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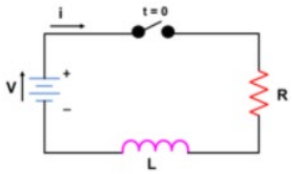
JEE-Main-26-08-2021-Shift-1 (Memory Based)

PHYSICS

Question: In an L R circuit at steady state energy stored in inductor is 64 J and power consumed by circuit is 640 J. Find time constant for this LR circuit.



Options:

(a) 0.2

(b) 0.1

(c) 0.8

(d) 0.25

Answer: (a)

Solution:

In steady state, energy stored in inductor

$$\frac{1}{2}LI^2 = 64J \ldots(1)$$

Power consumed in steady state,

$$I^2R = 640J \ldots(2)$$

Dividing (1) by (2), we get

$$\frac{L}{2R} = \frac{1}{10}$$

$$\Rightarrow \frac{L}{R} = \frac{2}{10} = 0.2$$

Question: If 'E' represents energy, 'G' represents gravitational constant, 'M' represents mass &

'L' represents angular momentum; find $\left[G^{-2}M^2E^{-3}L^3\right]$

Options:

(a) $\left[M^4L^{-6}T^7\right]$

(b) $\left[M^2L^2T^2\right]$

(c) $\left[M^{-3}L^{-3}T^{-4}\right]$

(d) $\left[M^{-4}L^{-2}T^2\right]$

Answer: (a)

Solution:

Dimensional formulae for given quantities

$G\left[M^{-1}L^{3}T^{-2}\right]$

$E\left[ML^{2}T^{-2}\right]$

$M[M]$

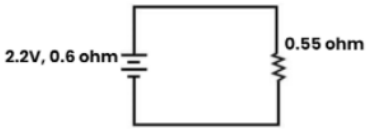
$L\left[ML^{2}T^{-1}\right]$

$\therefore\left[G^{-2}M^{2}E^{-3}L^{3}\right]$

$=\left[M^{-1}L^{3}T^{-2}\right]^{-2}[M]^{2}\left[ML^{2}T^{-2}\right]^{-3}\left[ML^{2}T^{-1}\right]^{3}$

$=\left[M^{4}L^{-6}T^{7}\right]$

Question: Find power loss in battery approximately.



Options:

(a) 2.2 W

(b) 2.4 W

(c) 4.2 W

(d) 2.8 W

Answer: (a)

Solution:

Equivalent resistance of circuit

$=0.55+0.6$

$=1.15\Omega$

Voltage/EMF of the circuit =2.2 V

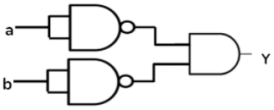
$\therefore \text{current(i)}=\dfrac{2.2}{1.15}\approx 1.91 Amp.$

Power loss in battery $=i^{2}R$

$=(1.91)^{2}(0.6)$

$\approx 2.19\approx 2.2W$

Question: Which logic gate is this?



Options:

(a) AND

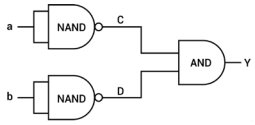
(b) OR

(c) NOR

(d) NAND

Answer: (c)

Solution:



$c = \overline{a \cdot a} = \overline{a}$

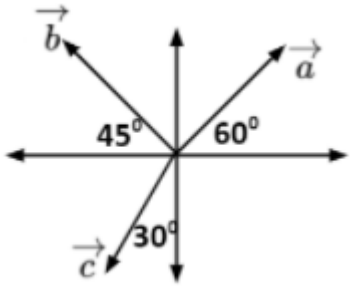
& $d = \overline{b \cdot b} = \overline{b}$

$Y = c \cdot d = \overline{a} \cdot \overline{b}$

$= \overline{a + b}$

$\therefore$ this circuit represents NOR gate

Question: Three vectors of equal magnitude are shown in figure. Find angle θ formed by $\vec{a} + \vec{b} - \vec{c}$ with x axis, $\tan\theta = ?$



Options:

(a) $\frac{\sqrt{6}+1}{\sqrt{2}-1}$

(b) $\frac{\sqrt{6}-1}{\sqrt{2}+1}$

(c) $\frac{1}{\sqrt{2}}$

(d) $\frac{\sqrt{2}-1}{\sqrt{6}+1}$

Answer: (a)

Solution:

Magnitude of each vector be

$\vec{a} = \frac{x}{2}\hat{i} + \frac{x\sqrt{3}}{2}\hat{j}$

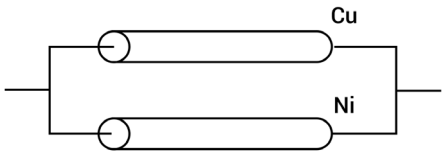
$\vec{b} = -\frac{x}{\sqrt{2}}\hat{i} + \frac{x}{\sqrt{2}}\hat{j}$

$$\vec{a} = -\frac{x}{2}\hat{i} - \frac{x\sqrt{3}}{2}\hat{j}$$

$$\vec{a} + \vec{b} - \vec{c} = \left(x - \frac{x}{\sqrt{2}}\right)\hat{i} + \left(x\sqrt{3} + \frac{x}{\sqrt{2}}\right)\hat{j}$$

$$\tan\theta = \frac{\sqrt{3} + \frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}} = \frac{\sqrt{6}+1}{\sqrt{2}-1}$$

Question: $\rho_{cu} = 12\mu\Omega/cm, \rho_{Ni} = 51\mu\Omega/cm.$ If both conductors have equal length, diameter 2
mm each and have equivalent resistance of 3^n, find I?



Answer: 0.97 cm

Solution:

$$\frac{1}{3} = \frac{1}{R_{Cu}} + \frac{1}{R_{Fe}}; \quad R_{Cu} = \frac{12\ell}{\pi} \times 10^2;$$

$$R_{Ni} = \frac{51\ell}{\pi} \times 10^2$$

$$\frac{1}{3} = \frac{\pi}{100\ell}\left[\frac{1}{12} + \frac{1}{51}\right]$$

$$= \frac{\pi}{100\ell} \times \frac{63}{612}$$

$$\ell = \frac{\pi \times 63}{204 \times 100} = \frac{21\pi}{68} cm \approx 0.97 cm$$

Question: What is the change in stopping potential when wavelength changes from 280 nm to
480 nm. $\phi = 2.65eV$.

