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# JEE

## (Main)

**PAPER-1 (B.E./B. TECH.)**

# 2021

## **COMPUTER BASED TEST (CBT)** **Memory Based Questions & Solutions**

**Date: 22 July, 2021 (SHIFT-2) | TIME : (3.00 p.m. to 6.00 p.m)**

**Duration: 3 Hours | Max. Marks: 300**






**SUBJECT: MATHEMATICS**

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**RESULT: JEE (Advanced),  
JEE (Main), NEET**

**HIGHEST No. of Classroom Selections**  
in JEE (Advanced) 2020 from any Institute of Kota

**5 AIRs in TOP-50 in JEE (Adv.) 2020 from Classroom**

**AIR-2  
(GEN-EWS)**  
**AIR-15**  
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KEJRIWAL**  
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**Zonal Topper  
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**AIR-25**  
**SAMARTH  
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**2nd Rank in  
IIT-Kharagpur Zone**  
**AIR-29**  
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**4505**

Classroom: 3441 | Distance: 1064

Eligible for JEE (Advanced) Through JEE (Main) 2020

**14755**

Classroom: 11047 | Distance: 3708

NEET 2020

**2646**

Classroom: 1833 | Distance: 813

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**PART : MATHEMATICS**

1 If  $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$  and B be a  $3 \times 3$  matrix whose entries are  $\{1,2,3,4,5\}$ , then the number of possible matrices B such that  $AB = BA$  are :  
 (1) 240 (2) 320 (3) 120 (4) 100

Ans. (3)

Sol.  $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Let  $B = \begin{bmatrix} a & b & c \\ p & q & r \\ x & y & z \end{bmatrix}$

Now  $AB = BA$

$$\Rightarrow \begin{bmatrix} p & q & r \\ a & b & c \\ x & y & z \end{bmatrix} = \begin{bmatrix} b & a & c \\ q & p & r \\ y & x & z \end{bmatrix}$$

$\Rightarrow p = b, a = q, r = c, x = y, \& z = z$

Hence number of such matrices are  $5! = 120$

2 The number of all possible numbers less than 10000 that can be formed using the digits 0, 2, 4, 6, 8 (without repetition) are :

Ans. 165

Sol. 1 digit numbers = 5  
 2 digit numbers =  $4 \cdot 4 = 16$   
 3 digit numbers =  $4 \cdot 4 \cdot 3 = 48$   
 4 digit numbers =  $4 \cdot 4 \cdot 3 \cdot 2 = 96$   
 Total =  $5 + 16 + 48 + 96 = 165$

3 If a circle  $36x^2 + 36y^2 - 108x + 120y + c = 0$  neither touches nor cuts the co-ordinate axis and point of intersection of lines  $x - 2y = 4$  and  $2x - y = 5$  also lies inside the circle, then the value of c lies in :  
 (1)  $81 < c < 156$  (2)  $100 < c < 156$  (3)  $81 < c < 150$  (4)  $100 < c < 150$

Ans. (2)

Sol. Intersection point of  $2x - y = 5$  and  $x - 2y = 4$  is  $(2, -1)$   
 So,  $(2, -1)$  lies inside the circle  $\Rightarrow S_1 < 0$   
 $36(2)^2 + 36(-1)^2 - 108(2) + 120(-1) + c < 0$   
 $c < 156 \dots\dots(i)$

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∴ circle  $36x^2 + 36y^2 - 108x + 120y + c = 0$  neither touches nor cuts the co-ordinate axis so

$$g^2 - c < 0 \Rightarrow \left(\frac{-3}{2}\right)^2 - \frac{c}{36} < 0 \Rightarrow c > 81 \dots \dots \dots \text{(ii)}$$

$$\text{and } f^2 - c < 0 \Rightarrow \left(\frac{5}{3}\right)^2 - \frac{c}{36} < 0 \Rightarrow c > 100 \dots \dots \text{(iii)}$$

From (i), (ii) and (iii)

$$100 < c < 156$$

4. If line  $2x + y = k$ , ( $k < 0$ ) is a tangent to both the curves  $x^2 - y^2 = 3$  and  $y^2 = \alpha x$ , then the value of  $\alpha$  is

