

1. If the ratio of coefficient of middle term in the expansion of $(1+x)^{20}$ and sum of coefficients of middle terms in the expansion of $(1+x)^{19}$ is λ , then λ is :

- (1) 2 (2) 1 (3) 3 (4) 4

Ans. (2)

Sol. $\frac{{}^{20}C_{10}}{{}^{19}C_9 + {}^{19}C_{10}} = \frac{{}^{20}C_{10}}{{}^{20}C_{10}} = 1$

2. If $f(x) = \begin{cases} \frac{\lambda |x^2 - 5x + 6|}{\mu (5x - 6 - x^2)} & ; x < 2 \\ \mu & ; x = 2 \\ e^{\frac{\tan(x-2)}{x-[x]}} & ; x > 2 \end{cases}$, is continuous at $x = 2$. Then the sum of λ and μ is:

- (1) $e+e^2$ (2) $e-e^2$ (3) e^2-e (4) e^2+e+1

Ans. (2)

Sol. RHL = $\lim_{x \rightarrow 2^+} e^{\frac{\tan(x-2)}{x-[x]}} = \lim_{x \rightarrow 2^+} e^{\frac{\tan(x-2)}{(x-2)}} = e$

LHL = $\lim_{x \rightarrow 2^-} \frac{\lambda |x^2 - 5x + 6|}{\mu (5x - 6 - x^2)}$

For $x < 2$, $|x^2 - 5x + 6| = x^2 - 5x + 6$

\therefore LHL = $\lim_{x \rightarrow 2^-} \frac{\lambda |x^2 - 5x + 6|}{\mu (5x - 6 - x^2)} = \frac{-\lambda}{\mu}$

Also, $f(2) = \mu$

For $f(x)$ to be continuous at $x = 2$,

RHL = LHL = $f(2)$

$\therefore e = \frac{-\lambda}{\mu} = \mu$

$\Rightarrow \mu = e$ and $\lambda = -e^2$

$\therefore \lambda + \mu = e - e^2$

$$\therefore \lambda + \mu = e - e^2$$

3. If $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$, $x \in [0, 2\pi]$ then sum of values of x is

- (1) 7π (2) 9π (3) 11π (4) 12π

Ans. (2)

Sol. $(\sin x + \sin 4x) + (\sin 2x + \sin 3x) = 0$

$$\Rightarrow 2\sin \frac{5x}{2} \cos \frac{3x}{2} + 2\sin \frac{5x}{2} \cos \frac{x}{2} = 0$$

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$$\Rightarrow 2\sin \frac{5x}{2} \left(\cos \frac{3x}{2} + \cos \frac{x}{2} \right) = 0$$

$$\Rightarrow 4\sin \frac{5x}{2} \cos x \cos \frac{x}{2} = 0$$

$$\Rightarrow \sin \frac{5x}{2} = 0 \quad \text{or } \cos x = 0 \quad \text{or } \cos \frac{x}{2} = 0$$

$$\Rightarrow \frac{5x}{2} = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi \quad \text{or } x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \text{or } \frac{x}{2} = \frac{\pi}{2}$$

$$\Rightarrow x = 0, \frac{2\pi}{5}, \frac{4\pi}{5}, \frac{6\pi}{5}, \frac{8\pi}{5}, \frac{10\pi}{5} \quad \text{or } x = \frac{\pi}{2}, \frac{3\pi}{2} \quad \text{or } x = \pi$$

4. The number of real solutions of the equation $e^{6x} + e^{4x} + 2e^{3x} + 12e^{2x} + e^x - 1 = 0$
(1) 0 (2) 1 (3) 6 (4) 8

Ans. (2)

Sol. Since $f(x) = e^{6x} + e^{4x} + 2e^{3x} + 12e^{2x} + e^x - 1$
 $\Rightarrow f'(x) = 6e^{6x} + 4e^{4x} + 6e^{3x} + 24e^{2x} + e^x > 0, \forall x \in \mathbb{R}$
Hence $f(x)$ is an increasing function
Now $\lim_{x \rightarrow -\infty} f(x) = -1$ and $f(0) = 1 + 1 + 2 + 12 + 1 - 1$
 $\Rightarrow f(0) > 0$
Hence $f(x) = 0$ has a root in $(-\infty, 0)$

