... THAT THIS QUESTION BOOKLET CONTAINS
120 QUESTIONS SERIALLY NUMBERED FROM 1 TO 120
PRINTED PAGES 32

1. The set of all integer values of x that satisfy the inequality $19 \le -3x \le 27$ is

(A) $\{-9,-8,-7,-6\}$ (B) $\{-9,-6\}$ (C) $\{-9,-8,-7\}$

(D) $\{-9,-8,-7,\ldots,4,5,6\}$ (E) $\varnothing$

2. Let X be the set $\left\{1, \pi, \left\{42, \sqrt{2}\right\}, \{1,3\}\right\}$. Which of the following statement(s) is/are true?

$P: \pi \in X$ $Q: \{1,3\} \subseteq X$ $R: \{1,\pi\} \subseteq X$

(A) P only (B) Q only (C) R only

(D) P and R only (E) P, Q and R

3. The value of θ in the range $0 \le \theta \le \frac{\pi}{2}$ which satisfies the equation $\sin\left(\theta + \frac{\pi}{6}\right) = \cos\theta$ is equal to

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{8}$ (E) $\frac{\pi}{5}$

4. If $\operatorname{cosec}\theta + \cot\theta = 5$, then the value of $\tan\theta$ is equal to

(A) $\frac{13}{24}$ (B) $\frac{5}{12}$ (C) $\frac{7}{12}$ (D) $\frac{1}{12}$ (E) $\frac{3}{12}$

5. The value of $\tan^{-1}\left(\frac{7}{4}\right)-\tan^{-1}\left(\frac{3}{11}\right)$ is equal to

(A) $\frac{-\pi}{3}$ (B) $\frac{-\pi}{4}$ (C) $\frac{\pi}{4}$ (D) $\frac{\pi}{3}$ (E) π

6. If $0<\theta<\frac{\pi}{2}$ and $\tan\theta=\frac{\sqrt{5}}{2}$, then $\cos\theta$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$ (E) $\frac{\sqrt{5}}{3}$

7. The value of $\sin^2\left(\cos^{-1}\left(\frac{3}{5}\right)\right)$ is equal to

(A) $\frac{4}{5}$ (B) $\frac{16}{25}$ (C) $\frac{9}{25}$ (D) $\frac{5}{3}$ (E) $\frac{25}{9}$

8. $\cos^4\frac{\pi}{12}-\sin^4\frac{\pi}{12}$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{\sqrt{3}+1}{2}$ (D) $\frac{\sqrt{3}-1}{2}$ (E) $\frac{\sqrt{2}}{2}$

9. $\tan\left(2\tan^{-1}\left(\frac{2}{5}\right)\right)$ is equal to

(A) $\frac{8}{5}$ (B) $\frac{10}{21}$ (C) $\frac{20}{21}$ (D) $\frac{21}{25}$ (E) $\frac{4}{25}$

10. The values of x in the interval $[0,\pi]$ such that $\sin 2x = \frac{\sqrt{3}}{2}$ are

(A) $\frac{\pi}{6}, \frac{\pi}{3}$ (B) $\frac{\pi}{6}, \frac{2\pi}{3}$ (C) $\frac{\pi}{3}, \frac{2\pi}{3}$ (D) $\frac{\pi}{6}, \frac{5\pi}{6}$ (E) $\frac{\pi}{3}, \frac{5\pi}{6}$

11. If $\sin\alpha + \sin\beta = \frac{\sqrt{6}}{2}$ and $\cos\alpha + \cos\beta = \frac{\sqrt{2}}{2}$, then $\cos(\alpha - \beta)$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $\frac{-1}{2}$ (D) $\frac{-3}{2}$ (E) 0

12. If $ay = x + b$ is the equation of the line passing through the points $(-5,-2)$ and $(4, 7)$, then the value of $2a+b$ is equal to

(A) 1 (B) 3 (C) 5 (D) -3 (E) -1

13. The y-intercept of the line passing through (2, 5) with slope $\frac{1}{2}$ is equal to

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

14. The equation of perpendicular bisector of the line segment joining the points (10, 0) and $(0, -4)$ is

(A) $5x+2y=21$ (B) $5x+2y=0$ (C) $2x-5y=21$

(D) $5x-2y=21$ (E) $2x+3y=21$

15. The equation of the line which is parallel to $x+\frac{1}{2}y=\frac{3}{2}$ and passing through (1, 3) is

(A) $2x+y=7$ (B) $2x+y+5=0$ (C) $2x+y=3$

(D) $2x+y=6$ (E) $2x+y=5$

16. If x-intercept of the straight line $ax+2ay=30$ is 10, then the y-intercept is

(A) 5 (B) 10 (C) 15 (D) 20 (E) 30

17. A straight line makes an angle α with the positive direction of x-axis, where $\cos\alpha = \frac{\sqrt{3}}{2}$. If it passes through $(0,-2)$, then its equation is