**Ans. 24.00**

**Sol.** Given slope of line ( $m$ ) =  $-2$

slope form of tangent to the curve  $x^2 - y^2 = 3$  is  $y = mx \pm \sqrt{a^2m^2 - b^2}$

$$\Rightarrow y = -2x \pm 3$$

On comparing, with the equation  $2x + y = k$ , ( $k < 0$ )  $\Rightarrow k = -3$

Now, slope form of tangent to the parabola  $y^2 = \alpha x$  is  $y = mx + \frac{\alpha}{4m}$

But  $m = -2$  so

$$y = -2x + \frac{\alpha}{4(-2)} \Rightarrow -3 = \frac{\alpha}{4 \times (-2)}$$

$$\alpha = 24$$

5. The number of all possible values of  $n \in \{1, 2, 3, \dots, 100\}$  which satisfy the condition  $11^n > 10^n + 9^n$  is:

**Ans. (96)**

**Sol.** Let  $11^n > 10^n + 9^n$   $n \in \{1, 2, 3, \dots, 100\}$

$$\Rightarrow 11^n - 9^n > 10^n$$

$$\Rightarrow (10+1)^n - (10-1)^n > 10^n$$

$$\Rightarrow 2 [{}^nC_1 10^{n-1} + {}^nC_3 10^{n-3} + {}^nC_5 10^{n-5} + \dots] > 10^n$$

$$\Rightarrow \frac{1}{5} [{}^nC_1 10^n + {}^nC_3 10^{n-2} + {}^nC_5 10^{n-4} + \dots] > 10^n$$

$$\Rightarrow \frac{1}{5} [{}^nC_1 + {}^nC_3 10^{-2} + {}^nC_5 10^{-4} + \dots] > 1$$

Clearly the above inequality is true for  $n \geq 5$

For  $n = 4$  we have  $\frac{1}{5} \left[ 4 + \frac{4}{10^2} \right] = \frac{4}{5} \left( \frac{101}{100} \right) < 1$ , Rejected

Hence, number of such  $n \in \{1, 2, 3, \dots, 100\}$  is equal to 96

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6. If an AP,  $S_{10} = 530$  &  $S_5 = 140$  (where  $S_n$  denotes the sum of first  $n$  terms of an AP), then the value of

$S_{20} - S_6$  is

- (1) 1562                      (2) 1862                      (3) 1762                      (4) 1662

Ans. (2)

Sol.  $S_{10} = 530$

$$\frac{10}{2}[2a + 9d] = 530$$

$$2a + 9d = 106 \quad \dots (1)$$

$$S_5 = 140$$

$$\frac{5}{2}[2a + 4d] = 140$$

$$2a + 4d = 56 \quad \dots (2)$$

$$5d = 50$$

$$d = 10$$

$$a = 8$$

Now,

$$S_{20} - S_6 =$$

$$10[2a + 19d] - 3[2a + 5d]$$

$$14a + 175d$$

$$14 \times 8 + (175)10 = 1862$$

7. The value of  $r$  for which the term independent of  $x$  in the expansion of  $\left(2x^r + \frac{1}{x^2}\right)^{10}$  is 180 is:

- (1) 7                      (2) 8                      (3) 9                      (4) 10

Ans. (2)

Sol.  $T_{k+1} = {}^{10}C_k (2x^r)^{10-k} (x)^{-2k} \Rightarrow {}^{10}C_k 2^{10-k} \cdot x^{10r-rk-2k}$

$$\text{Now, } 10r - rk - 2k = 0 \Rightarrow r = \frac{2k}{10-k}$$

$$\text{And } {}^{10}C_k (2)^{10-k} = 180 \Rightarrow k = 8$$

$$r = \frac{2 \times 8}{10-8} = 8$$

8. If  $f(x) = \begin{cases} -\frac{4}{3}x^3 + 2x^2 + 3 & ; x > 0 \\ 3xe^x & ; x \leq 0 \end{cases}$  then interval in which  $f(x)$  is increasing, is :

- (1)  $\left(-1, \frac{3}{2}\right)$                       (2)  $(0, 1)$                       (3)  $\left(\frac{-3}{2}, 1\right)$                       (4)  $\left(\frac{1}{2}, 2\right)$

