

(1) $\frac{\pi}{4}$ (2) $-\frac{\pi}{4}$ (3) $\frac{3\pi}{4}$ (4) $-\frac{3\pi}{4}$

Ans. (4)

Sol. Let $z = re^{i\theta}$ & $\omega = \frac{1}{r}e^{i\left(\theta-\frac{3\pi}{2}\right)}$

then $\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega} = \frac{1-2re^{-i\theta}.\frac{1}{r}e^{i\left(\theta-\frac{3\pi}{2}\right)}}{1+3re^{-i\theta}.\frac{1}{r}e^{i\left(\theta-\frac{3\pi}{2}\right)}}$

$= \frac{1-2e^{-i\frac{3\pi}{2}}}{1+3e^{-i\frac{3\pi}{2}}} = \frac{1-2i}{1+3i}$

$= -\frac{1}{2}-\frac{1}{2}i$

The $\arg\left(-\frac{1}{2}-\frac{1}{2}i\right) = -\frac{3\pi}{4}$

15. If $f(x) = \begin{cases} \sin x - e^x ; & x \le 0 \\ a + [-x] ; & 0 < x < 1 \\ 2x - b ; & x \ge 1 \end{cases}$ is continuous and differentiable function then find the value of a + b.

(where [•] is GIF)

Ans. (03.00)

Sol. Since f(x) is continuous at x = 0

So $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x) = f(0)$

16. If $A = [a_{ij}]_{3\times3}$ where $a_{ij} = \begin{cases} 1 & i = j \\ -x & |i-j| = 1 \\ 2x+1 & \text{otherwise} \end{cases}$ and f(x) = det (A), then the sum of local maximum and local minimum value of f(x) is:

(1) $\frac{20}{27}$ (2) $\frac{-20}{27}$ (3) $\frac{88}{27}$ (4) $\frac{-88}{27}$

Ans. (4)

Sol. $|A| = \begin{vmatrix} 1 & -x & 2x+1 \\ -x & 1 & -x \\ 2x+1 & -x & 1 \end{vmatrix} = 1 + x^2(2x+1) + x^2(2x+1) - (2x+1)^2 - x^2 - x^2$

$\Rightarrow f(x) = 4x^3 - 4x^2 - 4x$

$\Rightarrow f'(x) = 12x^2 - 8x - 4$

$= 4(3x^2 - 2x - 1)$

$= 4(x-1)(3x+1)$

+ $\frac{-1}{3}$ −   1   +

$\Rightarrow$ f(x) is maximum at $x = \frac{-1}{3}$ and minimum at x = 1

$\therefore$ maximum value $= \frac{20}{27}$ and minimum value $= -4$

$\therefore \text{sum} = \frac{20}{27} - 4 = -\frac{88}{27}$

17. The coefficient of $a^3 b^4 c^5$ in $(ab + bc + ac)^6$ is:

(1) 60 (2) 45 (3)40 (4) 90

Ans. (1)

Sol. $(ab + bc + ac)^6 = \sum_{p+q+r=6} \frac{6!}{p!q!r!}(ab)^p(bc)^q(ca)^r$

$= \sum_{p+q+r=6} \frac{6!}{p!q!r!} a^{p+r} b^{p+q} c^{q+r}$

18. If an invertible function f(x) is defined as $f(x) = 3x - 2$, g(x) is also an invertible function such that $f^{-1}(g^{-1}(x)) = x - 2$ then g(x) is

(1) $\frac{x-8}{3}$ (2) $\frac{x+8}{3}$ (3) $\frac{x-3}{8}$ (4) $\frac{x+3}{8}$

Ans. (2)

Sol. $f^{-1}(g^{-1}(x)) = x - 2$

$f(x-2) = g^{-1}(x)$

$3(x-2) - 2 = g^{-1}(x)$

$3x - 8 = g^{-1}(x)$

$g^{-1}(x) = 3x - 8$

or $x = 3g(x) - 8$

$g(x) = \frac{x+8}{3}$

19. $\int_{-1}^{1} \ell n\left(\sqrt{1-x} + \sqrt{1+x}\right)dx = ?$

(1) $\pi + \ell n2$ (2) $2\ell n2$ (3) $\frac{\pi}{2} - 1 + \ell n2$ (4) $\ell n2 - \frac{\pi}{2} - 1$

