FINAL JEE–MAIN EXAMINATION – JULY, 2021

(Held On Sunday 25th July, 2021)   **TIME : 9 : 00 AM to 12 : 00**

MATHEMATICS	TEST PAPER WITH ANSWER

SECTION-A

1. A spherical gas balloon of radius 16 meter subtends an angle 60° at the eye of the observer A while the angle of elevation of its center from the eye of A is 75°. Then the height (in meter) of the top most point of the balloon from the level of the observer's eye is :

(1) $8(2 + 2\sqrt{3} + \sqrt{2})$ (2) $8(\sqrt{6} + \sqrt{2} + 2)$

(3) $8(\sqrt{2} + 2 + \sqrt{3})$ (4) $8(\sqrt{6} - \sqrt{2} + 2)$

Official Ans. by NTA (2)

2. Let $f(x) = 3\sin^4 x + 10\sin^3 x + 6\sin^2 x - 3$, $x \in \left[-\dfrac{\pi}{6}, \dfrac{\pi}{2}\right]$. Then, f is :

(1) increasing in $\left(-\dfrac{\pi}{6}, \dfrac{\pi}{2}\right)$

(2) decreasing in $\left(0, \dfrac{\pi}{2}\right)$

(3) increasing in $\left(-\dfrac{\pi}{6}, 0\right)$

(4) decreasing in $\left(-\dfrac{\pi}{6}, 0\right)$

Official Ans. by NTA (4)

3. Let S_n be the sum of the first n terms of an arithmetic progression. If $S_{3n} = 3S_{2n}$, then the value of $\dfrac{S_{4n}}{S_{2n}}$ is :

(1) 6   (2) 4   (3) 2   (4) 8

Official Ans. by NTA (1)

4. The locus of the centroid of the triangle formed by any point P on the hyperbola $16x^2 - 9y^2 + 32x + 36y - 164 = 0$, and its foci is :

(1) $16x^2 - 9y^2 + 32x + 36y - 36 = 0$

(2) $9x^2 - 16y^2 + 36x + 32y - 144 = 0$

(3) $16x^2 - 9y^2 + 32x + 36y - 144 = 0$

(4) $9x^2 - 16y^2 + 36x + 32y - 36 = 0$

Official Ans. by NTA (1)

5. Let the vectors

$(2 + a + b)\hat{i} + (a + 2b + c)\hat{j} - (b + c)\hat{k}$, $(1 + b)\hat{i} + 2b\hat{j} - b\hat{k}$

and $(2 + b)\hat{i} + 2b\hat{j} + (1 - b)\hat{k}$ a, b, c, $\in \mathbf{R}$

be co-planar. Then which of the following is true?

(1) 2b = a + c   (2) 3c = a + b

(3) a = b + 2c   (4) 2a = b + c

Official Ans. by NTA (1)

6. Let f : $\mathbf{R} \rightarrow \mathbf{R}$ be defined as

$$f(x) = \begin{cases} \dfrac{\lambda|x^2 - 5x + 6|}{\mu(5x - x^2 - 6)}, & x < 2 \\ e^{\frac{\tan(x-2)}{x - [x]}} & , x > 2 \\ \mu & , x = 2 \end{cases}$$

where [x] is the greatest integer less than or equal to x. If f is continuous at x = 2, then $\lambda + \mu$ is equal to :

(1) e(–e + 1)   (2) e(e – 2)

(3) 1   (4) 2e – 1

Official Ans. by NTA (1)

7. The value of the definite integral

$\displaystyle\int_{\pi/24}^{5\pi/24} \dfrac{dx}{1 + \sqrt[3]{\tan 2x}}$ is :

(1) $\dfrac{\pi}{3}$ (2) $\dfrac{\pi}{6}$ (3) $\dfrac{\pi}{12}$ (4) $\dfrac{\pi}{18}$

Official Ans. by NTA (3)

8. If b is very small as compared to the value of a, so that the cube and other higher powers of $\dfrac{b}{a}$ can be neglected in the identity

$\dfrac{1}{a - b} + \dfrac{1}{a - 2b} + \dfrac{1}{a - 3b} + ... + \dfrac{1}{a - nb} = \alpha n + \beta n^2 + \gamma n^3$,

then the value of γ is :

(1) $\dfrac{a^2 + b}{3a^3}$ (2) $\dfrac{a + b}{3a^2}$ (3) $\dfrac{b^2}{3a^3}$ (4) $\dfrac{a + b^2}{3a^3}$

Official Ans. by NTA (3)

