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(Main)

PAPER-1 (B.E./B. TECH.)

2021

COMPUTER BASED TEST (CBT) Memory Based Questions & Solutions

Date: 27 July, 2021 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300






SUBJECT: MATHEMATICS

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**RESULT: JEE (Advanced),
JEE (Main), NEET**

≡ **HIGHEST** No. of Classroom Selections ≡
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5 AIRs in TOP-50 in JEE (Adv.) 2020 from Classroom




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PART : MATHEMATICS

1. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 4\hat{i} + 3\hat{j} + \hat{k}$ then the value of $((\vec{a} + \vec{b}) \times (\vec{a} - (\vec{a} - \vec{b}) \times \vec{b})) \times \vec{c}$ is :

- (1) $(30\hat{i} - 34\hat{j} + 36\hat{k})$ (2) $(30\hat{i} + 34\hat{j} + 36\hat{k})$
 (3) $(30\hat{i} + 34\hat{j} - 36\hat{k})$ (4) None of these

Ans. (1)

Sol. $\vec{a} + \vec{b} = 3\hat{i} + 2\hat{j} = \vec{r}_1$

$$\vec{r}_2 = \vec{a} - (\vec{a} - \vec{b}) \times \vec{b} = \vec{r}_2 = (\hat{i} + \hat{j} + \hat{k}) - (-\hat{i} + 2\hat{k}) \times (2\hat{i} + \hat{j} - \hat{k})$$

$$(\hat{i} + \hat{j} + \hat{k}) - (-\hat{k} - \hat{j} + 4\hat{j} - 2\hat{i}) = (3\hat{i} - 2\hat{j} + 2\hat{k})$$

$$\begin{aligned} \text{Now } ((\vec{a} + \vec{b}) \times (\vec{a} - (\vec{a} - \vec{b}) \times \vec{b})) \times \vec{c} &= (\vec{r}_1 \times \vec{r}_2) \times \vec{c} \\ &= (\vec{r}_1 \cdot \vec{c})\vec{r}_2 - (\vec{r}_2 \cdot \vec{c})\vec{r}_1 \\ &= 18(3\hat{i} - 2\hat{j} + 2\hat{k}) - 8(3\hat{i} + 2\hat{j}) \\ &= (30\hat{i} - 34\hat{j} + 36\hat{k}) \quad \text{Ans.} \end{aligned}$$

2 Which of the following is logically equivalent to $(p \vee q) \wedge (\sim p \rightarrow q)$ is :

- (1) $(p \vee q)$ (2) $(p \wedge q)$
 (3) $\sim p \vee q$ (4) $p \wedge \sim q$

Ans. (1)

Sol. We know $p \rightarrow q = \sim p \vee q$

$$\text{Hence } \sim p \rightarrow q = p \vee q$$

$$\Rightarrow (p \vee q) \wedge (p \vee q) = p \vee q \quad \text{Ans.}$$

3. If $S_1 = \{z : |z - 3 - 2i|^2 = 8\}$ and $S_2 = \{z : |z - \bar{z}| = 8\}$ and $S_3 = \{z : \text{re}(z) \geq 5\}$ then $S_1 \cap S_2 \cap S_3$ has

- (1) Infinite many element (2) Only one element
 (3) No any element (4) Two element


Ans. (2)

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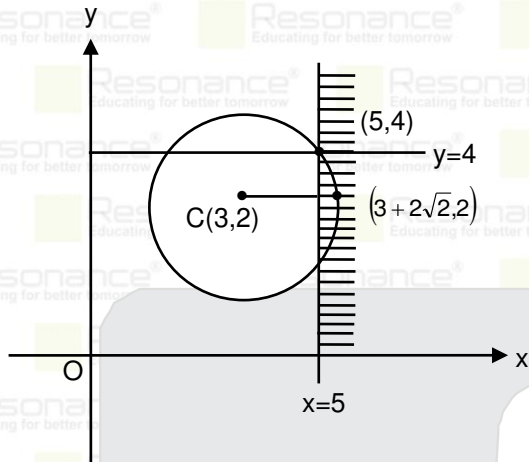
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Sol. Let $z = x+iy$



$$S_2 \Rightarrow |2iy| = 8 \Rightarrow |y| = 4 \\ y = \pm 4$$

$$S_3 \Rightarrow x \geq 5$$

$$S_1 \Rightarrow |z - 3 - 2i|^2 = 8 = (x-3)^2 + (y-2)^2 = 8$$

Is a circle with centre (3,2) and radius = $2\sqrt{2}$

\therefore Circle passes through (5,4)