5. If $\frac{1}{a-b} + \frac{1}{a-2b} + \dots + \frac{1}{a-nb} = \alpha n + \beta n^2 + \gamma n^3$ where a is so large than b such that cube and higher powers of $\frac{b}{a}$ may be neglected then value of γ is :

(1) $\frac{b^2}{3a^3}$ (2) $\frac{a^2 - b}{3a^3}$ (3) $\frac{a-b}{3a^3}$ (4) $\frac{b^2}{2a^2}$

Ans. (1)

Sol. $\frac{1}{a-b} + \frac{1}{a-2b} + \frac{1}{a-3b} + \dots + \frac{1}{a-nb}$
 $= \frac{1}{a} \left[\left(1 - \frac{b}{a}\right)^{-1} + \left(1 - \frac{2b}{a}\right)^{-1} + \left(1 - \frac{3b}{a}\right)^{-1} + \dots + \left(1 - \frac{nb}{a}\right)^{-1} \right]$

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$$= \frac{1}{a} \left[\left\{ 1 + \left(\frac{b}{a}\right) + \left(\frac{b}{a}\right)^2 + \dots \right\} + \left\{ 1 + \left(\frac{2b}{a}\right) + \left(\frac{2b}{a}\right)^2 + \dots \right\} + \left\{ 1 + \left(\frac{nb}{a}\right) + \left(\frac{nb}{a}\right)^2 + \dots \right\} \right]$$

$$= \frac{1}{a} \left[n + \frac{b}{a}(1+2+\dots+n) + \frac{b^2}{a^2}(1^2+2^2+\dots+n^2) \right]$$

$$= \frac{1}{a} \left[n + \frac{n(n+1)b}{2a} + \frac{n(n+1)(2n+1)b^2}{6a^2} \right]$$

$$= \frac{1}{a} \left[n + \frac{n^2 b}{2a} + \frac{n b}{2a} + \frac{2n^3 + 3n^2 + n}{6} \left(\frac{b^2}{a^2}\right) \right]$$

$$= n \left(\frac{1}{a} + \frac{b}{2a^2} + \frac{b^2}{6a^3} \right) + \left(\frac{b}{2a^2} + \frac{b^2}{2a^3} \right) n^2 + \frac{b^2}{3a^3} n^3$$

by comparing this result to $\alpha n + \beta n^2 + \gamma n^3$

we get $\gamma = \frac{b^2}{3a^3}$

6 In an A.P, if $S_{3n} = 3S_{2n}$, then ratio $\frac{S_{4n}}{S_{2n}}$ equals:

- (1) 8 (2) 6 (3) 4 (4) 2

Ans. (2)

Sol. $\frac{S_{3n}}{S_{2n}} = \frac{\frac{3n}{2}[2a + (3n-1)d]}{\frac{2n}{2}[2a + (2n-1)d]} = 3$

$$\Rightarrow 2a + (3n-1)d = 2[2a + (2n-1)d]$$

$$\Rightarrow 2a + (n-1)d = 0 \dots\dots(1)$$

Now $S_{4n} = \frac{4n}{2}[2a + (4n-1)d]$

$$= n \left(\frac{1}{a} + \frac{b}{2a^2} + \frac{b^2}{6a^3} \right) + \left(\frac{b}{2a^2} + \frac{b^2}{2a^3} \right) n^2 + \frac{b^3}{3a^3} n^3$$

by comparing this result to $\alpha n + \beta n^2 + \gamma n^3$

we get $\gamma = \frac{b^2}{3a^3}$

6. In an A.P, if $S_{3n} = 3S_{2n}$, then ratio $\frac{S_{4n}}{S_{2n}}$ equals:
- (1) 8 (2) 6 (3) 4 (4) 2

Ans. (2)

Sol. $\frac{S_{3n}}{S_{2n}} = \frac{\frac{3n}{2} [2a + (3n-1)d]}{\frac{2n}{2} [2a + (2n-1)d]} = 3$

$\Rightarrow 2a + (3n-1)d = 2[2a + (2n-1)d]$
 $\Rightarrow 2a + (n-1)d = 0 \dots\dots(1)$