(A) $\sqrt{3}x+y+2=0$ (B) $\sqrt{3}y+x+2=0$ (C) $\sqrt{3}y+x+2\sqrt{3}=0$

(D) $\sqrt{3}y-x+2\sqrt{3}=0$ (E) $\sqrt{3}x+y-2\sqrt{3}=0$

18. The equation of the circle is $3x^2+3y^2+6x-4y-1=0$. Then its radius is

(A) $\frac{1}{3}$ (B) $\frac{4}{3}$ (C) $\frac{2}{3}$ (D) $\frac{16}{3}$ (E) $\frac{8}{3}$

19. The end-points of a diameter of a circle are $(-1, 4)$ and $(5, 4)$. Then the equation of the circle is

(A) $(x-3)^2+y^2=9$ (B) $(x-3)^2+(y+4)^2=3$ (C) $(x-2)^2+(y-4)^2=9$

(D) $(x+3)^2+(y+4)^2=9$ (E) $(x-3)^2+(y-4)^2=4$

20. The two diameters of a circle are segments of the straight lines $x-y=5$ and $2x+y=4$. If the radius of the circle is 5, then the equation of the circle is

(A) $x^2+y^2-6x+4y=12$ (B) $x^2+y^2-3x+2y=12$ (C) $x^2+y^2-6x+2y=12$

(D) $x^2+y^2-8x+6y-18=0$ (E) $x^2+y^2-8x+6y-7=0$

Space for rough work

21. The equation of the parabola with vertex $(-6, 2)$, passing through $(-3, 5)$ and having axis parallel to x-axis is

(A) $(y+2)^2 = 3x+16$ (B) $(x+6)^2 = 3y-6$ (C) $(y+2)^2 = 4x+48$

(D) $(x-6)^2 = 4y-8$ (E) $(y-2)^2 = 3x+18$

22. One of the vertices of the major axis of an ellipse is (1, 1) and one of the vertices of its minor axis is (–2, –1). If the centre of the ellipse is (–2, 1), then the equation of the ellipse is

(A) $\frac{(x+2)^2}{9}+\frac{(y-1)^2}{4}=1$ (B) $\frac{(x+2)^2}{16}+\frac{(y-1)^2}{4}=1$ (C) $\frac{(x-2)^2}{9}+\frac{(y+1)^2}{4}=1$

(D) $\frac{(x-2)^2}{16}+\frac{(y+1)^2}{4}=1$ (E) $\frac{(x+2)^2}{9}+\frac{(y-1)^2}{2}=1$

23. The equation of the parabola with focus (3, 0) and directrix $x + 3 = 0$ is

(A) $y^2 = 3x-9$ (B) $y^2 = 4x-12$ (C) $y^2 = 12x$

(D) $y^2 = 12x-36$ (E) $y^2 = 12x-9$

24. The eccentricity of the ellipse $\frac{x^2}{36}+\frac{y^2}{16}=1$ is

(A) $\frac{\sqrt{5}}{3}$ (B) $\frac{\sqrt{5}}{6}$ (C) $\frac{\sqrt{30}}{6}$ (D) $\frac{\sqrt{10}}{6}$ (E) $\frac{\sqrt{30}}{7}$

25. The foci of a hyperbola are (8, 3) and (0, 3) and eccentricity is $\frac{4}{3}$. Then the length of the transverse axis is

(A) $\frac{32}{3}$ (B) 4 (C) 8 (D) $\frac{8}{3}$ (E) 6

26. The co-ordinates of the points P and Q are (2, 6, 4) and (8, –3, 1) respectively. If the point R lies on the line segment PQ such that $2|\overrightarrow{PR}| = |\overrightarrow{RQ}|$, then the co-ordinates of R are

(A) (4, –3, 3) (B) (4, 3, –3) (C) (2, –3, 1) (D) (4, 3, 3) (E) (2, 3, 3)

27. If $|\vec{a}|=2, \vec{b}=2\hat{i}-\hat{j}-3\hat{k}$ and the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$, then $\vec{a}\cdot\vec{b}$ is equal to