Answer: 1.845 volts

Solution:

$$\frac{hc}{\lambda} = \phi + eV$$

$$\frac{hc}{\lambda_1} - \frac{hc}{\lambda_2} = e\left(V_1 - V_2\right)$$

$$V_1 - V_2 = \frac{hc}{e}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$= \frac{1240eV\,nm}{e}\left(\frac{1}{280\,nm} - \frac{1}{480\,nm}\right) = 1.845V$$

Question: Compare the RMS speeds of the gases at same temperature

Options:

(a) $CO_2 > O_2 > H_2$

(b) $CO_2 < O_2 < H_2$

(c) $CO_2 < O_2 = H_2$

(d) $CO_2 > O_2 = H_2$

Answer: (b)

Solution:

$$V_{rms} = \sqrt{\frac{3RT}{M}}$$

Molecular mass of $CO_2 = 44gm$

Molecular mass of $O_2 = 32gm$

Molecular mass of $H_2 = 2gm$

$V_{rms} \propto \frac{1}{\sqrt{M}}$ If $T \rightarrow$ Same for all gases

Hence

$$(V_{rms})_{CO_2} < (V_{rms})_{O_2} < (V_{rms})_{H_2}$$

Question: Two spheres, each of radius r = 5 cm are thrown upwards at an internal of 3s. At what height they will collides? Initial speed is 30 m/s.

Options:

(a) 45 m

(b) 30 m

(c) 33.7 m

(d) 43.3 m

Answer: (c)

Solution:

Assuming balls as point mass objects.

Let first ball is thrown at t = 0

And second ball will be thrown at t = 3s.

After 3s first ball will be at

$$S = u \times 3 - \frac{1}{2}gt^2$$

$$S = 30 \times 3 - \frac{1}{2}10 \times 9$$

$$S = 90 - 45 = 45m$$

Velocity of ball (1) after 3s will be

$v = 30 - 10 \times 3 = 0$

So it will be highest point.

Now let they collide after time t from the time when second ball is throwing and 1st ball is at height point.

$S_1 = +\frac{1}{2} g t^2$

$S_2 = 30 \times t - \frac{1}{2} 10 \times t^2$

$30t = 45$

$t = 1.5s$

$S_1 = \frac{1}{2} 10 \times 1.5 \times 1.5$

$S_1 = \frac{15 \times 1.5}{2}$

$S_1 = 11.25$

$S_2 = 33.75m$

Nearly $S_2 = 33.7\,m$

Question: In a LCR circuit , find the capacitance in order to get maximum power. Given $X_L = 250\Omega$; f = 50Hz.

Options:

(a) $10\mu F$

(b) $12.7\mu F$

(c) $20\mu F$

(d) $27.3\mu F$

Answer: (b)

Solution:

$X_C = X_L$ for maximum power transfer

$X_c = \frac{1}{\omega C}$

$C = \frac{1}{\omega X_C}$

$= \frac{2}{2\pi f \times 250}$

$= \frac{1}{2 \times 3.14 \times 50 \times 250}$

$= 1.27 \times 10^{-5} = 12.7\mu F$

Question: There is a soap bubble of radius 6 cm and it has another soap bubble of radius 3 cm inside it. Find the equivalent radius of the soap bubble which has same excess pressure inside it as 3 cm bubble.

Options:

(a) 1 cm

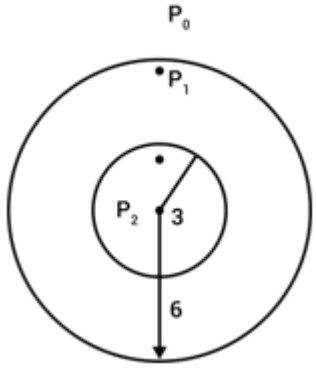
(b) 2 cm

(c) 3 cm

(d) 9 cm

Answer: (b)

Solution:



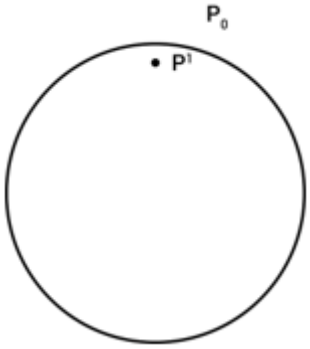
$$P_1 - P_0 = \frac{4T}{r_1}$$

$$P_2 - P_1 = \frac{4T}{r_2}$$

$$P_2 = \frac{4T}{r_2} + \frac{4T}{r_1} + P_0$$

Now new soap bubble of radius x having same excess pressure as 3 cm bubble

$$P' - P_0 = \frac{4T}{x}$$



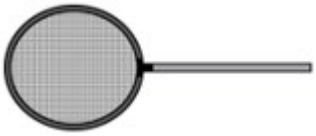
$$P' = P_2$$

$$\frac{4T}{r_2} + \frac{4T}{r_1} + P_0 - P_0 = \frac{4T}{x}$$

$$\frac{1}{r_1} + \frac{1}{r_2} = \frac{1}{x}$$

$$x = \frac{r_1 r_2}{r_1 + r_2} = \frac{6 \times 3}{9} = 2cm$$

Question: Badminton is in shape of ring connected with a handle (a rod). Radius of ring is r, length of rod is 6r. If mass of ring is M, and of rod is m, find moment of inertia of system about the axis passing from a point at $\frac{r}{2}$ from end and perpendicular to ring plane.