\Rightarrow There is exactly one point (5,4) in $S_1 \cap S_2 \cap S_3$

4. If $e^{-x} \int_3^x \{3t^2 + 2t + 4f'(t)\} dt = f(x)$ and $f'(4) = \frac{\alpha e^\beta - 224}{(e^\beta - 4)^2}$ then value of $(\alpha + \beta)$ is

Ans. 16.00

Sol. Put $x = 3 \Rightarrow f(3) = 0$

$$e^{-x} [t^3 + t^2 + 4(F(t))_3^x] = F(x)$$

$$e^{-x} (x^3 + x^2 + 4F(x) - (27 + 9 + 4F(3))) = F(x)$$

$$\Rightarrow F(x) = e^{-x} (x^3 + x^2 - 36 + 4F(x))$$

$$\Rightarrow e^x F(x) = x^3 + x^2 - 36 + 4F(x)$$

$$F(x) = \frac{x^3 + x^2 - 36}{(e^x - 4)}$$

$$F'(x) = \frac{(3x^2 + 2x)(e^x - 4) - (e^x)(x^3 + x^2 - 36)}{(e^x - 4)^2}$$

$$F'(4) = \frac{56(e^4 - 4) - e^4(44)}{(e^4 - 4)^2}$$

$$F'(4) = \frac{12e^4 - 224}{(e^4 - 4)^2}$$

Hence $\alpha = 12$ and $\beta = 4$

$$\alpha + \beta = 16$$

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5. The coefficient of x^7 and x^{-7} in the expansion of $\left(x^2 + \frac{1}{bx}\right)^{11}$ and $\left(x + \frac{1}{bx^2}\right)^{11}$ respectively are equal then the value of b is :
 (1) 1 (2) -1 (3) 0 (4) 2

Ans. (1)

Sol.

$$\left(x^2 + \frac{1}{bx}\right)^{11}$$

$$T_{r+1} = {}^{11}C_r (x^2)^{11-r} \left(\frac{1}{bx}\right)^r = {}^{11}C_r (x)^{22-3r} \left(\frac{1}{b}\right)^r$$

$$\text{for } x^4 \Rightarrow 22 - 3r = 7 \Rightarrow r = 5$$

$$\therefore \text{coefficient of } x^7 = {}^{11}C_5 \left(\frac{1}{b}\right)^5$$

$$\text{similarly } \left(x + \frac{1}{bx^2}\right)^{11}$$

$$T_{r+1} = {}^{11}C_r (x)^{11-r} \left(\frac{1}{bx^2}\right)^r = {}^{11}C_r \left(\frac{1}{b}\right)^r (x)^{11-3r}$$

$$\text{For } x^{-7} \Rightarrow 11 - 3r = -7 \Rightarrow r = 6$$

$$\therefore \text{coefficient } x^{-7} = {}^{11}C_6 \left(\frac{1}{b}\right)^6$$

$${}^{11}C_6 \left(\frac{1}{b}\right)^6 = {}^{11}C_5 \left(\frac{1}{b}\right)^5 \Rightarrow b = 1$$

6. Evaluate $\lim_{x \rightarrow 2} \frac{x^2 f(2) - 4f(x)}{x - 2}$, if $f(2) = 4$ & $f'(2) = 1$

Ans. (12)

$$\text{Sol. } \lim_{x \rightarrow 2} \frac{x^2 f(2) - 4f(x)}{x - 2} = \lim_{x \rightarrow 2} \frac{2xf(2) - 4f'(x)}{1} = 2 \cdot (2) \cdot f(2) - 4f'(2) = 16 - 4 = 12$$