(A) $14\sqrt{2}$ (B) $2\sqrt{7}$ (C) $\sqrt{30}$ (D) $\sqrt{7}$ (E) $\sqrt{14}$

28. If α is the angle made by the vector $\vec{a} = 5\hat{i} + 3\hat{j} + 4\hat{k}$ with the positive x-axis, then $\cos\alpha =$

(A) $\frac{5}{12}$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{5}}{5}$ (E) $\frac{\sqrt{2}}{10}$

29. If $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{a} - \vec{b}| = \sqrt{7}$, then $\vec{a} \cdot \vec{b}$ is equal to

(A) 7 (B) 8 (C) 9 (D) 10 (E) 12

30. If $\vec{a} = \hat{i} + \lambda\hat{j} - 2\hat{k}$, $\vec{b} = 2\hat{i} - 3\hat{j} + 5\hat{k}$ and $\vec{a} \cdot \vec{b} = -20$, then the value of λ is equal to

(A) 2 (B) −2 (C) −4 (D) 4 (E) 5

31. If $\vec{a} = \hat{i} - 3\hat{j} + \alpha\hat{k}$, $\vec{b} = \hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{a} \times \vec{b} = -2\hat{i} + \hat{j} + \beta\hat{k}$, then the value of β is equal to

(A) −2 (B) 2 (C) −1 (D) 1 (E) −3

32. The values of α so that the vectors $\alpha \hat{i}+(\alpha-1)\hat{j}+3\hat{k}$ and $(\alpha+2)\hat{i}+\alpha\hat{j}-2\hat{k}$ are perpendicular, are

(A) $\frac{3}{2}, -2$ (B) $2, \frac{3}{2}$ (C) $-2, \frac{-3}{2}$ (D) $2, \frac{-3}{2}$ (E) $-4, \frac{3}{2}$

33. If $|\vec{u}| = 5$, $|\vec{v}| = 4$ and the angle between $\vec{u}$ and $\vec{v}$ is $\frac{\pi}{6}$, then $|\vec{u} \times \vec{v}|$ is equal to

(A) $10\sqrt{3}$ (B) $10\sqrt{2}$ (C) 20 (D) $5\sqrt{2}$ (E) 10

34. If the point $P(x,1,4)$ lies on the line $\vec{r} = \hat{i}+3\hat{j}+4\hat{k}+\lambda(2\hat{i}-\hat{j})$, then the value of x is equal to

(A) 2 (B) –2 (C) 3 (D) –3 (E) 5

35. The equation of the plane through the point (2, 1, 3) and perpendicular to the vector $4\hat{i}+5\hat{j}+6\hat{k}$ is

(A) $4x+5y+6z=28$ (B) $2x+y+3z=17$ (C) $4x+5y+6z=33$

(D) $8x+5y+18z=21$ (E) $4x+5y+6z=31$

36. The angle between the line $\vec{r} = \hat{i}+2\hat{j}+t(3\hat{i}+2\hat{j}-\hat{k})$ and the plane $2x-3y-z=1$ is

(A) $\sin^{-1}\left(\frac{1}{196}\right)$ (B) $\sin^{-1}\left(\frac{1}{14}\right)$ (C) $\cos^{-1}\left(\frac{1}{14}\right)$ (D) $\cos^{-1}\left(\frac{13}{14}\right)$ (E) $\sin^{-1}\left(\frac{13}{14}\right)$

37. If the line $\vec{r} = 2\hat{i}+\hat{j}+t(3\hat{i}+\hat{j}-2\hat{k})$ is parallel to the plane $2x+4y+az=8$, then the value of a is equal to

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

38. The angle between the lines $\vec{r} = \hat{i}+4\hat{k}+\lambda(2\hat{i}+\hat{j}-\hat{k})$ and $\vec{r} = 2\hat{i}-\hat{j}+3\hat{k}+\mu(3\hat{i}+\hat{k})$ is

(A) $\cos^{-1}\left(\frac{\sqrt{5}}{6}\right)$ (B) $\cos^{-1}\left(\frac{\sqrt{15}}{6}\right)$ (C) $\cos^{-1}\left(\frac{1}{12}\right)$ (D) $\cos^{-1}\left(\frac{\sqrt{15}}{15}\right)$ (E) $\cos^{-1}\left(\frac{\sqrt{3}}{30}\right)$

39. The Cartesian equation of the line passing through (7, 5, 3) and perpendicular to the plane $3x+2y+z=6$ is

(A) $\frac{x-7}{3}=\frac{y-5}{2}=\frac{z-3}{1}$ (B) $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z-1}{3}$ (C) $\frac{x-3}{7}=\frac{y-2}{5}=\frac{z}{3}$