Now $\frac{S_{4n}}{S_{2n}} = \frac{\frac{4n}{2} [2a + (4n-1)d]}{\frac{2n}{2} [2a + (2n-1)d]}$
 $= \frac{2[2a + (4n-1)d]}{[2a + (2n-1)d]}$

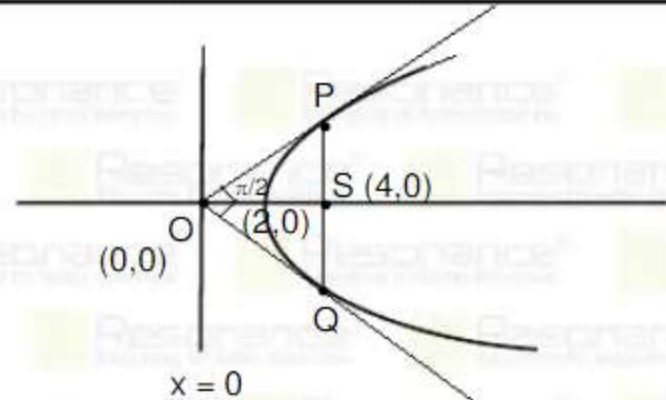
Put, $2a = -(n-1)d$, we have, $\frac{S_{4n}}{S_{2n}} = \frac{2[3nd]}{nd} = 6$

7. A parabola whose vertex is at 2 unit distance from origin on positive x-axis and distance between focus and origin is 4 unit. The tangent drawn from the origin to the parabola meet the parabola at P and Q, then the area of ΔOPQ is

Ans. 16 square unit

Sol.

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Equation of parabola
 $(y - 0)^2 = 4(2)(x - 2)$
 origin lie on directrix $x = 0$
 so ΔOPQ is right angle triangle.
 Equation of chord of contact PQ is
 $T = 0$
 $x = 4$
 It is latus rectum of parabola
 so area of $OPQ = \frac{1}{2} \times 4 \times 8 = 16$ square unit

8. The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3} - x^{1/3} + 1} - \frac{x-1}{x - x^{1/2}}\right)^{10}$ is :
- (1) 4 (2) 120 (3) 210 (4) 310

Sol. $\left(x^{1/3} + 1 - \frac{\sqrt{x} + 1}{\sqrt{x}}\right)^{10}$
 $(x^{1/3} - x^{-1/2})^{10}$
 $T_{r+1} = {}^{10}C_r (x^{1/3})^{10-r} (-x^{-1/2})^r$
 $\frac{10-r}{3} - \frac{r}{2} = 0 \Rightarrow 20 - 2r - 3r = 0$
 $\Rightarrow r = 4$
 $T_5 = {}^{10}C_4 = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = 210$

9. The area of the region bounded by the inequalities $2x^2 < y < 4 - 2x, x > 0$ is :
- (1) $\frac{11}{3}$ (2) $\frac{7}{3}$ (3) $\frac{2}{3}$ (4) $\frac{5}{3}$

9. The area of the region bounded by the inequalities $2x^2 < y < 4 - 2x$, $x > 0$ is :

- (1) $\frac{11}{12}$ (2) $\frac{7}{3}$ (3) $\frac{2}{3}$ (4) $\frac{5}{2}$

Ans. (2)

Sol.

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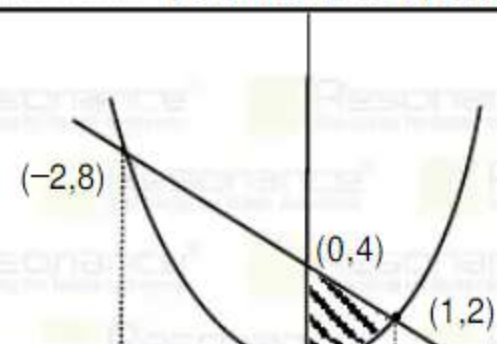
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focus of ellipse and its radius is $\frac{2}{\sqrt{3}}$. If circle cuts the ellipse at two different points P and Q then the

value of PQ^2 is

- (1) $\frac{8}{3}$ (2) $\frac{4}{3}$ (3) $\frac{16}{3}$ (4) $\frac{5}{3}$

Ans. (3)