Ans. (B)

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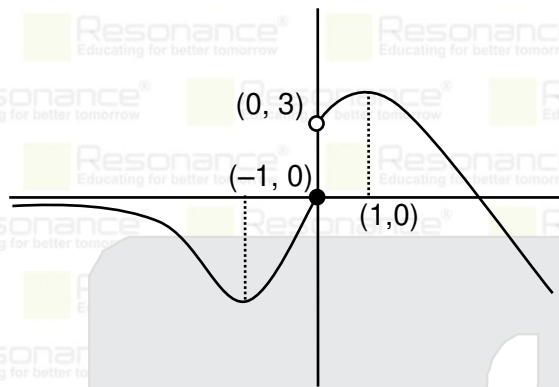
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Sol.  $f'(x) = \begin{cases} -4x^2 + 4x, & x > 0 \\ 3(x.e^x + e^x), & x \leq 0 \end{cases}$



9. If  $z^2 + 3\bar{z} = 0$  has  $n$  solutions, then the value of  $\sum_{k=0}^{\infty} \frac{1}{n^k}$  is :

(1)  $\frac{3}{4}$

(2)  $\frac{4}{3}$

(3)  $\frac{5}{5}$

(4)  $\frac{1}{2}$

Ans. (2)

Sol. Let  $z = x + iy$

$$(x + iy)^2 + 3(x - iy) = 0$$

$$x^2 - y^2 + 2ixy + 3x - 3iy = 0$$

$$x^2 - y^2 + 3x = 0 \text{ \& } 2xy - 3y = 0$$

Case-1:  $y = 0$

$$x^2 - y^2 + 3x = 0$$

$$\Rightarrow x = 0 \text{ or } x = -3$$

Solutions are  $z=0$  and  $z=-3$

Case-2:  $x = \frac{3}{2}$

$$x^2 - y^2 + 3x = 0$$

$$\Rightarrow y = \frac{3\sqrt{3}}{2} \text{ or } y = \frac{-3\sqrt{3}}{2}$$

$$\text{Solutions are } z = \frac{3}{2} + i\frac{3\sqrt{3}}{2} \text{ and } z = \frac{3}{2} - i\frac{3\sqrt{3}}{2}$$

Total number of solutions =  $n = 4$

$$\text{So } \sum_{k=0}^{\infty} \frac{1}{4^k} = \frac{1}{1 - \frac{1}{4}} = \frac{4}{3} \text{ Ans.}$$

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10 The number of solutions of the equation  $\sin^7 x + \cos^7 x = 1$  in the interval  $[0, 4\pi]$  is :

- (1) 7 (2) 9 (3) 14 (4) 5

Ans. (4)

Sol.  $\sin^2 x + \cos^2 x = 1$ ,  $\sin^2 x \leq 1$  and  $\cos^2 x \leq 1$

$$\sin^7 x \leq \sin^2 x$$

$$\cos^7 x \leq \cos^2 x$$

$$\text{so, } \sin^7 x + \cos^7 x \leq 1$$

$$\sin^7 x + \cos^7 x = 1 \text{ when } \sin^7 x = \sin^2 x \text{ \& } \cos^7 x = \cos^2 x$$

$$\text{Case-1 : } \sin x = 0, \cos x = 1 \Rightarrow x = 0, 2\pi, 4\pi$$

$$\text{Case-2 : } \sin x = 1, \cos x = 0 \Rightarrow x = \frac{\pi}{2}, \frac{5\pi}{2}$$

Total number of solutions = 5

11. The sum of all natural numbers belonging to the set  $\{1, 2, 3, \dots, 100\}$ , whose HCF with 2304 is 1, is

- (1) 2449 (2) 1633 (3) 1449 (4) 2633

Ans. (2)