(D) $\frac{x-7}{3}=\frac{y-5}{1}=\frac{z-3}{2}$ (E) $\frac{x-4}{4}=\frac{y-3}{3}=\frac{z-2}{2}$

40. The acute angle between the planes $2x-y-3z=7$ and $x+2y+2z=0$ is

(A) $\cos^{-1}\left(\frac{-\sqrt{14}}{14}\right)$ (B) $\pi-\cos^{-1}\left(\frac{-\sqrt{14}}{7}\right)$ (C) $\cos^{-1}\left(\frac{\sqrt{14}}{11}\right)$

(D) $\pi-\cos^{-1}\left(\frac{-\sqrt{14}}{21}\right)$ (E) $\pi-\cos^{-1}\left(\frac{\sqrt{14}}{7}\right)$

41. The vector equation of the line joining the points (2, 1, 3) and $(-2, 4, 1)$ is

(A) $\vec{r}=2\hat{i}+\hat{j}+3\hat{k}+\lambda(-4\hat{i}+3\hat{j}-2\hat{k})$ (B) $\vec{r}=2\hat{i}+\hat{j}+3\hat{k}+\lambda(4\hat{i}+3\hat{j}+2\hat{k})$

(C) $\vec{r}=-2\hat{i}+\hat{j}+3\hat{k}+\lambda(-4\hat{i}-3\hat{j}-2\hat{k})$ (D) $\vec{r}=2\hat{i}+\hat{j}+3\hat{k}+\lambda(3\hat{i}-4\hat{j}-2\hat{k})$

(E) $\vec{r}=-4\hat{i}+3\hat{j}-2\hat{k}+\lambda(2\hat{i}+\hat{j}+3\hat{k})$

42. A bag contains 5 yellow, 3 green, 2 blue and 7 white balls. If 4 balls are chosen at random, then the probability that none of them are white is

(A) $\frac{3}{37}$ (B) $\frac{7}{34}$ (C) $\frac{5}{34}$ (D) $\frac{5}{37}$ (E) $\frac{3}{34}$

43. An urn contains 25 marbles which are numbered from 1 to 25 and a marble is chosen at random two times with replacement. Then the probability that both times the marble has the same number is

(A) $\frac{1}{25}$ (B) $\frac{24}{25}$ (C) $\frac{1}{625}$ (D) $\frac{624}{625}$ (E) $\frac{2}{25}$

44. If A and B are two events such that $P(A) = 0.2$, $P(B) = 0.55$ and $P(A \cap B) = 0.1$, then $P\left(B \cap A^C\right)$ is equal to

(A) 0.25 (B) 0.35 (C) 0.45 (D) 0.65 (E) 0.75

45. Two dice are rolled. If A is the event that sum of the numbers is 4 and B is the event that at least one of the dice shows a 3, then $P(A|B)$ is equal to

(A) $\frac{3}{11}$ (B) $\frac{2}{11}$ (C) $\frac{1}{4}$ (D) $\frac{1}{6}$ (E) $\frac{1}{11}$

46. Assume that n distinct values $x_1, x_2, \ldots, x_n$ occur with frequencies $f_1, f_2, \ldots, f_n$ respectively. If $\overline{x} = 7$ and $\sum_{i=1}^{8} f_i x_i = 315$, then $\sum_{i=1}^{8} f_i =$

(A) 35 (B) 45 (C) 48 (D) 42 (E) 40

47. The variance of the data $x_1, x_2, \ldots, x_{50}$ with $\sum_{i=1}^{50} x_i = 650$ and $\sum_{i=1}^{50} x_i^2 = 10000$ is

(A) 30 (B) 40 (C) 39 (D) 41 (E) 31

48. If X is a random variable with $E(X) = 6$ and $V(X) = 3$, then $E\left(X^2\right)$ is equal to

(A) 33 (B) 36 (C) 39 (D) 42 (E) 27

49. Let $f(x) = \frac{4x+3}{x+2}$. Then the value of $f^{-1}(-2)$ is equal to

(A) $\frac{7}{5}$ (B) $\frac{-7}{6}$ (C) $\frac{-7}{5}$ (D) $\frac{7}{6}$ (E) $\frac{5}{6}$

50. If $f(x) = \begin{cases} 2x & \text{for } x < 1 \\ 5a - x & \text{for } x \geq 1 \end{cases}$ is continuous on $\mathbb{R}$, then the value of a is equal to