7. If $\sin\theta + \cos\theta = \frac{1}{2}$, then the value of $16(\sin 2\theta + \cos 4\theta + \sin 6\theta)$ is :

- (1) 23 (2) -23 (3) 36 (4) -43

Ans. (2)

$$\text{Sol. } (\sin\theta + \cos\theta)^2 = \frac{1}{4}$$

$$1 + \sin 2\theta = \frac{1}{4}$$

$$\sin 2\theta = \frac{-3}{4}$$

$$16(\sin 2\theta + \cos 4\theta + \sin 6\theta) = 16(\sin 2\theta + 1 - 2\sin^2 2\theta + 3\sin 2\theta - 4\sin^3 2\theta)$$

$$= 16(4\sin 2\theta + 1 - 2\sin^2 2\theta - 4\sin^3 2\theta) = -23$$

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8. If mean and variance of eight observation 10, 13, 6, 7, a, 12, b, 12 are 9 and $\frac{37}{4}$ respectively, then the value of $(a-b)^2$ is s
 (1) 25 (2) 36 (3) 16 (4) 49

Ans. (3)

Sol. $9 = \frac{10+13+6+7+a+12+b+12}{8}$

$\Rightarrow a + b = 12 \dots\dots\dots (i)$

$\sum x_i^2 = 100+169+36+49+a^2+144+b^2+144$
 $= a^2 + b^2 + 642$

$\sigma^2 = \frac{\sum x_i^2}{8} - (\bar{x})^2 = \frac{37}{4}$

$\frac{a^2 + b^2 + 642}{8} - 81 = \frac{37}{4}$

$\Rightarrow a^2 + b^2 = 80 \Rightarrow a^2 + (12-a)^2 = 80 \Rightarrow a^2 - 12a + 32 = 0$

$\Rightarrow a = 4 \quad \text{or} \quad a = 8$

$b = 8 \quad \text{or} \quad b = 4$

$\Rightarrow (a - b)^2 = 16$ Ans.

9. If matrix $A = \begin{bmatrix} 2 & 4 \\ -1 & 1 \end{bmatrix}$ and $A^{-1} = \alpha I + \beta A$, $\alpha, \beta \in \mathbb{R}$. Then the value of $(\alpha - 6\beta)$ is equal to :

(1) $\frac{3}{2}$

(2) $\frac{1}{2}$

(3) $\frac{-1}{2}$

(4) $\frac{-3}{2}$

Ans. (1)

Sol. Equation of $|A-\lambda I| = 0$

$\begin{vmatrix} 2-\lambda & 4 \\ -1 & 1-\lambda \end{vmatrix} = 0$

$\Rightarrow (2 - \lambda)(1 - \lambda) + 4 = 0 \Rightarrow \lambda^2 - 3\lambda + 2 + 4 = 0 \Rightarrow \lambda^2 - 3\lambda + 6 = 0$

$A^2 - 3A + 6I = 0$

Multiply both side by A^{-1}

$A - 3I + 6A^{-1} = 0$

$A^{-1} = \frac{-A}{6} + \frac{I}{2} \dots\dots\dots (1)$

$A^{-1} = \alpha I + \beta A \dots\dots\dots (2)$

Comparing equation (1) and (2)

$\alpha = \frac{1}{2}, \beta = \frac{-1}{6}$

$\alpha - 6\beta = \frac{1}{2} + 1 = \frac{3}{2}$

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10. If α and β are the roots of the equation $x^2 - 20^{1/4}x + 5^{1/2} = 0$, then the value of $\alpha^8 + \beta^8$ is equal to :

(1) 25

(2) 50

(3) 75

(4) 100

Ans. (2)

Sol. $x^2 - 20^{1/4}x + 5^{1/2} = 0$

$$(x^2 + \sqrt{5})^2 = \sqrt{20}x^2$$

$$x^4 + 5 + 2\sqrt{5}x^2 = 2\sqrt{5}x^2$$

$$x^4 = -5$$

$$x^8 = 25$$

$$\alpha^8 = 25, \beta^8 = 25$$

$$\alpha^8 + \beta^8 = 50$$

11. Let n be a two digit natural number. If n is selected at random then find the probability such that $2^n - 2$ is a multiple of 3.