Sol. Let equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$)

it passes through $(\sqrt{\frac{3}{2}}, 1) \Rightarrow \frac{3}{2a^2} + \frac{1}{b^2} = 1$ (1)

Given $e = \frac{1}{\sqrt{3}} \Rightarrow b^2 = a^2(1 - e^2) = \frac{2}{3}a^2$ (2)

Solve (1) & (2) we get $a^2 = 3, b^2 = 2$

\therefore Ellipse is $\frac{x^2}{3} + \frac{y^2}{2} = 1$ (3)

Focus $(\pm ae, 0) = (\pm\sqrt{3} \cdot \frac{1}{\sqrt{3}}, 0) = (\pm 1, 0)$

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$$\text{Required area} = \frac{1}{2}(2+4) \times 1 - \int_0^1 2x^2 dx = 3 - \frac{2}{3} = \frac{7}{3} \text{ square units}$$

10. An ellipse with eccentricity $\frac{1}{\sqrt{3}}$ passes through the point $\left(\sqrt{\frac{3}{2}}, 1\right)$. A circle is drawn whose centre is the focus of ellipse and its radius is $\frac{2}{\sqrt{3}}$. If circle cuts the ellipse at two different points P and Q then the value of PQ^2 is

- (1) $\frac{8}{3}$ (2) $\frac{4}{3}$ (3) $\frac{16}{3}$ (4) $\frac{5}{3}$

Ans. (3)

Sol. Let equation of ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$)

$$\text{it passes through } \left(\sqrt{\frac{3}{2}}, 1\right) \Rightarrow \frac{3}{2a^2} + \frac{1}{b^2} = 1 \dots(1)$$

$$\text{Given } e = \frac{1}{\sqrt{3}} \Rightarrow b^2 = a^2(1 - e^2) = \frac{2}{3}a^2 \dots\dots(2)$$

Solve (1) & (2) we get $a^2 = 3, b^2 = 2$

$$\therefore \text{ Ellipse is } \frac{x^2}{3} + \frac{y^2}{2} = 1 \dots\dots (3)$$

$$\text{Focus } (\pm ae, 0) = \left(\pm\sqrt{3} \cdot \frac{1}{\sqrt{3}}, 0\right) = (\pm 1, 0)$$

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11. The statement $(p \rightarrow q) \wedge (p \rightarrow \sim q)$ is logically equivalent to:
 (1) p (2) q (3) $\sim p$ (4) $\sim q$

Ans. (3)
Sol. $= (p \rightarrow q) \wedge (p \rightarrow \sim q)$
 $= (\sim p \vee q) \wedge (\sim p \vee \sim q)$
 $= \sim p \vee (q \wedge \sim q)$
 $= \sim p \vee f$
 $= \sim p$

12. In class 10th, 11th and 12th of a school there are 5, 6 and 8 students respectively. The number of ways of selection of 10 students such that at least two students are selected from each of the classes and at most 5 students together can be selected from class 10th & 11th are $k \times 100$ then the value of k is.

Ans. 238
Sol. Total student (5) (6) (8)
 Class 10th 11th 12th

2	2	6	→	$5C_2 \times 6C_2 \times 8C_6$
2	3	5	→	$5C_2 \times 6C_3 \times 8C_5$
3	2	5	→	$5C_3 \times 6C_2 \times 8C_5$

Total number of ways $= 5C_2 \times 8C_3 (6C_3 + 6C_2) + 5C_2 \times 6C_2 \times 8C_6$
 $= 23800$

13. If $\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots + \infty\right)^{\log_{90} 25} \left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots\right)$ is ℓ then ℓ^2 is

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13. If $\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots + \infty\right)^{\log_{0.25}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots\right)}$ is ℓ then ℓ^2 is

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Ans. (3)

Sol. $\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots\right)^{\log_{0.25}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots\right)} = \ell$

$\left(1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots\right)^{\log_{\frac{1}{4}}\frac{1}{2}} = \ell$

Let $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots = x$

$(x - 1) = \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots$ (1)