(A) $\frac{1}{5}$ (B) $\frac{2}{5}$ (C) $\frac{3}{5}$ (D) $\frac{4}{5}$ (E) 1

51. $\lim_{t \to 0} \frac{\sin 2t}{8t^2 + 4t}$ is equal to

(A) $\frac{1}{2}$ (B) $\frac{2}{5}$ (C) $\frac{1}{6}$ (D) $\frac{1}{3}$ (E) 1

52. $\lim_{x \to 0} \frac{x}{\sqrt{9-x}-3}$ is equal to

(A) 6 (B) 3 (C) –3 (D) –6 (E) 0

53. Let $f(x) = \begin{cases} 3x+2, & \text{if } x < -2 \\ x^2-3x-1, & \text{if } x \geq -2 \end{cases}$. Then $\lim_{x \to -2^-} f(x)$ and $\lim_{x \to -2^+} f(x)$ are respectively

(A) –4, 3 (B) 6, 3 (C) –6, 3 (D) –4, 9 (E) 9, –4

54. $\lim_{x \to -3} \frac{x^2+16x+39}{2x^2+7x+3}$ is equal to

(A) 2 (B) $\frac{8}{3}$ (C) $\frac{-8}{3}$ (D) –2 (E) 0

55. Let $f(x) = 6\sqrt[3]{x^5}$. If $f'(x) = ax^p$, where a and p are constants, then the value of p is equal to

(A) $\frac{3}{5}$ (B) $\frac{-2}{5}$ (C) $\frac{2}{3}$ (D) $\frac{-2}{3}$ (E) $\frac{2}{5}$

56. Let $y = (\tan x)^{\sin x}$ for $0 < x < \frac{\pi}{2}$. If $\frac{dy}{dx} = (\tan x)^{\sin x}\left((\cos x)\log(\tan x) + g(x)\right)$, then $g(x) =$

(A) $\sin x \sec^2 x$ (B) $\sec x \operatorname{cosec} x$ (C) $\sec x$

(D) $\operatorname{cosec} x$ (E) $\sin x \tan x$

57. If $f(x) = \left(x^3 + \sin \pi x\right)^5$, then $f'(1)$ is equal to

(A) 2^5 (B) $5(2^4)$ (C) 15 (D) $5(3+\pi)$ (E) $5(3-\pi)$

58. If $h(x) = 4x^3 - 5x + 7$ is the derivative of $f(x)$, then $\lim_{t \to 0} \frac{f(1+t) - f(1)}{t}$ is equal to

(A) 5 (B) 6 (C) 7 (D) 8 (E) 0

59. Let $f(x) = \begin{cases} e^x, & \text{if } x \le 1 \\ mx + 6, & \text{if } x > 1 \end{cases}$ be differentiable at $x = 1$. Then the value of m is

(A) 6 (B) e (C) -6 (D) $-e$ (E) 1

60. $\lim_{t \to 0} \dfrac{\tan^2\left(\frac{\pi}{3}+t\right)-3}{t}$ is equal to

(A) $4\sqrt{3}$ (B) 24 (C) $16\sqrt{3}$ (D) $8\sqrt{3}$ (E) 16

61. If the tangent line to the graph of a function f at the point $x = 3$ has x-intercept $\frac{5}{3}$ and y-intercept -10, then $f'(3)$ is equal to

(A) 3 (B) 5 (C) $\frac{5}{3}$ (D) 6 (E) -10

62. The slope of tangent line to the curve $4x^2+2xy+y^2=12$ at the point $(1, 2)$ is

(A) 2 (B) 1 (C) -1 (D) -2 (E) 0

63. Let $f(x)=\sqrt{x}+5$ for $1 \le x \le 9$. Then the value of c whose existence is guaranteed by the Mean Value Theorem is

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

64. The derivative of a function f is given by $f'(x)=\dfrac{x-5}{\sqrt{x^2+4}}$. Then the interval in which f is increasing, is

(A) $(5,\infty)$ (B) $(0,\infty)$ (C) $(-4,\infty)$ (D) $(-\infty,-4)$ (E) $(-\infty,5)$

65. Let $f(x)=x^2\log x, x>0$. Then the minimum value of f is

(A) $\frac{1}{\sqrt{e}}$ (B) $2e$ (C) $-2e$ (D) $\sqrt{e}$ (E) $\frac{-1}{2e}$

66. A cube is expanding in such a way that its edge is increasing at a rate of 2 inches per second. If its edge is 5 inches long, then the rate of change of its volume is