Sol.  $2304 = 2^8 \cdot 3^2$

Hence n can not be multiple of 2 or 3

Then sum is

$$\Rightarrow n(1) - (n(2) + n(3) - n(6))$$

(where n(a) means the sum of all numbers belonging to the set  $\{1, 2, 3, \dots, 100\}$ , which are divisible by a)

$$\Rightarrow \frac{100 \times 101}{2} - \frac{2 \times 50 \times 51}{2} - 3 \times \frac{33 \times 34}{2} + 6 \times \frac{17 \times 16}{2}$$

$$\Rightarrow 5050 - 2550 - 1683 + 816$$

$$\Rightarrow 1633$$

12. If  $f(x)$  is a continuous function defined as  $f(x) = \begin{cases} \frac{x^3}{(1 - \cos 2x)^2} \ln \left( \frac{1 + \alpha x e^x}{(1 + x e^x)^2} \right) & ; x < 0 \\ \alpha & ; x \geq 0 \end{cases}$ , then the value

of  $\alpha$  is

- (1)  $-\frac{2}{3}$  (2)  $\frac{2}{3}$  (3)  $\frac{1}{3}$  (4)  $-\frac{1}{3}$

Ans. (1)

$$\text{Sol.} = \lim_{x \rightarrow 0^-} \frac{x^3}{(1 - \cos 2x)^2} \ln \left( \frac{1 + \alpha x e^x}{(1 + x e^x)^2} \right)$$

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$$= \lim_{x \rightarrow 0^-} \frac{x^3 \times x}{4 \sin^4 x} \frac{\ln(1 + \alpha x e^x) - 2 \ln(1 + x e^x)}{x}$$

$$= \lim_{x \rightarrow 0^-} \frac{1}{4} \frac{\ln(1 + \alpha x e^x) - 2 \ln(1 + x e^x)}{x}$$

$$= \lim_{x \rightarrow 0^-} \frac{1}{4} \left\{ \frac{\ln(1 + \alpha x e^x) \alpha e^x}{\alpha e^x} - \frac{2 \ln(1 + x e^x) e^x}{\alpha e^x} \right\}$$

$$= \lim_{x \rightarrow 0^-} \frac{1}{4} \{ \alpha e^x - 2e^x \}$$

$$= \frac{1}{4} (\alpha - 2)$$

Now,  $\lim_{x \rightarrow 0^+} \alpha = \alpha$

$\therefore f(x)$  is continuous function so

$$\frac{\alpha - 2}{4} = \alpha \Rightarrow \alpha = \frac{-2}{3}$$

13. If the domain of  $f(x) = \frac{\cos^{-1} \sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1} \left( \frac{2x - 1}{2} \right)}}$  is  $(\alpha + \beta]$ , then the value of  $\alpha + \beta$  is :

(1)  $\frac{1}{2}$

(2)  $\frac{3}{2}$

(3) 1

(4) 2

Ans. (2)

Sol.  $0 \leq x^2 - x + 1 \leq 1$  and  $0 < \frac{2x - 1}{2} \leq 1$

$$\Rightarrow x(x - 1) \leq 0 \quad \& \quad 1 < 2x \leq 3$$

$$\Rightarrow x \in [0, 1] \cap x \in \left( \frac{1}{2}, \frac{3}{2} \right]$$

$$\Rightarrow x \in \left( \frac{1}{2}, 1 \right]$$

Hence  $\alpha + \beta = \frac{1}{2} + 1 = \frac{3}{2}$

14. Let  $E_1$  and  $E_2$  be two Ellipses with same eccentricity. Where  $E_1 \equiv \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ , ( $a > b$ ) and  $E_2$  is such that it passes through the end points of major axis of  $E_1$  also end points of minor axis of  $E_1$  are foci of  $E_2$ . Find the eccentricity of  $E_1$