$\frac{1}{3}(x - 1) = \frac{2}{3^2} + \frac{6}{3^3} + \dots$ (2)

From (1) - (2), we get

$\frac{2}{3}(x - 1) = \frac{2}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \dots$

$$\left(1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \dots\right)^{-1} = \ell$$

Let $1 + \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots = x$

$$(x - 1) = \frac{2}{3} + \frac{6}{3^2} + \frac{10}{3^3} + \dots \quad \dots (1)$$

$$\frac{1}{3}(x - 1) = \frac{2}{3^2} + \frac{6}{3^3} + \dots \quad \dots (2)$$

From (1) - (2), we get

$$\frac{2}{3}(x - 1) = \frac{2}{3} + \frac{4}{3^2} + \frac{4}{3^3} + \dots$$

$$\frac{2}{3}(x - 1) = \frac{2}{3} + \frac{4}{3^2} \left(\frac{1}{1 - \frac{1}{3}}\right)$$

$$\frac{2}{3}(x - 1) = \frac{2}{3} + \frac{4}{3^2} \cdot \frac{3}{2}$$

$$\frac{2}{3}(x - 1) = \frac{2}{3} + \frac{2}{3}$$

$$x - 1 = 2 \quad \& \quad x = 3$$

So, $3^{\log_{\frac{1}{4}} \frac{1}{2}} = \ell$

$$3^{\frac{1}{2}} = \ell$$

$$\ell^2 = 3$$

14. A hyperbola with equation $\frac{(x-1)^2}{16} - \frac{(y+2)^2}{9} = 1$ is given. A triangle is formed with two vertices as the focus

of the hyperbola and third vertex lies on hyperbola. The locus of centroid of the triangle is:

(1) $16(x-1)^2 - 9(y+2)^2 = 16$ (2) $9(x-1)^2 - 16(y+2)^2 = 16$

(3) $9(x-1)^2 + 16(y+2)^2 = 16$ (4) $16(x-1)^2 + 9(y+2)^2 = 16$

Ans. (2)

Sol. Given $\frac{(x-1)^2}{16} - \frac{(y+2)^2}{9} = 1$

Let $x - 1 = X$

$y + 2 = Y$

$$b^2 = a^2 (e^2 - 1) \Rightarrow \frac{9}{16} = e^2 - 1 \Rightarrow e = \frac{5}{4}$$

$$\text{Focus } (\pm ae, 0) \Rightarrow X = \pm ae, Y = 0$$

$$x - 1 = \pm 5, y + 2 = 0$$

$$x = 6, -4, y = -2$$

Hence focus S(-4, -2), S'(6, -2)

Let any point on hyperbola $x - 1 = 4\sec\theta, y + 2 = 3\tan\theta \Rightarrow P(1 + 4\sec\theta, -2 + 3\tan\theta)$

$$\text{Hence centroid is } \equiv \left(\frac{-4 + 6 + 1 + 4\sec\theta}{3}, \frac{-2 - 2 - 2 + 3\tan\theta}{3} \right)$$

$$h = \frac{3 + 4\sec\theta}{3} \Rightarrow \sec\theta = \frac{3h - 3}{4}$$

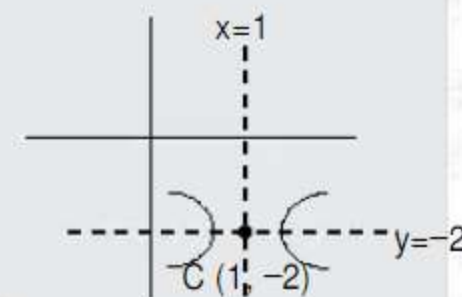
$$k = \frac{-6 + 3\tan\theta}{3} \Rightarrow \tan\theta = k + 2$$

$$\sec^2\theta - \tan^2\theta = 1$$

$$\left(\frac{3h - 3}{4} \right)^2 - (k + 2)^2 = 1$$

$$\text{Locus is } \frac{9(x - 1)^2}{16} - \frac{(y + 2)^2}{1} = 1$$

$$\Rightarrow 9(x - 1)^2 - 16(y + 2)^2 = 16$$



15. Evaluate $\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1 + \sqrt{\tan 2x}}$:

- (1) $\frac{\pi}{24}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{12}$ (4) $\frac{\pi}{6}$