Options:

(a) $\frac{37}{4}mr^2 + \frac{173}{4}MR^2$

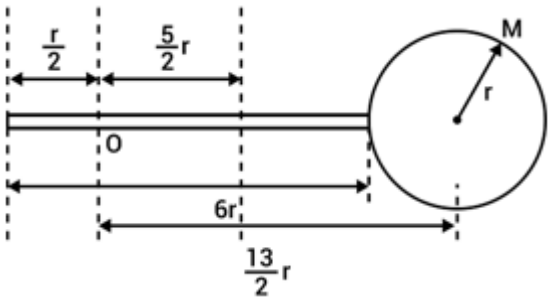
(b) $\frac{37mr^2}{4} + Mr^2$

(c) $3mr^2 + 16Mr^2$

(d) $3mr^2 + \frac{229}{4}Mr^2$

Answer: (a)

Solution:



$$I_{O(\text{ring})} = Mr^2 + M\left(\frac{13}{2}r\right)^2$$

$$= Mr^2 + \frac{169}{4}Mr^2$$

$$= \frac{173}{4}Mr^2$$

$$I_{O(\text{rod})} = \frac{m(6r)^2}{12} + m\left(\frac{5}{2}r\right)^2$$

$$= 3mr^2 + \frac{25}{4}mr^2$$

$$= \frac{37}{4}mr^2$$

$$I_{O(\text{Badminton})} = I_{O(\text{ring})} + I_{O(\text{rod})}$$

$$= \frac{37}{4}mr^2 + \frac{173}{4}Mr^2$$

Question: In a standard YDSE, distance between slits is d and distance between screen and slit plane is D. The location of 1st maxima for red light is at Y_1 distance from central maxima and for violet light is Y_2 distance from central maxima. Find difference between wavelengths of red and violet lights.

Options:

(a) $\frac{d}{D}(y_1 + y_2)$

(b) $\frac{d}{2D}(y_1 - y_2)$

(c) $\frac{d}{D}(Y_1 - Y_2)$

(d) $\frac{d}{2D}(y_1 + y_2)$

Answer: (c)

Solution:

In YDSE

$$Y = \frac{m\lambda d}{D}$$

For 1st maxima m = 1

For red light $Y_1 = 1 \times \frac{\lambda_{red} D}{d}$

$$\lambda_{red} = \frac{Y_1 d}{D}$$

For violet light $Y_2 = \frac{1 \times \lambda_{violet} D}{d}$

$$\lambda_{violet} = \frac{Y_2 d}{D}$$

$$\lambda_{red} - \lambda_{violet} = \frac{d}{D}(Y_1 - Y_2)$$

Question: A rocket of mass 1000 kg starts by burning fuel at relative velocity 500 m/s. If it accelerates at a rate of 20 m/s^2, find the rate at which fuel is burning in kg/s

Options:

(a) 20 kg/s

(b) 10 kg/s

(c) 30 kg/s

(d) 60 kg/s

Answer: (d)

Solution:

Mass of rocket (m) = 1000 kg.

Velocity v = 500 m/s

Acceleration a = 20 m/s^2

$$m(g+a) = v\frac{dm}{dt}$$

$$\frac{dm}{dt} = \frac{m(g+a)}{v} = \frac{1000 \times 30}{500}$$

$$\frac{dm}{dt} = 60kg/s$$

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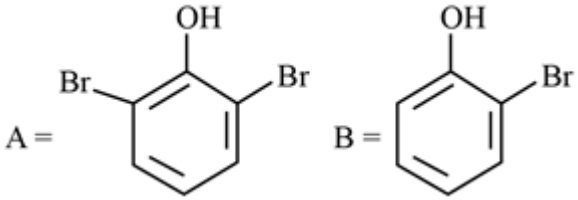
CHEMISTRY

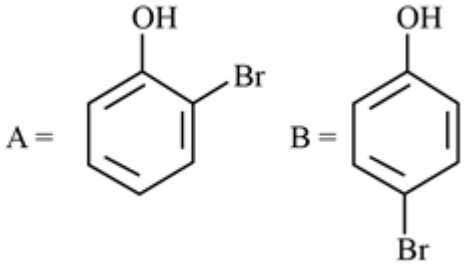
Question: By which of the following process deionized water can be obtained?
Options:
(a) Calgon's process
(b) Synthetic resin method
(c) Clark's method
(d) Permutit
Answer: (b)
Solution: Pure de-mineralised (de-ionized) water free from all soluble mineral salts is obtained by passing water successively through a cation exchange (in the H^+ form) and an anion-exchange (In the OH^- form) resins.

Question: How many electrons are present in 4f orbital of Gd^{2+}?
Options:
(a) 7
(b) 8
(c) 6
(d) 5
Answer: (a)
Solution: Electronic configuration of Gd^{2+} is $[Xe]4f^7$

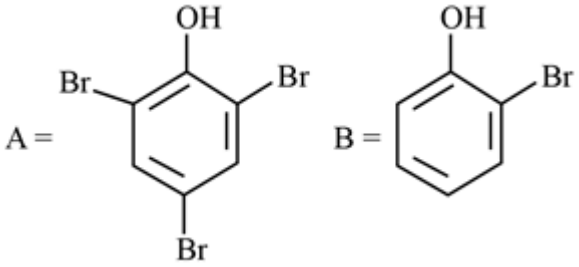
Question: Reaction of phenol with Br_2 and H_2O gives A and reaction of phenol with Br_2 and CS_2 at less than 5°C gives B. Find the product A and B
Options:
(a)



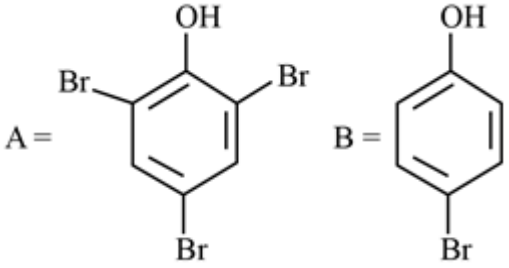
(b)



(c)

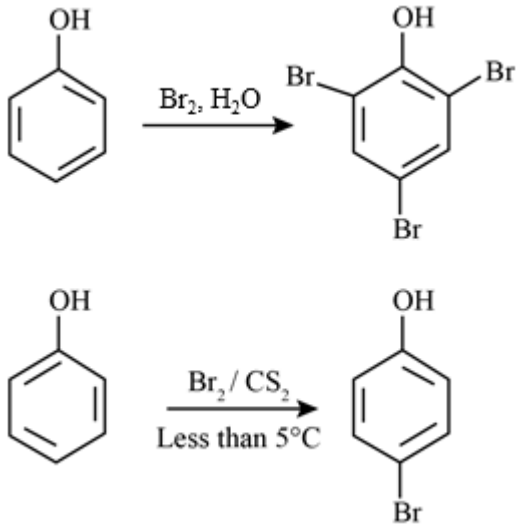


(d)



Answer: (d)

Solution:



Question: S1: Ellingham diagram is used to check which metal oxide is to be reduced by which compound.
S2: In Ellingham diagram as we move from left to right, ΔS always increases.