(1)  $\sqrt{\frac{\sqrt{5} - 3}{2}}$

(2)  $\sqrt{\frac{\sqrt{5} + 3}{2}}$

(3)  $\frac{\sqrt{5} + 1}{2}$

(4)  $\frac{\sqrt{5} - 1}{2}$

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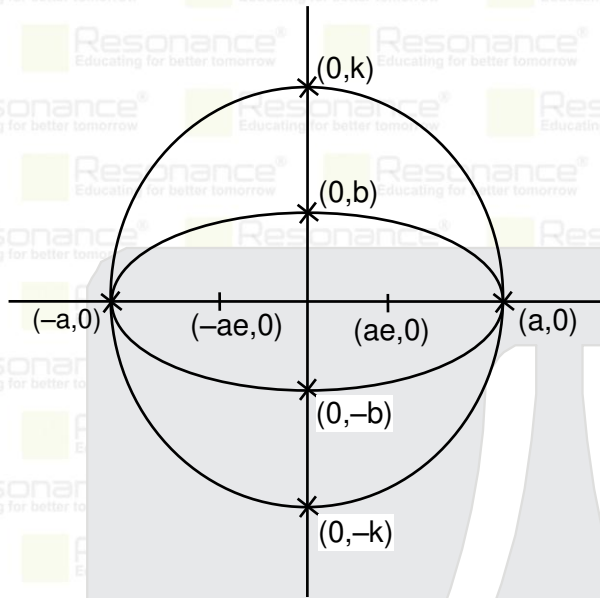
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Ans. (4)

Sol.



Eccentricity of  $E_1$  is  $e \Rightarrow e^2 = 1 - \frac{b^2}{a^2}$

Eccentricity of  $E_2$  is  $e \Rightarrow e^2 = 1 - \frac{a^2}{k^2}$

So,  $e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{a^2}{k^2} \Rightarrow k = \frac{a^2}{b}$  .....(i)

Also  $ke = b$  .....(ii)

From equation (i) and (ii)  $e = \frac{b^2}{a^2}$

Since  $e^2 = 1 - \frac{b^2}{a^2} \Rightarrow e^2 = 1 - e$

$\Rightarrow e^2 + e - 1 = 0$

$\Rightarrow e = \frac{\sqrt{5} - 1}{2}$

15. Value of  $x$  for which  $[e^x]^2 + [e^{x+1}] - 3 = 0$  is

- (1)  $[e, e^2]$       (2)  $[0, e]$       (3)  $[e, \ln 2]$       (4)  $[0, \ln 2]$

Ans. (4)

Sol.  $[e^x]^2 + [e^{x+1}] - 3 = 0$

$[e^x]^2 + [e^x] - 2 = 0$

Let  $[e^x] = t$

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$$t^2 + t - 2 = 0$$

$$(t+2)(t-1) = 0$$

$$t = 1, -2$$

$$[e^x] : 1, -2 \text{ (-2 is not possible)}$$

$$[e^x] = 1$$

$$x \in [0, \ln 2]$$

16. Evaluates  $\int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}} dx$

(1)  $e^x \sqrt{\frac{1+x}{1-2x}} + C$       (2)  $e^x \sqrt{\frac{1+x}{1-x}} + C$       (3)  $e^x \sqrt{\frac{1-x}{1+x}} + C$       (4)  $e^x \left( \frac{1+x}{1-x} \right) + C$

Ans. (2)

Sol.  $I = \int \frac{e^x(2-x^2)}{(1-x)\sqrt{1-x^2}} dx$

$$= \int e^x \left\{ \frac{1}{(1-x)^{3/2}(1+x)^{1/2}} + \left( \frac{1+x}{1-x} \right)^{\frac{1}{2}} \right\} dx$$

$$f(x) = \left( \frac{1+x}{1-x} \right)^{\frac{1}{2}} \Rightarrow f'(x) = \frac{1}{(1-x)^{3/2}(1+x)^{1/2}}$$

$$\Rightarrow I = \int e^x (f'(x) + f(x)) dx$$

$$= e^x f(x) + C = e^x \sqrt{\frac{1+x}{1-x}} + C$$

17. Four dice are rolled and the outcomes are put in  $2 \times 2$  matrices. Find the probability that such a matrix will be non singular and all its entries are different.