Ans. (3)

Locus is $\frac{x^2}{16} - \frac{y^2}{1} = 1$
 $\Rightarrow 9(x-1)^2 - 16(y+2)^2 = 16$

15. Evaluate $\int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1 + \sqrt{\tan 2x}}$:

(1) $\frac{\pi}{24}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{12}$ (4) $\frac{\pi}{6}$

Ans. (3)

Sol. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

$I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1 + \sqrt{\tan 2x}} \dots\dots(1)$

By property

$I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{dx}{1 + \sqrt{\cot 2x}}$

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$$I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{\sqrt[3]{\tan 2x} dx}{1 + \sqrt[3]{\tan 2x}} \dots\dots(2)$$

By adding (1) & (2)

$$2I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} \frac{(1 + \sqrt[3]{\tan 2x}) dx}{1 + \sqrt[3]{\tan 2x}}$$

$$2I = \int_{\frac{\pi}{24}}^{\frac{5\pi}{24}} dx = \frac{\pi}{6}$$

$$\therefore I = \frac{\pi}{12}$$

16. If a set of matrix $M = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \{\pm 1, \pm 2, \pm 3\} \right\}$ and $A \in M$ then the number of such matrices A whose determinant value is 15.

Ans. (16)

Sol. $|A| = (ad - bc) = 15$

where $a, b, c, d \in \{\pm 1, \pm 2, \pm 3\}$

Case - I $ad = 9$ & $bc = -6$

$ad = (3,3)$ or $(-3, -3)$ $bc = (2, -3), (-2, 3), (-3,2), (3, -2)$

Total = $2 \times 4 = 8$ matrix

Case - II $ad = 6$ and $bc = -9$

Similarly, Total = $4 \times 2 = 8$ matrix

Total such matrix = $8 + 8 = 16$ matrix

17. If $\frac{dy}{dx} = 1 + xe^{y-x}$, $-\sqrt{2} < x < \sqrt{2}$ and $y(0) = 0$ then minimum value of y is

(1) $(1 - \sqrt{3}) - \ln(\sqrt{3} - 1)$ (2) $(1 + \sqrt{3}) - \ln(\sqrt{3} - 1)$

(3) $(1 - \sqrt{3}) - \ln(\sqrt{3} + 1)$ (4) $(1 + \sqrt{3}) - \ln(\sqrt{3} + 1)$

12
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- (2) $(1 + \sqrt{3}) - \ln(\sqrt{3} - 1)$
- (3) $(1 - \sqrt{3}) - \ln(\sqrt{3} + 1)$
- (4) $(1 + \sqrt{3}) - \ln(\sqrt{3} + 1)$

Ans. (1)

Sol. $\frac{dy}{dx} = 1 + xe^{y-x}$ (1)

$$e^{-y} \frac{dy}{dx} = e^{-y} + xe^{-x}$$

$$\text{Put } e^{-y} = t \Rightarrow e^{-y} \frac{dy}{dx} = -\frac{dt}{dx}$$

$$-\frac{dt}{dx} = t + xe^{-x}$$

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$$\frac{dt}{dx} + t = -xe^{-x} \dots\dots(2)$$

$$\text{I.F.} = e^{\int 1 \cdot dx} = e^x$$

Solution of equation (2) is

$$te^x = \int (-xe^{-x}) \cdot e^x dx + c$$

$$te^x = -\frac{x^2}{2} + c$$

$$e^{x-y} = -\frac{x^2}{2} + c \dots\dots(3)$$

$$\therefore y(0) = 0 \Rightarrow 1 = c \Rightarrow e^{x-y} = \left(\frac{2-x^2}{2}\right)$$

$$x - y = \ln\left(\frac{2-x^2}{2}\right)$$

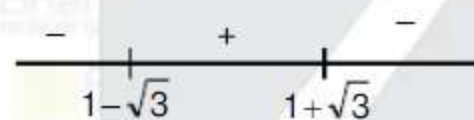
$$y = x - \ln\left(\frac{2-x^2}{2}\right)$$

$$\text{Now, } \frac{dy}{dx} = 1 + x\left(\frac{2}{2-x^2}\right)$$

$$\Rightarrow \left(\frac{2-x^2+2x}{2-x^2}\right) = 0$$

$$\Rightarrow -\left(\frac{x^2-2x-2}{2-x^2}\right) = 0$$

$$x = 1 \pm \sqrt{3}$$



$$\Rightarrow y_{\min} \text{ at } x = 1 - \sqrt{3} \Rightarrow y_{\min} = (1 - \sqrt{3}) - \ln(\sqrt{3} - 1)$$