(1) $\frac{1}{4}$

(2) $\frac{1}{3}$

(3) $\frac{1}{2}$

(4) 0

Ans. (3)

Sol. Total number of cases = ${}^{90}C_1 = 90$

$$\text{Now } 2^n - 2 = (3 - 1)^n - 2$$

$$= {}^nC_0 3^n - {}^nC_1 3^{n-1} + \dots + (-1)^{n-1} {}^nC_{n-1} 3 + (-1)^n {}^nC_n - 2$$

$$= 3(3^{n-1} - n \cdot 3^{n-2} + \dots + (-1)^{n-1} \cdot n) + (-1)^n - 2$$

So $(2^n - 2)$ is multiple of 3 only when n is odd

So number of favourable cases = 45, Hence required probability = $\frac{45}{90} = \frac{1}{2}$

12. If tangents to the circle $(x - 1)^2 + (y - 3)^2 = 2^2$ are drawn from the point $(-1, 1)$ such that the points of contact of tangents are A and B. Also a point P lies on the circle such that $AB = AP$, then the area of $\triangle ABP$ is :

(1) 2

(2) $\sqrt{3}$

(3) 4

(4) $2\sqrt{2}$

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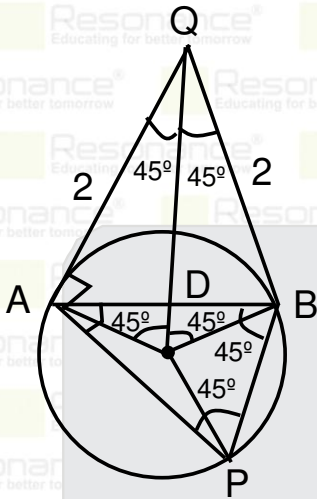
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Ans. (3)

Sol.



$$AQ = \sqrt{S_1} = \sqrt{(-1-1)^2 + (1-3)^2} - 4 = 2$$

$$AD = 2\sin 45^\circ$$

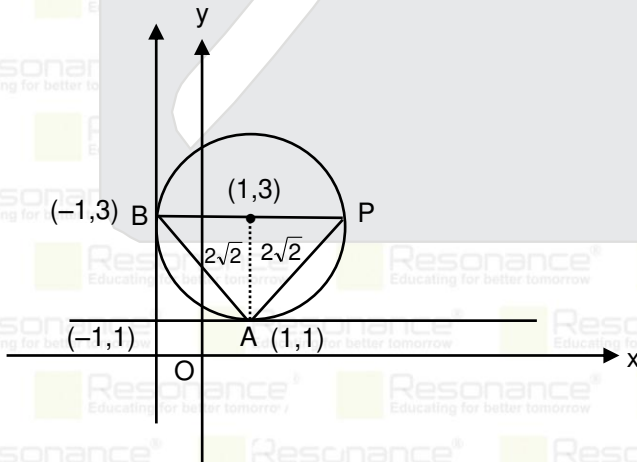
$$AB = 4\sin 45^\circ$$

$$AP = 4\sin 45^\circ$$

$$\text{Area of triangle ABP} = \frac{1}{2} (4\sin 45^\circ) (4\sin 45^\circ)$$

$$= 4$$

Alternate :



$$\Delta PAB = \frac{1}{2} \times 4 \times 2 = 4$$

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13. If $\log_3 2, \log_3(2^x - 5), \log_3(2^x - \frac{7}{2})$ are in A.P. then the value of x is

- (1) 4 (2) 2 (3) 3 (4) 5

Ans. (3)

Sol. $2\log_3(2^x - 5) = \log_3 2 + \log_3(2^x - \frac{7}{2})$

$$\Rightarrow \log_3(2^x - 5)^2 = \log_3(2(2^x - \frac{7}{2}))$$

$$\Rightarrow (2^x)^2 - 10(2^x) + 25 = 2(2^x) - 7$$

$$\Rightarrow (2^x)^2 - 12(2^x) + 32 = 0$$

$$\Rightarrow 2^x = 4 \text{ or } 8$$

$$\Rightarrow x = 2 \text{ or } 3$$

For $x = 2, \log(2^x - 5)$ is not defined

$$\Rightarrow x = 3$$

14. If $\sec y \frac{dy}{dx} = \sin(x+y) + \sin(x-y); y(0) = 0$ then the value of $5y'(\frac{\pi}{2})$ is

- (1) 3 (2) 4 (3) 2 (4) 1

Ans. (3)

Sol. $\sec y \frac{dy}{dx} = 2\sin(x) \cos(y) \dots\dots\dots(1)$

$$\int \sec^2 y dy = \int 2\sin x dx$$

$$\tan y = -2\cos x + c$$

$$\text{When } x = 0, y = 0 \Rightarrow c = 2$$

$$\tan y = -2\cos x + 2 \dots\dots\dots(2)$$

$$\therefore \sec^2 y \frac{dy}{dx} = 2\sin x$$

$$(1 + \tan^2 y) \frac{dy}{dx} = 2\sin x \dots\dots\dots(3)$$

By equation (2) & (3)

$$\frac{dy}{dx} = \frac{2\sin x}{1 + (2 - 2\cos x)^2}$$

$$f'(\frac{\pi}{2}) = \frac{2}{1 + (2)^2} = \frac{2}{5} \Rightarrow 5f'(\frac{\pi}{2}) = 2$$