Options:

(a) Both S1 and S2 are correct.
(b) S1 is correct but S2 is incorrect.
(c) S1 is incorrect but S2 is correct.
(d) Both S1 and S2 are incorrect.

Answer: (b)

Solution: ΔH (enthalpy change) and the ΔS (entropy change) values for any chemical reaction remain nearly constant even on varying temperature. So the only dominant variable becomes T. However. ΔS depends much on the physical state of the compound. Since entropy depends on disorder or randomness in the system. It will increase if a compound melt (s $\rightarrow$ l) or vaporises (l $\rightarrow$ g) since molecular randomness increases on changing the phase from solid to liquid or from liquid to gas.

Question: S1: In Bohr's model velocity of electron increases with decrease in positive charge of nucleus as electrons are not held tightly.
S2: Velocity decreases with an increase in principal quantum number.
Options:
(a) Both S1 and S2 are correct.
(b) S1 is correct but S2 is incorrect.
(c) S1 is incorrect but S2 is correct.
(d) Both S1 and S2 are incorrect.
Answer: (c)
Solution:

$$v = 2.18 \times 10^6 \frac{Z}{n}$$

As positive charge on the nucleus (Z) decreases, velocity will also decrease. As principal quantum number (n) increases, the velocity decreases.

Question: S1: Frenkel defect is interstitial as well as vacancy effect
S2: In Frenkel defect, solids show colour because of F-centre.
Options:
(a) Both S1 and S2 are correct.
(b) S1 is correct but S2 is incorrect.
(c) S1 is incorrect but S2 is correct.
(d) Both S1 and S2 are incorrect.
Answer: (b)
Solution: In Frenkel Defect the smaller ion (usually cation) is dislocated from its normal site to an interstitial site. It creates a vacancy defect at its original site and an interstitial defect at its new location. F-centres are shown by crystals showing metal excess defect.

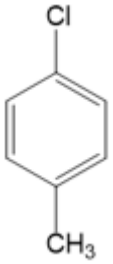
Question: On heating novolac with formaldehyde which of the following polymers will form?
Options:
(a) Melamine
(b) Resin
(c) Bakelite
(d) Polystyrene
Answer: (c)
Solution: Bakelite is a polymer made up of the monomers phenol and formaldehyde. This phenol-formaldehyde resin is a thermosetting polymer.

Question: S1: Methyl orange is suitable indicator for titration of strong acid and weak base
S2: Phenolphthalein is not suitable indicator for titration of acetic acid and NaOH
Options:
(a) Both S1 and S2 are correct.
(b) S1 is correct but S2 is incorrect.
(c) S1 is incorrect but S2 is correct.
(d) Both S1 and S2 are incorrect.
Answer: (b)
Solution: Methyl orange indicator is used in the strong acid weak base titration.

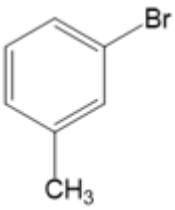
Phenolphthalein changes colour at a pH above 7. So, it is quite good as an indicator for titrations of strong acids with strong bases. It is also suitable for titrations of weak acids and strong bases, which have an equivalence point at a pH above 7.

Question: Which of the following will dissolve in water and give colour?
Options:
(a) Cu_2Cl_2
(b) $CuCl_2$
(c) AgBr
(d) $ZnCl_2$
Answer: (b)
Solution: $CuCl_2$ gives blue-green coloured aqueous solution

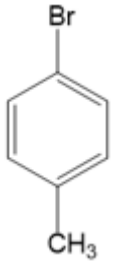
Question: Which of the following reagents gives yellow precipitate for the following sequence -
(i) NaOH, (ii) dil. HNO_3, (iii) $AgNO_3$?
Options:
(a)



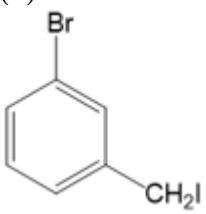
(b)



(c)



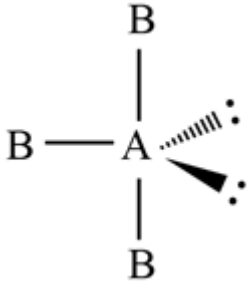
(d)



Answer: (d)
Solution: Silver nitrate test is carried out to detect the type of halide ion in a given solution. The solution is first acidified by adding dilute HNO_3 followed by silver nitrate

solution. F^-, Cl^-, Br^- & I^- react as given below.
$Ag^+_{(aq)} + F^-_{(aq)} \rightarrow AgF$ (No precipitate as it is soluble.)
$Ag^+_{(aq)} + Cl^-_{(aq)} \rightarrow AgCl_{(s)}$ (white precipitate.)
$Ag^+_{(aq)} + Br^-_{(aq)} \rightarrow AgBr_{(s)}$ (pale cream precipitate.)
$Ag^+_{(aq)} + I^-_{(aq)} \rightarrow AgI_{(s)}$ (pale yellow precipitate.)