(1)  $\frac{71}{81}$       (2)  $\frac{80}{81}$       (3)  $\frac{30}{71}$       (4)  $\frac{25}{71}$

Ans. (2)

Sol.  $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$|x| = ad - bc = 0$$

$$\left. \begin{matrix} (1,6) & (3,2) \\ (3,4) & (6,2) \end{matrix} \right\} 8 + 8 \text{ possibilities}$$

$$\text{Required probability} = 1 - \frac{16}{6^4} = \frac{80}{81}$$

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18. Find the values of  $\lambda$  &  $\mu$  for which the system of equations

$$x + y + z = 6$$

$$3x + 5y + 5z = 26$$

$$x + 2y + \lambda z = \mu \text{ has no solution}$$

(1)  $\lambda = 2, \mu \neq 10$

(2)  $\lambda \neq 2, \mu = 10$

(3)  $\lambda \neq 3, \mu = 10$

(4)  $\lambda \neq 2, \mu \neq 10$

Ans. (1)

Sol. For no solution  $\Delta = 0$

$$\Delta = 0$$

$$\begin{vmatrix} 1 & 1 & 1 \\ 3 & 5 & 5 \\ 1 & 2 & \lambda \end{vmatrix} = 0$$

$$\Rightarrow 1(5\lambda - 10) - 1(3\lambda - 5) + 1(6 - 5) = 0$$

$$\Rightarrow 2\lambda - 4 = 0$$

$$\Rightarrow \lambda = 2$$

$$\Delta_1 = \begin{vmatrix} 6 & 1 & 1 \\ 26 & 5 & 5 \\ \mu & 2 & 2 \end{vmatrix} = 0$$

$$\Delta_2 = \begin{vmatrix} 1 & 6 & 1 \\ 3 & 26 & 5 \\ 1 & \mu & 2 \end{vmatrix} = 1(52 - 5\mu) - 6(6 - 5) + 1(3\mu - 26)$$

$$= 52 - 5\mu - 6 + 3\mu - 26$$

$$\Delta_2 = 20 - 2\mu$$

$$\Delta_3 = \begin{vmatrix} 1 & 1 & 6 \\ 3 & 5 & 26 \\ 1 & 2 & \mu \end{vmatrix} = 1(5\mu - 52) - 1(3\mu - 26) + 6(6 - 5)$$

$$\Delta_3 = 2\mu - 20$$

Case-I

$$\lambda = 2, \mu = 10 \Rightarrow \Delta = 0, \Delta_1 = 0, \Delta_2 = 0, \Delta_3 = 0$$

system of equations are

$$x + y + z = 6$$

$$3x + 5y + 5z = 26$$

$$x + 2y + 2z = 10 \text{ has infinite many solutions}$$

Case - II

$$\lambda = 2, \mu \neq 10 \Rightarrow \Delta = 0, \Delta_1 = 0, \Delta_2 \neq 0, \Delta_3 \neq 0$$

system has not solution

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19. The number of elements in the set  $\{x \in \mathbb{R} : (|x| - 3)|x - 4| = 6\}$  is equal to

- (1) 3 (2) 4 (3) 2 (4) 1

Ans. (3)

Sol.  $(|x| - 3)|x - 4| = 6$

Case - 1

$$x \geq 4$$

$$(x - 3)(x - 4) = 6$$

$$x^2 - 7x + 6 = 0$$

$$(x - 1)(x - 6) = 0$$

$$x = 1, 6 \Rightarrow x = 6$$

Case - 2

$$0 < x < 4$$

$$(x - 3)(4 - x) = 6$$

$$x^2 - 7x + 18 = 0$$

$D < 0$ , No solution

Case - 3

$$x \leq 0$$

$$(-x - 3)(4 - x) = 6$$

$$(x + 3)(x - 4) = 6$$

$$x^2 - x - 18 = 0$$

$$x = \frac{1 \pm \sqrt{73}}{2}$$

$$x = \frac{1 - \sqrt{73}}{2}$$

20. If  $\int_0^{100\pi} \frac{\sin^2 x}{e^{\left(\frac{x}{\pi} - \left[\frac{x}{\pi}\right]\right)}} dx = \frac{\alpha\pi^3}{1 + 4\pi^2}$ ,  $\alpha \in \mathbb{R}$  where  $[x]$  is greatest integer function, then  $\alpha$  is

- (1)  $50(e - 1)$  (2)  $150(e^{-1} - 1)$  (3)  $200(1 - e^{-1})$  (4)  $100(1 - e)$

Ans. (3)