(A) 150 in^3/sec (B) 75 in^3/sec (C) 50 in^3/sec

(D) 30 in^3/sec (E) 45 in^3/sec

67. $\int x^5 e^{1-x^6} dx =$

(A) $\frac{1}{6}e^{1-x^6}+C$ (B) $-e^{1-x^6}+C$ (C) $\frac{-1}{6}e^{1-x^6}+C$

(D) $\frac{x^5}{5}e^{1-x^6}+C$ (E) $\frac{x^6}{6}e^{1-x^6}+C$

68. $\int (5-4x)e^{-x} dx =$

(A) $e^{-x}(4x-1)+C$ (B) $e^{-x}(9-4x)+C$ (C) $e^{-x}(4x-5)+C$

(D) $e^{-x}(4x-9)+C$ (E) $e^{-x}(5-4x)+C$

69. $\int \frac{\cos(\tan x)}{\cos^2 x} dx =$

(A) $(\tan x)\sin(\tan x)+C$ (B) $\sin(\tan x)+C$ (C) $\sec(\tan x)+C$

(D) $(\cos x)\sin(\tan x)+C$ (E) $\cos^2(\tan x)+C$

70. $\int \frac{1}{e^{2x}-1} dx =$

(A) $2\log\left|e^{2x}-1\right|-x+C$ (B) $x-\frac{1}{2}\log\left|e^{2x}-1\right|+C$ (C) $x+\frac{1}{2}\log\left|e^{2x}-1\right|+C$

(D) $x-\log\left|e^{2x}-1\right|+C$ (E) $\frac{1}{2}\log\left|e^{2x}-1\right|-x+C$

71. $\int \sin 2x \cos x \, dx =$

(A) $\frac{-1}{3}\cos^3 x+C$ (B) $\frac{-2}{3}\cos^3 x+C$ (C) $\frac{2}{3}\cos^3 x+C$

(D) $\frac{1}{3}\cos^3 x+C$ (E) $\frac{-4}{3}\cos^3 x+C$

72. $\int \frac{1}{(1+\cot^2 x)\sin^2 x} dx =$

(A) $\tan^{-1}(\sin x)+C$ (B) $\tan^{-1}(\cos x)+C$ (C) $\cot^{-1}(\sin x)+C$

(D) $\cot^{-1}(\cos x)+C$ (E) $x+C$

73. $\int \frac{4x^9}{x^{10}-10}\,dx =$

(A) $\frac{1}{5}\log\left|x^{10}-10\right|+C$ (B) $\frac{2}{5}\log\left|x^{10}-10\right|+C$ (C) $\frac{1}{10}\log\left|x^{10}-10\right|+C$

(D) $\frac{-2}{5}\log\left|x^{10}-10\right|+C$ (E) $\frac{-1}{10}\log\left|x^{10}-10\right|+C$

74. The value of $\int_0^{\sqrt{3}} \frac{6}{9+x^2}\,dx$ is equal to

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{4}$ (D) $\frac{2\pi}{3}$ (E) 1

75. The value of $\int_{-5}^{5} (4-|x|)\,dx$ is equal to

(A) 18 (B) 10 (C) 12 (D) 16 (E) 15

76. The area of the region bounded by the curves $y=x^2$ and $y=\sqrt{x}$ is (in square units)

(A) $\frac{2}{3}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{5}{6}$ (E) 1

81. The integrating factor of the differential equation $xy' + 2y - 7x^3 = 0$ is

(A) $\log|x|$ (B) x^2 (C) $\frac{1}{x^2}$ (D) $\frac{1}{2}\log|x|$ (E) x

82. The general solution of the differential equation $4xy + 12x + (2x^2 + 3)y' = 0$ is

(A) $\frac{2x^2+3}{y+3} = C$ (B) $\frac{y-3}{2x^2+3} = C$ (C) $\frac{y+2}{2x^2+3} = C$

(D) $(y-3)(2x^2+3) = C$ (E) $(y+3)(2x^2+3) = C$

83. The constraints of a linear programming problem are $x + 2y \leq 10$ and $6x + 3y \leq 18$. Which of the following points lie in the feasible region?

(A) (0, 6) (B) (4, 3) (C) (5, 7) (D) (1, 7) (E) (1, 3)

84. Let $f:[-4, 2] \to \mathbb{R}$ be given by $f(x)=\sqrt{16-x^2}$. Then the range of the function f is

(A) $[0,2]$ (B) $\left[0,2\sqrt{3}\right]$ (C) $[0,4]$ (D) $\left[2\sqrt{3},4\right]$ (E) $[-2,2]$

85. Let $f(x)=x^2$ and $g(x)=\sqrt{9+x}$. Then the value of $(f \circ g - g \circ f)(4)$ is equal to

(A) 6 (B) $\sqrt{6}$ (C) $\sqrt{8}$ (D) 8 (E) 5

86. Let A and B be subsets of the universal set U. If $n(A)=24$, $n(A \cap B)=8$ and $n(U)=63$, then $n(A' \cup B')$ is equal to