15. A circle with centre (2, 3) passes through the origin. A diameter PQ is such that it is perpendicular to the line joining the centre of circle and origin. The coordinates of points P and Q are :

- (1) (-1, 5) & (5, 1) (2) (-2, 5) & (-5, 1) (3) (3, 6) & (5, 1) (4) (-1, 1) & (5, -1)

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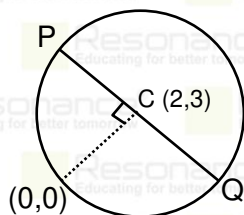
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Ans. (1)

Sol. $(x-2)^2 + (y-3)^3 = 13$



$$y = \frac{3}{2}x$$

line perpendicular to the above line and passing through (2,3) is $3y + 2x = 13$

Coordinates of P,Q $\Rightarrow (2 \pm \sqrt{13} \cos\theta, 3 \pm \sqrt{13} \sin\theta)$

$$\Rightarrow \left(2 \pm \sqrt{13} \left(\frac{-3}{\sqrt{13}} \right), 3 \pm \sqrt{13} \left(\frac{2}{\sqrt{13}} \right) \right)$$

$$\Rightarrow (-1, 5) \text{ \& } (5, 1)$$

16. Evaluate $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^n \frac{(2j-1) + 8n}{((2j-1) + 4n)}$

(1) $1 + 2\ln\left(\frac{3}{2}\right)$

(2) $1 - 2\ln\left(\frac{3}{2}\right)$

(3) $2\ln\left(\frac{3}{2}\right) - 1$

(4) $3\ln\left(\frac{2}{3}\right) + 5$

Ans. (1)

Sol. $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^n \frac{\left(2\frac{j}{n} - \frac{1}{n} + 8\right)}{\left(2\frac{j}{n} - \frac{1}{n} + 4\right)} \cdot \frac{1}{n}$

$$= \int_0^1 \frac{2x+8}{2x+4} dx$$

$$= \int_0^1 dx + \int_0^1 \frac{4}{2x+4} dx$$

$$= 1 + 4 \cdot \frac{1}{2} [\ln(2x+4)]_0^1$$

$$= 1 + 2[\ln 6 - \ln 4]$$

$$= 1 + 2\ln\left(\frac{3}{2}\right)$$

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17. If $f(x) = \begin{cases} (1 + |\sin x|)^{\frac{a}{|\sin x|}} & -\frac{\pi}{4} < x < 0 \\ b & x = 0 \\ e^{\frac{\cot(4x)}{\cot(2x)}} & 0 < x < \frac{\pi}{4} \end{cases}$ is continuous, then find the value of $(a + \ln b)$.

Ans. (1)

Sol. RHL = $f(0^+) = \lim_{x \rightarrow 0^+} e^{\frac{\cot 4x}{\cot 2x}} = \lim_{x \rightarrow 0^+} e^{\frac{\tan 2x}{\tan 4x}} = e^{\frac{1}{2}}$

LHL = $f(0^-) = \lim_{x \rightarrow 0^-} (1 - \sin x)^{\frac{a}{-\sin x}}$

$= \lim_{x \rightarrow 0^-} e^{\frac{-a}{-\sin x} (-\sin x)} = e^a$

For $f(x)$ to be continuous

$f(0^+) = f(0^-) = f(0)$

$\Rightarrow e^{\frac{1}{2}} = e^a = b \Rightarrow a = \frac{1}{2} \text{ \& } b = e^{\frac{1}{2}} \Rightarrow \ln b = \frac{1}{2}$

$\Rightarrow a + \ln b = 1$

18. Evaluate $I = \int_{-\pi/4}^{\pi/4} \frac{dx}{(1 + e^{x \cos x})(\sin^4 x + \cos^4 x)}$

(1) $\frac{\pi}{2}$

(2) $\frac{\pi}{2\sqrt{2}}$

(3) $\frac{\pi}{\sqrt{2}}$

(4) $\frac{\pi}{4}$

Ans. (2)