9. Let $y = y(x)$ be the solution of the differential equation $\dfrac{dy}{dx} = 1 + x\,e^{y-x}, -\sqrt{2} < x < \sqrt{2}, y(0) = 0$

then, the minimum value of $y(x)$, $x \in \left(-\sqrt{2}, \sqrt{2}\right)$ is equal to :

(1) $\left(2 - \sqrt{3}\right) - \log_e 2$

(2) $\left(2 + \sqrt{3}\right) + \log_e 2$

(3) $\left(1 + \sqrt{3}\right) - \log_e\left(\sqrt{3} - 1\right)$

(4) $\left(1 - \sqrt{3}\right) - \log_e\left(\sqrt{3} - 1\right)$

Official Ans. by NTA (4)

10. The Boolean expression

$(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to :

(1) $\sim q$ (2) q

(3) p   (4) $\sim p$

Official Ans. by NTA (4)

11. The area (in sq. units) of the region, given by the set $\{(x, y) \in \mathbf{R} \times \mathbf{R} \mid x \geq 0, 2x^2 \leq y \leq 4 - 2x\}$ is :

(1) $\dfrac{8}{3}$ (2) $\dfrac{17}{3}$

(3) $\dfrac{13}{3}$ (4) $\dfrac{7}{3}$

Official Ans. by NTA (4)

12. The sum of all values of x in $[0, 2\pi]$, for which $\sin x + \sin 2x + \sin 3x + \sin 4x = 0$, is equal to :

(1) 8π (2) 11π

(3) 12π (4) 9π

Official Ans. by NTA (4)

13. Let $g : \mathbf{N} \to \mathbf{N}$ be defined as

$g(3n + 1) = 3n + 2$,

$g(3n + 2) = 3n + 3$,

$g(3n + 3) = 3n + 1$, for all $n \geq 0$.

Then which of the following statements is true ?

(1) There exists an onto function $f : \mathbf{N} \to \mathbf{N}$ such that $fog = f$

(2) There exists a one–one function $f: \mathbf{N} \to \mathbf{N}$ such that $fog = f$

(3) $gogog = g$

(4) There exists a function $f : \mathbf{N} \to \mathbf{N}$ such that $gof = f$

Official Ans. by NTA (1)

14. Let $f : [0, \infty) \to [0, \infty)$ be defined as

$f(x) = \int_0^x [y]\,dy$

where $[x]$ is the greatest integer less than or equal to x. Which of the following is true?

(1) f is continuous at every point in $[0, \infty)$ and differentiable except at the integer points.

(2) f is both continuous and differentiable except at the integer points in $[0, \infty)$.

(3) f is continuous everywhere except at the integer points in $[0, \infty)$.

(4) f is differentiable at every point in $[0, \infty)$.

Official Ans. by NTA (1)

15. The values of a and b, for which the system of equations

$$2x + 3y + 6z = 8$$
$$x + 2y + az = 5$$
$$3x + 5y + 9z = b$$

has no solution, are :

(1) $a = 3, b \neq 13$ (2) $a \neq 3, b \neq 13$

(3) $a \neq 3, b = 3$ (4) $a = 3, b = 13$

Official Ans. by NTA (1)

16. Let 9 distinct balls be distributed among 4 boxes, B_1, B_2, B_3 and B_4. If the probability than B_3 contains exactly 3 balls is $k\left(\dfrac{3}{4}\right)^9$ then k lies in the set :

(1) $\{x \in \mathbf{R} : |x - 3| < 1\}$ (2) $\{x \in \mathbf{R} : |x - 2| \leq 1\}$

(3) $\{x \in \mathbf{R} : |x - 1| < 1\}$ (4) $\{x \in \mathbf{R} : |x - 5| \leq 1\}$

Official Ans. by NTA (1)

17. Let a parabola P be such that its vertex and focus lie on the positive x–axis at a distance 2 and 4 units from the origin, respectively. If tangents are drawn from O(0, 0) to the parabola P which meet P at S and R, then the area (in sq. units) of ΔSOR is equal to :

(1) $16\sqrt{2}$ (2) 16

(3) 32   (4) $8\sqrt{2}$

Official Ans. by NTA (2)

18. The number of real roots of the equation

$e^{6x} - e^{4x} - 2e^{3x} - 12e^{2x} + e^x + 1 = 0$ is :

(1) 2   (2) 4

(3) 6   (4) 1

Official Ans. by NTA (1)