(A) 43 (B) 55 (C) 35 (D) 32 (E) 45

87. Let $f(x)=[x], x \in \mathbb{R}$, where $[x]$ denotes the greatest integer $\leq x$. Then the images of the elements -4.6 and 2.7 are respectively

(A) –5, 2 (B) –5, 3 (C) – 4, 2 (D) –3, 3 (E) –4, 3

88. For any two positive rational numbers m and n, a binary operation $*$ is defined by $m * n = \frac{m+n}{3}$, then $\frac{7}{2} * \frac{5}{2}$ is equal to

(A) 4 (B) 6 (C) 2 (D) 8 (E) 9

89. The function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = 7 - 3x$ is

(A) not one–one (B) not onto (C) even (D) one–one and onto (E) odd

90. A relation R on $\{0, 1, 2\}$ is given by $R = \{(0, 0), (1, 1), (0, 1), (2, 2), (1, 2)\}$. Then the relation R is

(A) reflexive (B) symmetric (C) transitive
(D) symmetric and transitive (E) equivalence

91. Let z_1, z_2 and z_3 be three distinct points in the complex plane such that the segment joining z_1 and z_2 is perpendicular to the segment joining z_1 and z_3. If $|z_1 - z_2| = 5$ and $|z_1 - z_3| = 12$ then $|z_2 - z_3|$ is equal to

(A) 17 (B) 7 (C) 13 (D) 14 (E) 9

92. If $\frac{z}{i} = 11 - 13i$, then $z + \overline{z}$ is equal to

(A) −22 (B) 22 (C) 25 (D) 26 (E) −26

93. Let $\alpha = 2-3i$ be a root of the equation $z^2 - 4z + k = 0$, where k is a real number. If β is the other root, then the value of $\alpha^2 + \beta^2$ is

(A) 26 (B) –5 (C) 5 (D) 10 (E) –10

94. If $z = 2 - i\sqrt{3}$, then $|z^4|$ is equal to

(A) 7 (B) $\sqrt{7}$ (C) $7\sqrt{7}$ (D) 49 (E) $49\sqrt{7}$

95. The imaginary part of $z = \frac{2+i}{3-i}$ is

(A) $\frac{5}{8}$ (B) $\frac{-5}{8}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$ (E) $\frac{3}{8}$

96. The area of the triangle on the complex plane formed by the points $z, z+iz$ and iz is 128. Then the value of $|z|$ is

(A) 12 (B) 16 (C) 18 (D) 17 (E) 19

97. If the real part of the complex number $z = \frac{p+2i}{p-i}$, $p \in \mathbb{R}$, $p > 0$ is $\frac{1}{2}$, then the value of p is equal to

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) $\sqrt{5}$ (D) $\frac{\sqrt{3}}{2}$ (E) 1

98. The value of $\sqrt{(-25)}+3\sqrt{(-4)}+2\sqrt{(-9)}$ is equal to

(A) $13i$ (B) $-13i$ (C) $11i$ (D) $-17i$ (E) $17i$

99. The value of $\sum_{k=5}^{36} \frac{1}{k^2-k}$ is

(A) $\frac{7}{36}$ (B) $\frac{1}{9}$ (C) $\frac{2}{9}$ (D) $\frac{1}{12}$ (E) $\frac{5}{36}$

100. If $a_1, a_2, a_3, \ldots, a_n$ are in A. P. with $a_1 = 3$, $a_n = 39$ and $a_1 + a_2 + \ldots + a_n = 210$, then the value of n is equal to

(A) 8 (B) 10 (C) 11 (D) 13 (E) 15

101. Let t_n, $n = 1, 2, 3, \ldots$ be the n^{th} term of the A. P. 5, 8, 11,…. Then the value of n for which $t_n = 305$ is

(A) 101 (B) 100 (C) 103 (D) 99 (E) 95

102. If the first term of a G. P. is 1 and the sum of 3rd and 5th terms is 90, then the positive common ratio of the G. P. is

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Space for rough work

103. In an A.P. the difference between the last and the first terms is 632 and the common difference is 4. Then the number of terms in the A. P. is