Sol. $f(a + b - x) = f(x)$

$I = \int_{-\pi/4}^{\pi/4} \frac{e^{x \cos x}}{(1 + e^{x \cos x})(\sin^4 x + \cos^4 x)} dx$

$2I = \int_{-\pi/4}^{\pi/4} \frac{1}{\sin^4 x + \cos^4 x}$

$2I = \int_{-\pi/4}^{\pi/4} \frac{1}{1 - \frac{1}{2} \sin^2 2x} dx$

$2I = \int_{-\pi/4}^{\pi/4} \frac{2 \sec^2 2x}{\sec^2 2x + 1} dx$ (put $\tan 2x = t$)

$2I = \frac{4}{2} \int_0^\infty \frac{dt}{2 + t^2} = \left(\frac{1}{\sqrt{2}} \tan^{-1} \frac{t}{\sqrt{2}} \right)_0^\infty$

$I = \frac{\pi}{2\sqrt{2}}$

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19. If the domain of the function $f(x) = \log_5(\log_4(\log_3(18x-x^2-77)))$ is (a,b) , then find the value of

$$\int_a^b \frac{\sin^3 x \, dx}{\sin^3 x + \sin^3(a+b-x)}$$

Ans. (1)

Sol. $\log_4 \log_3 (18x - x^2 - 77) > 0$

$$18x - x^2 - 77 > 3$$

$$x^2 - 18x + 80 < 0$$

$$(x-8)(x-10) < 0$$

$$x \in (8, 10)$$

$$\therefore a = 8 \text{ and } b = 10$$

$$\therefore I = \int_a^b \frac{\sin^3 x \, dx}{\sin^3 x + \sin^3(a+b-x)} = \int_8^{10} \frac{\sin^3 x \, dx}{\sin^3 x + \sin^3(a+b-x)}$$

Using the property

$$\int_a^b f(x) \, dx = \int_a^b f(a+b-x) \, dx, \text{ we get}$$

$$2I = \int_8^{10} 1 \, dx = 2$$

$$\therefore I = 1 \text{ Ans.}$$

20. Find area bounded by $y = \max\{0, \ln x\}$ and $y < 2^x$ where $\frac{1}{2} < x < 1$:

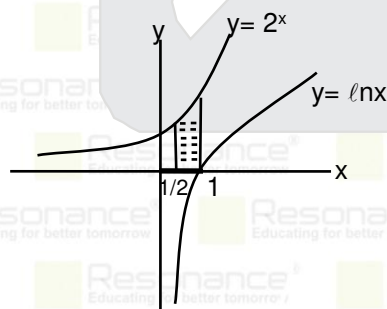
(1) $\frac{2 + \sqrt{2}}{\ln 2}$

(2) $\frac{2 - \sqrt{2}}{\ln 2}$

(3) $\frac{1 - \sqrt{2}}{\ln 2}$

(4) $\frac{1 + \sqrt{2}}{\ln 2}$

Ans. (2)
Sol.



$$\text{Area is} = \int_{\frac{1}{2}}^1 (2^x - 0) \, dx = \left(\frac{2^x}{\ln 2} \right)_{\frac{1}{2}}^1 = \frac{2 - \sqrt{2}}{\ln 2}$$

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21. If $f(x) = \begin{vmatrix} \sin^2 x & -2 + \cos^2 x & \cos 2x \\ 2 + \sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1 + \cos 2x \end{vmatrix}$, $x \in (0, \pi]$. Then the maximum value of $f(x)$ is ?

Ans. 06.00

Sol. $R_1 \rightarrow R_1 - R_2$

$R_2 \rightarrow R_2 - R_3$

$$= \begin{vmatrix} -2 & -2 & 0 \\ 2 & 0 & -1 \\ \sin^2 x & \cos^2 x & 1 + \cos 2x \end{vmatrix}$$

$$= -2(\cos^2 x) + 2(2 + 2\cos 2x + \sin^2 x)$$

$$= -2\cos^2 x + 4 + 4\cos 2x + 2\sin^2 x$$

$$= -2(\cos^2 x - \sin^2 x) + 4\cos 2x + 4$$

$$= 4 + 2\cos 2x = 2(2 + \cos 2x), \text{ maximum value of } f(x) = 6$$

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