Question: Interhalogen compound AB_3 has T-shaped structure, how many lone pairs are present on A?
Options:
(a) 2
(b) 3
(c) 1
(d) 0
Answer: (a)
Solution:

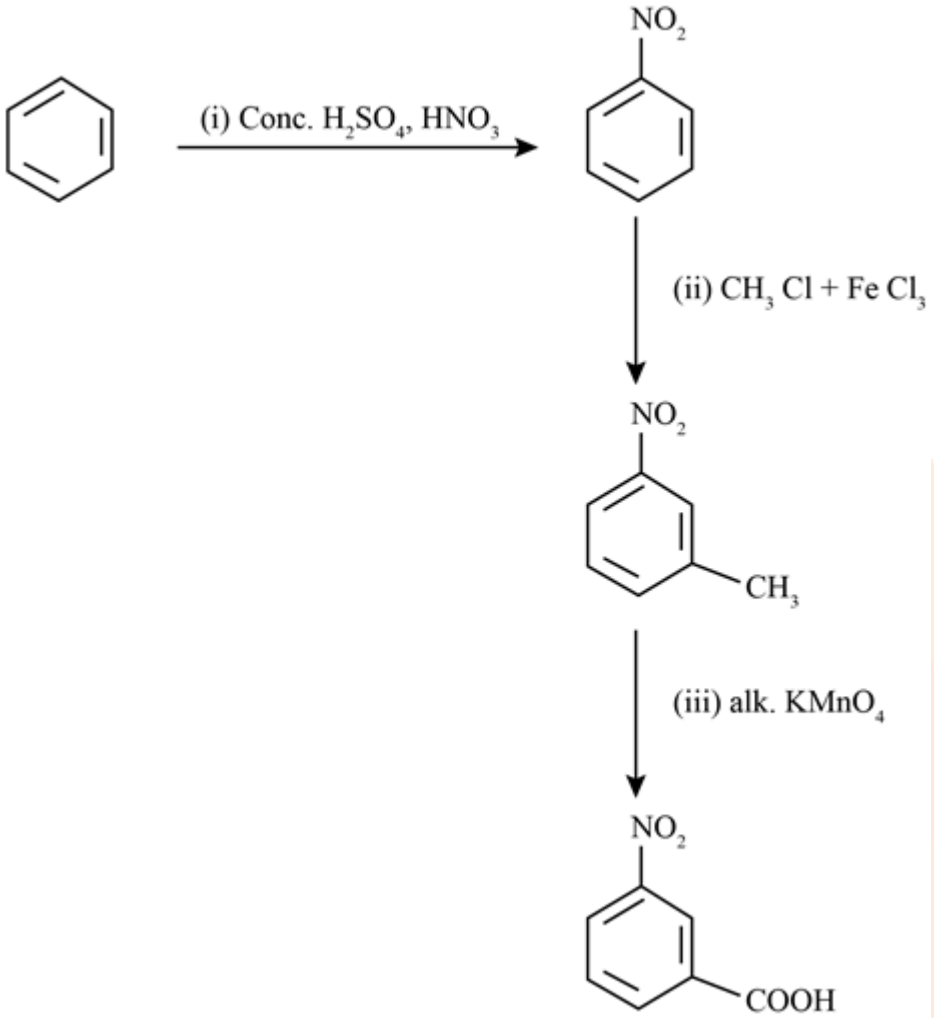


Question: Which is a violet compound among these?
Options:
(a) $[Fe(CN)_5NOS]^{4-}$
(b) $[Fe(CN)_6]^{4-}$
(c) $[Fe(CN)_6]^{3-}$
(d) $Fe(SCN)_3$
Answer: (a)
Solution: $[Fe(CN)_5NOS]^{4-}$ Nitroprusside ion is of violet colour

Question: Which of the following is true for adsorption of gas on a solid surface?
Options:
(a) $\Delta H > 0, \Delta S > 0$
(b) $\Delta H > 0, \Delta S < 0$
(c) $\Delta H < 0, \Delta S > 0$
(d) $\Delta H < 0, \Delta S < 0$
Answer: (d)
Solution: Adsorption is an exothermic process. In other words, ΔH of adsorption is always negative. When a gas is adsorbed, the freedom of movement of its molecules become restricted. This amounts to decrease in the entropy of the gas after adsorption, i.e., ΔS is negative.

Question: Which of the following is the correct sequential method to convert benzene to 3-nitrobenzoic acid?
Options:
(a) (i) $CH_3Cl + FeCl_3$, (ii) Conc. H_2SO_4, HNO_3 (iii) alkaline $KMnO_4$
(b) (i) Conc. H_2SO_4, HNO_3, (ii) $CH_3Cl + FeCl_3$ (iii) alkaline $KMnO_4$

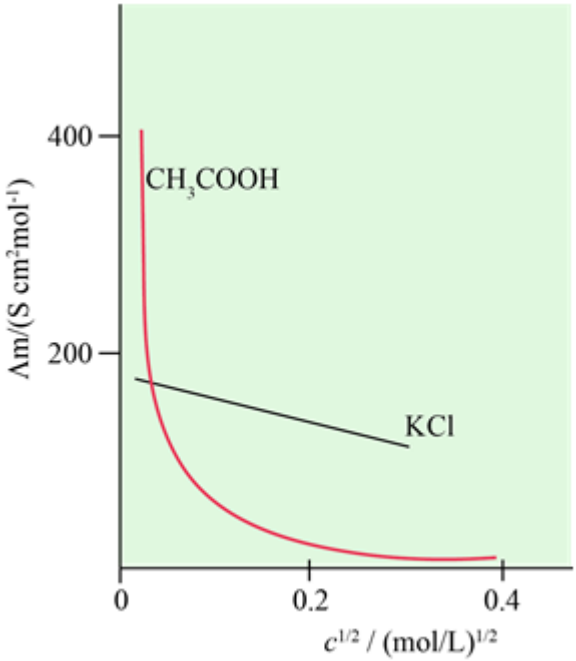
(c) (i) alkaline $KMnO_4$, (ii) $CH_3Cl + FeCl_3$ (iii) conc. H_2SO_4, HNO_3
(d) (i) alkaline $KMnO_4$, (ii) conc. H_2SO_4, HNO_3 (iii) $CH_3Cl + FeCl_3$
Answer: (b)
Solution:



Question: Which of the following statements are correct?
Options:
(a) F_2 is better reducing agent than Cl_2
(b) Cl_2 is more reactive than ClF
(c) F_2 is more reactive than ClF
(d) The fluorine atom can expand its octet
Answer: (c)
Solution: In general, interhalogen are more reactive than halogens (except fluorine)

Question: S1: The y-intercept in the molar conductance vs. concentration graph is always greater for strong electrolyte than weak electrolyte.
S2: On decreasing the conc., molar conductivity decrease
Options:
(a) Both S1 and S2 are correct.
(b) S1 is correct but S2 is incorrect.
(c) S1 is incorrect but S2 is correct.
(d) Both S1 and S2 are incorrect.