(A) 157 (B) 160 (C) 158 (D) 159 (E) 140

104. If the 10^{th} and 12^{th} terms of an A. P. are respectively 15 and 21, then the common difference of the A. P. is

(A) –6 (B) 4 (C) 6 (D) –3 (E) 3

105. The first term of a G. P. is 3 and the common ratio is 2. Then the sum of first eight terms of the G.P. is

(A) 763 (B) 189 (C) 381 (D) 765 (E) 655

106. A covid-19 vaccination reduces the probability of getting covid-19 infection from 0.4 to 0.1. In a city, 45% people are vaccinated. Then the probability that a non-vaccinated person chosen at random in the city gets covid-19 infection is

(A) 0.55 (B) 0.45 (C) 0.32 (D) 0.22 (E) 0.18

107. The number of ways a committee of 3 women and 5 men can be formed from a panel of 8 men and 5 women is

(A) 940 (B) 1120 (C) 560 (D) 760 (E) 520

108. A set contains 9 elements. Then the number of subsets of the set which contains at most 4 elements is

(A) 32 (B) 64 (C) 128 (D) 256 (E) 512

109. If p and q are positive integers such that ${}^{(p+q)}P_2 = 42$ and ${}^{(p-q)}P_2 = 20$, then the values of p and q are respectively

(A) 5, 2 (B) 4, 3 (C) 7, 2 (D) 6, 1 (E) 7, 5

110. The number of 3-digit numbers that can be formed from the digits 0, 2, 3, 5, 7 is (repetition is allowed)

(A) 125 (B) 100 (C) 105 (D) 150 (E) 60

111. If x^{22} is in the $(r+1)^{\text{th}}$ term of the binomial expansion of $\left(3x^3 - x^2\right)^9$, then the value of r is equal to

(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

112. The term independent of x in the binomial expansion of $\left(x+\frac{2}{x^3}\right)^{20}$ is

(A) $\binom{20}{5}2^{15}$ (B) $\binom{20}{15}2^{10}$ (C) $\binom{20}{10}2^{5}$

(D) $\binom{20}{10}2^{10}$ (E) $\binom{20}{5}2^{5}$

113. Let $A+B=\begin{bmatrix}4 & 1 & 4\\ 1 & 4 & 4\end{bmatrix}$ and $B=\begin{bmatrix}1 & 0 & -2\\ -1 & 3 & 0\end{bmatrix}$, then $A=$

(A) $\begin{bmatrix}3 & 1 & 2\\ 0 & 3 & 4\end{bmatrix}$ (B) $\begin{bmatrix}5 & 1 & 2\\ 0 & 7 & 4\end{bmatrix}$ (C) $\begin{bmatrix}3 & -1 & -2\\ 2 & 1 & 4\end{bmatrix}$

(D) $\begin{bmatrix}5 & 1 & 6\\ 2 & 1 & 4\end{bmatrix}$ (E) $\begin{bmatrix}3 & 1 & 6\\ 2 & 1 & 4\end{bmatrix}$

114. The value of the determinant $\begin{vmatrix}4 & 4^2 & 4^3\\ 3 & 3^2 & 3^3\\ 2 & 2^2 & 2^3\end{vmatrix}$ is

(A) 52 (B) −24 (C) 24 (D) 48 (E) −48

115. If $\begin{vmatrix}1 & 2 & 1\\ 0 & x & -3\\ 2 & -1 & x\end{vmatrix}=0$, then the values of x are

(A) 5, −3 (B) 5, 3 (C) −5, 3 (D) 2, 3 (E) −2, −3

116. If $AB = \begin{bmatrix} 4 & 3 \\ 5 & 4 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$, then $B =$

(A) $\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$ (E) $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

117. The matrix $\begin{bmatrix} -2 & 1 & 0 \\ 3 & 4 & 1 \\ -4 & \lambda & 0 \end{bmatrix}$ is non-singular for $\lambda \neq$

(A) 2 (B) –2 (C) 4 (D) –4 (E) 0

118. Let $\begin{vmatrix} x-1 & 2 & 1 \\ 2 & x-1 & 2 \\ 1 & x+2 & x-1 \end{vmatrix} = ax^3 + bx^2 + cx + d$, where a, b, c and d are constants. Then the value of d is

(A) –8 (B) 6 (C) 0 (D) –6 (E) 16

119. If the inequality $-13 \leq x \leq 5$ is expressed in the form $|x-a| \leq b$, then the values of a and b are respectively

(A) 4, 8 (B) –4, 9 (C) 4, 9 (D) 5, 9 (E) –5, 9

120. The solution set of the inequality $5(4x+6) < 25x+10$ is

(A) $(4, \infty)$ (B) $(-\infty, 4)$ (C) $(-\infty, 5)$ (D) $(5, \infty)$ (E) $(-4, 4)$

Space for rough work