18. A balloon with radius 16 cm is at a certain height above the ground such that the angle of elevation of its centre is 75° from a point on the ground. If the balloon subtends an angle of 60° at that point, then the height of its topmost point from the ground is:

$$\frac{1-\sqrt{3}}{1+\sqrt{3}}$$

$\Rightarrow y_{\min}$ at $x = 1 - \sqrt{3} \Rightarrow y_{\min} = (1 - \sqrt{3}) - (n(\sqrt{3} - 1))$

18. A balloon with radius 16 cm is at a certain height above the ground such that the angle of elevation of its centre is 75° from a point on the ground. if the balloon subtends an angle of 60° at that point, then the height of its topmost point from the ground is:

- (1) $8(\sqrt{6} - \sqrt{2} + 2)$ (2) $8(\sqrt{6} - 2\sqrt{2} + 2)$
 (3) $8(\sqrt{6} + \sqrt{2} + 2)$ (4) $8(\sqrt{6} - 2\sqrt{2} + 2)$

Ans. (3)

Sol. In triangle EOD

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$$ED = \frac{16}{\tan 30^\circ} = 16\sqrt{3}$$



$$\text{Height of top most point} = 8\sqrt{6} + 8\sqrt{2} + 16 = 8(\sqrt{6} + \sqrt{2} + 2)$$

19. If $S = \{n : n \in \mathbb{N} \text{ and } 1 \leq n \leq 100, \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}^n \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \forall a, b, c, d \in \mathbb{R}\}$ then the number of 2 digit numbers in S is
- (1) 25 (2) 22 (3) 24 (4) 20

Ans. (2)

Sol. Let $A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}^n$ and $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$AB = IB$$

$$(A - I)B = 0$$

$$A = I$$

$$\begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}^n = I$$

$$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$n =$ multiple of 4 (4, 8,100)

Number of two digit numbers in S = 22 (12, 16,96)

20. If $|z - (3i + 2)| < 2$ then the min value of $|2z - 6 + 5i|$ is

- (1) $(5\sqrt{5} - 2)$ (2) $(5\sqrt{5} - 4)$ (3) $(5\sqrt{5} + 2)$ (4) $\left(\frac{5\sqrt{5}}{2} + 2\right)$

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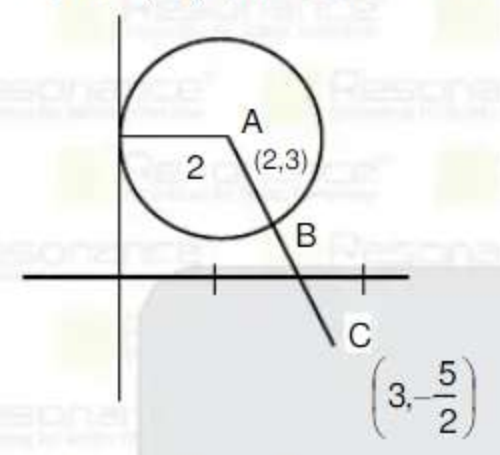
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Ans. (2)

Sol. $|(x - 2) + (y - 3)i| < 2$

Point A (2,3) and radius = 2



$$|(2x - 6) + (2y + 5)i|$$

$$x = 3$$

$$y = -\frac{5}{2}$$

point C $(3, -\frac{5}{2})$

$$AC = \sqrt{(3-2)^2 + (-\frac{5}{2}-3)^2}$$

$$= \sqrt{1 + \frac{121}{4}}$$

$$= \sqrt{\frac{125}{4}}$$

$$\text{Min distance } 2BC = 2\left(\frac{5\sqrt{5}}{2} - 2\right) = (5\sqrt{5} - 4)$$