19. Let an ellipse $E : \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$, $a^2 > b^2$, passes through $\left(\sqrt{\dfrac{3}{2}}, 1\right)$ and has eccentricity $\dfrac{1}{\sqrt{3}}$. If a circle, centered at focus $F(\alpha, 0)$, $\alpha > 0$, of E and radius $\dfrac{2}{\sqrt{3}}$, intersects E at two points P and Q, then PQ^2 is equal to :

(1) $\dfrac{8}{3}$ (2) $\dfrac{4}{3}$ (3) $\dfrac{16}{3}$ (4) 3

Official Ans. by NTA (3)

20. Let the foot of perpendicular from a point P(1, 2, –1) to the straight line $L : \dfrac{x}{1} = \dfrac{y}{0} = \dfrac{z}{-1}$ be N. Let a line be drawn from P parallel to the plane x + y + 2z = 0 which meets L at point Q. If α is the acute angle between the lines PN and PQ, then cosα is equal to _____.

(1) $\dfrac{1}{\sqrt{5}}$ (2) $\dfrac{\sqrt{3}}{2}$ (3) $\dfrac{1}{\sqrt{3}}$ (4) $\dfrac{1}{2\sqrt{3}}$

Official Ans. by NTA (3)

SECTION-B

1. Let y = y(x) be solution of the following differential equation

$e^y \dfrac{dy}{dx} - 2e^y \sin x + \sin x \cos^2 x = 0$, $y\left(\dfrac{\pi}{2}\right) = 0$

If $y(0) = \log_e(\alpha + \beta e^{-2})$, then $4(\alpha + \beta)$ is equal to _____.

Official Ans. by NTA (4)

2. If the value of

$\left(1 + \dfrac{2}{3} + \dfrac{6}{3^2} + \dfrac{10}{3^3} + \ldots\ldots\text{upto } \infty\right)^{\log_{(0.25)}\left(\frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots\ldots\text{upto } \infty\right)}$

is l, then l^2 is equal to _____.

Official Ans. by NTA (3)

3. Consider the following frequency distribution :

class :	10–20	20–30	30–40	40–50	50–60
Frequency :	α	110	54	30	β

If the sum of all frequencies is 584 and median is 45, then $|\alpha - \beta|$ is equal to _____.

Official Ans. by NTA (164)

4. Let $\vec{p} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{q} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. If a vector $\vec{r} = (\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k})$ is perpendicular to each of the vectors $(\vec{p} + \vec{q})$ and $(\vec{p} - \vec{q})$, and $|\vec{r}| = \sqrt{3}$, then $|\alpha| + |\beta| + |\gamma|$ is equal to _____.

Official Ans. by NTA (3)

5. The ratio of the coefficient of the middle term in the expansion of $(1 + x)^{20}$ and the sum of the coefficients of two middle terms in expansion of $(1 + x)^{19}$ is _____.

Official Ans. by NTA (1)

6. Let $M = \left\{ A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \in \{\pm 3, \pm 2, \pm 1, 0\} \right\}$.

Define $f : M \to \mathbf{Z}$, as f(A) = det(A), for all A $\in$ M, where $\mathbf{Z}$ is set of all integers. Then the number of A $\in$ M such that f(A) = 15 is equal to _____.

Official Ans. by NTA (16)

7. There are 5 students in class 10, 6 students in class 11 and 8 students in class 12. If the number of ways, in which 10 students can be selected from them so as to include at least 2 students from each class and at most 5 students from the total 11 students of class 10 and 11 is 100 k, then k is equal to _____.

Official Ans. by NTA (238)

8. If α, β are roots of the equation $x^2 + 5(\sqrt{2})x + 10 = 0$, $\alpha > \beta$ and $P_n = \alpha^n - \beta^n$ for each positive integer n, then the value of $\left(\dfrac{P_{17}P_{20} + 5\sqrt{2}P_{17}P_{19}}{P_{18}P_{19} + 5\sqrt{2}P_{18}^2}\right)$ is equal to _____.

Official Ans. by NTA (1)

9. The term independent of 'x' in the expansion of $\left(\dfrac{x+1}{x^{2/3} - x^{1/3} + 1} - \dfrac{x-1}{x - x^{1/2}}\right)^{10}$, where x $\neq$ 0, 1 is equal to _____.

Official Ans. by NTA (210)

10. Let

$S = \left\{ n \in \mathbf{N} \left| \begin{pmatrix} 0 & i \\ 1 & 0 \end{pmatrix}^n \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \forall a, b, c, d \in \mathbf{R} \right. \right\}$,

where $i = \sqrt{-1}$. Then the number of 2–digit numbers in the set S is _____.

Official Ans. by NTA (11)