Answer: (d)
Solution:



Limiting molar conductance for weak electrolyte may be greater than that for strong electrolyte.
On decreasing the concentration, molar conductance increases.

Question: Which of the following represents correct formula of hydroxyapatite?
Options:
(a) $3Ca_3(PO_4)_2.Ca(OH)_2$
(b) $3Ca_3(PO_4)_2.CaF_2$
(c) $CaSO_4$
(d) $CaCO_3$
Answer: (a)
Solution: Hydroxyapatite $3Ca_3(PO_4)_2.Ca(OH)_2$ is the compound present in the enamel on the surface of the teeth

Question: The number of properties on which the standard reduction electrode potential of halogens depends is:
a) Electron gain enthalpy
b) Sublimation energy
c) Dissociation enthalpy
d) Hydration energy
Answer: 3.00
Solution: Standard reduction electrode potentials of halogens are dependent on the parameters indicated below

$$\frac{1}{2}X_2(g) \xrightarrow{\frac{1}{2}\Delta_{diss}H^{\Theta}} X(g) \xrightarrow{\Delta_{eg}H^{\Theta}} X^-(g) \xrightarrow{\Delta_{hyd}H^{\Theta}} X^-(aq)$$

Question: Number of compounds which have greater freezing point than 0.1 M ethanol (Assume molarity = molality)

a) 0.1 M Na_2SO_4
b) 0.1 M $Ba_3(PO_4)_2$
c) 0.1 M HCl
d) 0.1 M NaOH
Answer: 0.00
Solution: $\Delta T_f = i\ K_f\ m$
Since all solutions have the same concentration, ΔT_f will depend only on i.
i values are as follows:
Ethanol = 1
Na_2SO_4 = 3
$Ba_3(PO_4)2$ = 5
HCl = 2
NaOH = 2
Thus, all the given solutions will have more depression in freezing point than ethanol, i.e., they will have lesser freezing point than 0.1 M ethanol.

JEE-Main-26-08-2021-Shift-1 (Memory Based)

MATHEMATICS

Question: If $a=\hat{i}+\hat{j}+\hat{k}, b=\hat{j}-\hat{k}, a\times c=b, a.c=3$. Find $[a, b, c]$

Options:

(a) 2

(b) 6

(c) -2

(d) -6

Answer: (c)

Solution:

$\overline{a}=\hat{i}+\hat{j}+\hat{k}, \overline{b}=\hat{j}-\hat{k}$

Let $\overline{c}=l\hat{i}+m\hat{j}+n\hat{k}$

$\overline{a}\times\overline{c}=\overline{b}; \overline{a}\cdot\overline{c}=3$

$\Rightarrow(\overline{a}\times\overline{c})\times\overline{a}=\overline{b}\times\overline{a}$

$\left(|\overline{a}|^2\right)\overline{c}-(\overline{a}\cdot\overline{c})\overline{a}=\overline{b}\times\overline{a}$

$\overline{c}=\overline{a}+\frac{1}{3}(\overline{b}\times\overline{a}) \quad ....(1)$

$\because \overline{b}\times\overline{a}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & -1\\ 1 & 1 & 1\end{vmatrix}=2\hat{i}-\hat{j}-\hat{k}$

$\therefore \overline{c}=\left(\hat{i}+\hat{j}+\hat{k}\right)+\frac{1}{3}\left(2\hat{i}-\hat{j}-\hat{k}\right)$

$=\frac{5\hat{i}+2\hat{j}+2\hat{k}}{3}$

$\therefore \begin{bmatrix}\overline{a} & \overline{b} & \overline{c}\end{bmatrix}=\begin{vmatrix}1 & 1 & 1\\ 0 & 1 & -1\\ \frac{5}{3} & \frac{2}{3} & \frac{2}{3}\end{vmatrix}$

$=\frac{-10}{3}+\frac{2}{3}+\frac{2}{3}=-2$

Question: A circle centre = (-15, 0) and radius $= \frac{15}{2}$) Chord to circle through (-30, 0) & tangent to $y^2 = 30x$. Length the of chord = ?

Options:

(a)

(b)

(c)

(d)

Answer: $\frac{15}{\sqrt{5}}$

Solution:

$y^2 = 30x$

$4a = 30 \Rightarrow a = \frac{15}{2}$

Equation of tangent

$y = mx + \frac{a}{m}$

$\Rightarrow y = mx + \frac{15}{2m}$

Passes through (-30, 0)

$\Rightarrow 0 = -30m + \frac{15}{2m}$

$\Rightarrow 30m = \frac{15}{2m}$

$\Rightarrow m^2 = \frac{1}{4} \Rightarrow m = \pm\frac{1}{2}$

$y = \frac{x}{2} + 15$

Distance of line from centre is

$$\frac{\left|\frac{-15}{2} + 15\right|}{\sqrt{1^2 + \frac{1}{2^2}}} = \frac{\frac{15}{2}}{\frac{\sqrt{5}}{2}}$$

$$=\frac{15}{\sqrt{5}}$$

$$y=\frac{-x}{2}-15$$

Distance of line from centre is

$$\frac{\left|\frac{15}{2}-15\right|}{\sqrt{1^2+\frac{1}{1^2}}}=\frac{15}{\sqrt{5}}$$