Ans. (4)

Sol. $f(x) = \ell n\left(\sqrt{1-x} + \sqrt{1+x}\right)$ $x \in [-1, 1]$ is an even function

$\Rightarrow I = 2\int_{0}^{1} \ell n\left(\sqrt{1-x} + \sqrt{1+x}\right)dx$

Put $x = \cos 2\theta \Rightarrow dx = -2\sin 2\theta\, d\theta$

$\therefore \cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\Rightarrow I = -4\int_{\frac{\pi}{4}}^{0} \left[\ell n\left\{(\sin\theta + \cos\theta)\sqrt{2}\right\}\right] \sin 2\theta\, d\theta$

$= 4\int_{0}^{\frac{\pi}{4}} \left[\ell n\left\{(\sin\theta + \cos\theta)\sqrt{2}\right\}\right] \sin 2\theta\, d\theta$

$= -\frac{\ }{2} - 1 + \ell n2$

20. The probability of selecting integers $a \in [-5, 30]$, such that $x^2 + 2(a+4)x - 5a + 64 > 0$ for all $x \in R$ is :

(1) $\frac{7}{9}$ (2) $\frac{4}{9}$ (3) $\frac{5}{9}$ (4) $\frac{1}{3}$

Ans. (3)

Sol. $x^2 + 2(a+4)x - (5a - 64) > 0$

$D < 0$

$\therefore 4(a+4)^2 + 4(5a - 64) < 0$

$\Rightarrow (a+4)^2 + (5a - 64) < 0$

$\Rightarrow a^2 + 13a - 48 < 0$

$a = \frac{-13 \pm \sqrt{169 + 192}}{2}$

So, $a \in [-16, 3]$

Total integers are 20 $\therefore$ Probability $= \frac{20}{36} = \frac{5}{9}$

21. If $\int_0^a e^{x-[x]} dx = 10e - 9$, then the value of 'a' is (where [•] is GIF)

(1) $9 + \ell n2$ (2) $10 + \ell n2$ (3) 10 (4) 9

Ans. (2)

Sol. Let $a = 10 + K$, $0 \le K < 1$

$\int_0^a e^{\{x\}} dx = 10e - 9$

$\int_0^{10} e^{\{x\}} dx + \int_{10}^{10+K} e^{\{x\}} dx = 10e - 10 + 1 \Rightarrow \int_{10}^{10+K} e^{\{x\}} dx = 1 \Rightarrow \int_0^K e^{\{x\}} dx = 1$

$e^K - 1 = 1$

$K = \ell n2$

so, $a = 10 + \ell n2$

22. If $\vec{A}.\vec{B} = |\vec{A} \times \vec{B}|$, then $|\vec{A} - \vec{B}|$ is

(1) $\sqrt{A^2 + B^2 + \sqrt{2}AB}$ (2) $\sqrt{A^2 + B^2 - \sqrt{2AB}}$ (3) $\sqrt{A^2 + B^2 + \sqrt{2AB}}$ (4) $\sqrt{A^2 + B^2 - \sqrt{2}AB}$

Ans. (4)

Sol. $\vec{A}.\vec{B} = |\vec{A} \times \vec{B}| \Rightarrow \cos\theta = \sin\theta \Rightarrow \tan\theta = 1$

$\theta = \frac{\pi}{4}$

$|\vec{A} - \vec{B}|^2 = A^2 + B^2 - 2\vec{A}.\vec{B}$

$= A^2 + B^2 - 2AB\cos\left(\frac{\pi}{4}\right)$

$= A^2 + B^2 - \sqrt{2}AB$

$\Rightarrow |\vec{A} - \vec{B}| = \sqrt{A^2 + B^2 - \sqrt{2}AB}$