Sol.  $\int_0^{100\pi} \frac{\sin^2 x}{e^{\left\{\frac{x}{\pi}\right\}}} dx$

$$\Rightarrow 100 \int_0^{\pi} \frac{\sin^2 x}{e^{x/\pi}} dx$$


$$\Rightarrow 50 \int_0^{\pi} e^{-x/\pi} [1 - \cos 2x] dx$$

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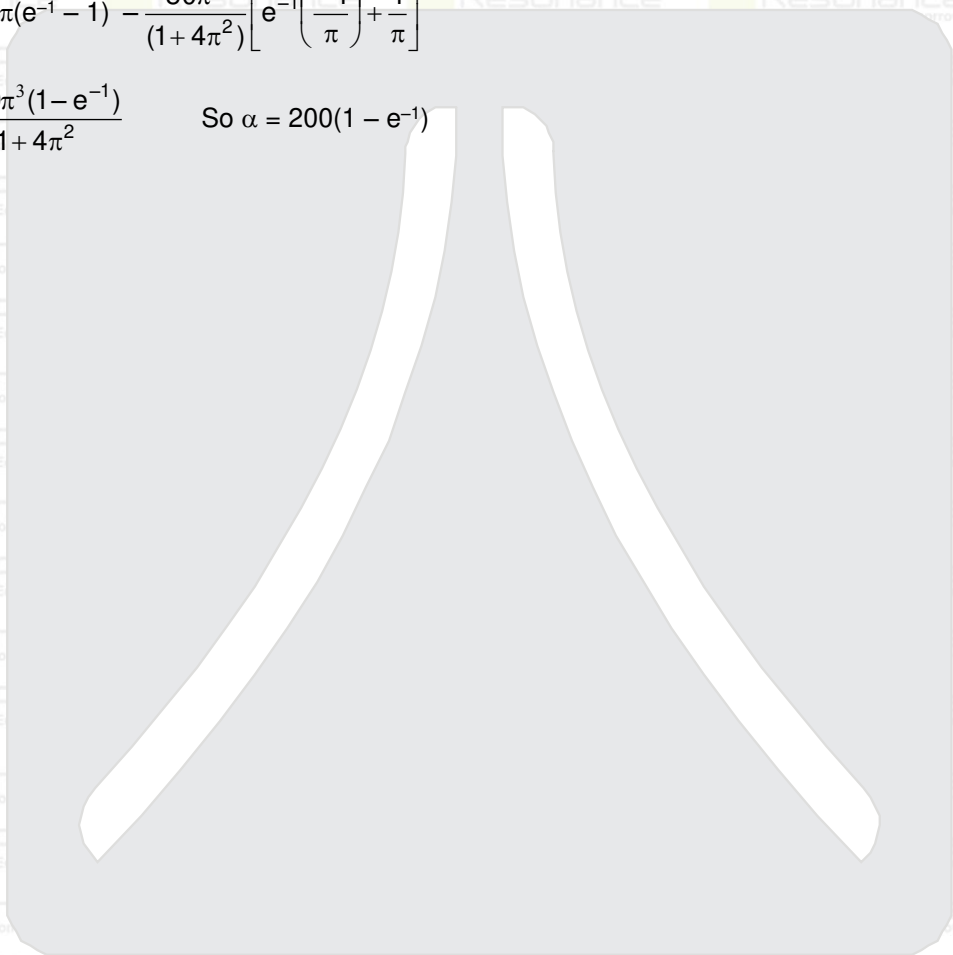
$$\Rightarrow 50 \left[ e^{-x/\pi} \times (-\pi) \right]_0^\pi - 50 \int_0^\pi e^{-x/\pi} \cos 2x dx$$

$$\Rightarrow 50 \times (-\pi)(e^{-1} - 1) - \frac{50 \times \left[ e^{-x/\pi} \left( \frac{-1}{\pi} \times \cos 2x + 2 \sin 2x \right) \right]_0^\pi}{\left( \frac{1}{\pi^2} + 4 \right)}$$

$$\Rightarrow -50\pi(e^{-1} - 1) - \frac{50\pi^2}{(1 + 4\pi^2)} \left[ e^{-1} \left( \frac{-1}{\pi} \right) + \frac{1}{\pi} \right]$$

$$\Rightarrow \frac{200\pi^3(1 - e^{-1})}{1 + 4\pi^2}$$

So  $\alpha = 200(1 - e^{-1})$



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