Length of chord = $2\sqrt{\left(\frac{15}{2}\right)^2-\left(\frac{15}{\sqrt{5}}\right)^2}$

$$=2\times15\sqrt{\frac{1}{4}-\frac{1}{5}}$$

$$=30\sqrt{\frac{1}{20}}=\frac{30}{\sqrt{20}}=\frac{15}{\sqrt{5}}$$

Question: If $\int_{\frac{-1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}}\left(\left(\frac{x-1}{x+1}\right)^2+\left(\frac{x+1}{x-1}\right)^2-2\right)^{\frac{1}{2}}dx$

Options:

(a) $\ln 16$

(b) $2\ln 16$

(c) $3\ln 16$

(d) $4\ln 16$

Answer: (a)

Solution:

$$\int_{\frac{-1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}}\left(\left(\frac{x-1}{x+1}\right)^2+\left(\frac{x+1}{x-1}\right)^2-2\right)^{\frac{1}{2}}dx$$

$$=\int_{\frac{-1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}}\left|\left(\frac{x-1}{x+1}\right)-\left(\frac{x+1}{x-1}\right)\right|dx$$

$$= \int_{\frac{-1}{\sqrt{2}}}^{\frac{1}{\sqrt{2}}} \left| \frac{-4x}{x^2-1} \right| dx$$

$$I = 8 \int_{0}^{\frac{1}{\sqrt{2}}} \frac{x\,dx}{1-x^2}$$

$$= -4 \int_{1}^{\frac{1}{2}} \frac{dt}{t}$$

$$= 4 \ln 2 = \ln 16$$

Question: Mean & standard deviation of 20 observations are 10 & 2.5. It is observed that are entry was taken 25 instead of 35. Find correct means & variance.

Options:

(a) 10.5, 26

(b) 10.5, 25

(c) 11, 26

(d) 11, 25

Answer: (a)

Solution:

$$n = 20, \overline{x} = 10, \sigma = 2.5$$

$$\overline{x}_{new} = \frac{20\overline{x} - 25 + 35}{20} = 10.5$$

$$2.5 = \sqrt{\frac{\sum x_i^2}{n} - (\overline{x})^2}$$

$$\sum x_i^2 = 2125$$

$$\therefore \left(\sum x_i^2 \right)_{new} = 2125 - 625 + 1225 = 2725$$

$$\therefore \sigma_{new} = \sqrt{\frac{2725}{20} - (10.5)^2} = 26$$

Question: Find $\sum r^2 . {}^{20}C_r$

Options:

(a) 2^{20}

(b) 2^{21}

(c) 210×2^{19}

(d)

Answer: (c)

Solution:

$(1+x)^{20} = C_0 + C_1 x + C_2 x^2 + ... + C_{20} x^{20}$

Differentiate w.r.t. x

$20(1+x)^{19} = C_1 + 2C_2 x + ... + 20C_{20} x^{15}$

Multiply by x

$20x(1+x)^{15} = C_1 x + 2C_2 x^2 + 20C_{20} x^{20}$

Differentiate w.r.t x

$20(1+x)^{19} + 20 \cdot x \cdot 19(1+x)^{18} = C_1 + 2^2 C_2 x + ... + 20^2 C_{20} x^{19}$

Put $x = 1$

$20 \cdot 2^{15} + 20 \cdot 19 \cdot 2^{18} = C_1 + 2^2 C_2 + ... + 20^2 C_{20}$

$\Rightarrow \sum r^2 \cdot {}^{20}C_r = 20 \cdot 2^{19} + 20 \cdot 19 \cdot 2^{18}$

$= 2^{18} \cdot 20[2+19]$

$= 20 \cdot 21 \cdot 2^{18}$

Question: $a + ar + ar^2 + ... \infty = 15$

$a^2 + (ar)^2 + (ar^2)^2 + ... \infty = 150$. Find $ar^3 + ar^4 + ar^6 + ... \infty$

Options:

(a) $\frac{1}{2}$

(b) $\frac{2}{5}$

(c)

(d)

Answer: (a)

Solution:

$a + ar + ar^2 + ... \infty = 15$

$\Rightarrow \frac{a}{1-r} = 15 \quad ...(1)$

$$a^2+(ar)^2+(ar^2)^2+\ldots\infty=150$$

$$\Rightarrow \frac{a^2}{1-r^2}=150 \quad \ldots(2)$$

Square (1) and divide by 2

$$\frac{\frac{a^2}{1-r^2}}{\frac{a^2}{(1-r)^2}}=\frac{150}{15.15}$$

$$\Rightarrow \frac{1-r}{1+r}=\frac{2}{3}$$

$$\Rightarrow 3-3r=2+2r$$

$$\Rightarrow 1=5r$$

$$\Rightarrow r=\frac{1}{5}$$

Substitute in (1)

$$\frac{a}{1-\frac{1}{5}}=15$$

$$\frac{5a}{4}=15$$

$$a=12$$

$$ar^2+ar^4+ar^6+\ldots\infty$$

$$=\frac{ar^2}{1-r^2}=\frac{12\times\left(\frac{1}{5}\right)^2}{1-\left(\frac{1}{5}\right)^2}$$

$$=\frac{\frac{12}{25}}{\frac{24}{25}}=\frac{1}{2}$$

Question: $P(A)=p, P(B)=2p$ P(exactly one of them occur) = $\frac{5}{9}$ then $P_{\max}=?$

Options:

(a)

(b)

(c)

(d)

Answer: ()

Solution:

$P(A)=p, P(B)=2p$

$P(A\cup B)-P(A\cap B)=\frac{5}{9}$

$3p-2p(A\cap B)=\frac{5}{9}$

$\therefore P(A\cap B)=\frac{3}{2}p-\frac{5}{18}$

Also, $P(A\cup B)\le 1$

$\frac{3p}{2}+\frac{5}{18}\le 1$

$\frac{3p}{2}\le\frac{13}{18}$

$\therefore p\le\frac{13}{27}$

$\therefore P_{\max}=\frac{13}{27}$

Question: If $f(x)=\cos\left[2\tan^{-1}\left(\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)\right]$

Options:

(a) $f'(x)(x-1)^2-2(f(x))^2=0$

(b) $f'(x)(x-1)^2+2(f(x))^2=0$

(c) $f'(x)(x+1)^2+2(f(x))^2=0$

(d) $f'(x)(x+1)^2-2(f(x))^2=0$

Answer: ()

Solution:

$$f(x)=\cos\left[2\tan^{-1}\left(\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)\right]$$

$$=\cos\left(2\tan^{-1}\sqrt{x}\right)$$

$$=\left(\frac{2}{1+x}\right)-1$$

$$\therefore f'(x)=\frac{-2}{(1+x)^2}$$

$$\Rightarrow f'(x)(1+x)^2=-2$$

Question: The value of $\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{2n-1}\frac{n^2}{n^2+4r^2}$ is

Options:

(a)

(b)

(c)

(d)

Answer: $\left(\frac{1}{2}\right)\tan^{-1}4$

Solution:

$$\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{2n-1}\frac{n^2}{n^2+4r^2}$$

$$=\int_0^2\frac{dx}{1+4x^2}$$

$$= \frac{1}{4}\int_0^2 \frac{dx}{\left(\frac{1}{2}\right)^2 + x^2}$$

$$= \left[\frac{1}{2}\tan^{-1}(2x)\right]_0^2$$

$$= \frac{1}{2}\tan^{-1} 4$$

Question: Find the value of $\frac{1}{1+x} + \frac{2}{1+x^2} + \frac{2^2}{1+x^4} + ... \frac{2^{100}}{1+x^{200}}$

Options:

(a)

(b)

(c)

(d)

Answer:

Solution:

$$\frac{1}{1+x} + \frac{2}{1+x^2} + \frac{2^2}{1+x^4} + ... \frac{2^{100}}{1+x^{200}}$$

$$\frac{1}{1-x} + \frac{1}{1+x} + \frac{2}{1+x^2} + \frac{2^2}{1+x^4} + ... + \frac{2^{100}}{1+x^{200}} - \frac{1}{1-x}$$

$$= \frac{2}{1-x^2} + \frac{2}{1+x^2} + \frac{2^2}{1+x^4} + ... + \frac{2^{100}}{1+x^{200}} - \frac{1}{1-x}$$

$$= \frac{2^2}{1-x^4} + \frac{2^2}{1+x^4} + ... + \frac{2^{100}}{1+x^{200}} - \frac{1}{1-x}$$

In similar way we will get

$$\frac{2^{200}}{1-x^{400}} - \frac{1}{1-x}$$

Question: If sum of distances of point P from (0, 0) (0, 1) (1, 0), (1, 1) = 18 Locus is at circle of diameter. find d^2.

Answer: 16.00

Solution:

$$x^2+y^2+x^2(y-1)^2+y^2+(x-1)^2+(y-1)^2=18$$

$$x^2+y^2+x^2+y^2-2y+1+x^2-2x+1+y^2+x^2-2x+1+y^2-2y+1=18$$

$$4x^2+4y^2-4x-4y+4=18$$

$$x^2+4y^2-4x-4y-14=0$$

$$x^2+y^2-x-y-\frac{7}{2}=0$$

$$r=\sqrt{\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2+\frac{7}{2}}$$

$$=2$$

$$d=4$$

$$d^2=16$$

Question: Ellipse: $\frac{x^2}{8}+\frac{y^2}{4}=1$ tangent at P(2nd quard) $\perp$ to $x+dy=0$ eccentricity $=e.SS'$ is foci. Find $\left(s-e^2\right)*\Delta SPS'$

Answer: 6.00

Solution:

$$\frac{x^2}{8}+\frac{y^2}{4}=1$$

$$e^2=1-\frac{4}{8}=\frac{1}{2}$$

Equation of tangent at $P(x_1,y_1)$ with slope 2 is

$$2x-y+6=0 \quad ....(1)$$

Also, $\frac{xx_1}{8}+\frac{yy_1}{4}=1 \quad ....(2)$

From (1) and (2)

$$\frac{x_1}{16}=\frac{y_1}{-4}=\frac{-1}{6}$$

$\therefore (x_1, y_1) = \left(\frac{-8}{3}, \frac{2}{3}\right)$

$\therefore \left(5 - e^2\right) \cdot \Delta SPS' = \left(5 - \frac{1}{2}\right) \cdot aey$

$= \frac{9}{2} \times 2 \times \frac{2}{3} = 6$

Question: If $\ln(x + y) = 4 \times y$. Find $\frac{d^2y}{dx^2}$ at $x = 0$.

Answer: 40.00

Solution:

$\ln(x + y) = 4xy$

For $x = 0$

$\ln^y = 0$

$\Rightarrow y = 1$

$\ln(x + y) = 4xy$

$\Rightarrow \frac{1}{x + y}\left(1 + \frac{dy}{dx}\right) = 4y + 4x\frac{dy}{dx} \quad ...(1)$

At $x = 0$ and $y = 1$

$\Rightarrow \frac{1}{1}\left(1 + \frac{dy}{dx}\right) = 4 + 0$

$\Rightarrow \left.\frac{dy}{dx}\right|_{x=0} = 3$

Differentiate Equation (1)

$\frac{1}{x + y}\left(1 + \frac{dy}{dx}\right) = 4y + 4x\frac{dy}{dx}$

$\Rightarrow \frac{-1}{(x + y)^2}\left(1 + \frac{dy}{dx}\right)^2 + \left(\frac{1}{x + y}\right)\left(\frac{d^2y}{dx^2}\right) = 4\frac{dy}{dx} + 4\frac{dy}{dx} + 4x\frac{d^2y}{dx^2}$

Put $x = 0, y = 1$ and $\frac{dy}{dx} = 3$

$$\Rightarrow \frac{-1}{(0+1)^2}(1+3)^2+\left(\frac{1}{0+1}\right)\left(\frac{d^2y}{dx^2}\right)=4x^3+4\times 3+0$$

$$\Rightarrow -16+\frac{d^2y}{dx^2}=24$$

$$\Rightarrow \frac{d^2y}{dx^2}=40$$