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JEE-Main-27-08-2021-Shift-2 (Memory Based)

PHYSICS

Question: Determine the resistance of the given resistor with the given color sequence (Violet, Green, Red, Gold).

Options:

(a) $(1400 \pm 5\%)\Omega$

(b) $(2500 \pm 10\%)\Omega$

(c) $(2800 \pm 5\%)\Omega$

(d) $(7500 \pm 5\%)\Omega$

Answer: (d)

Solution:

Violet = 7

Green = 5

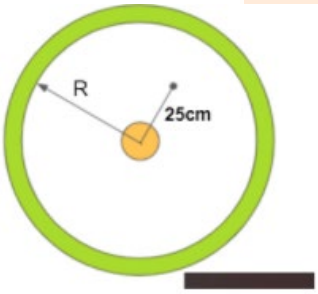
Red = 10^2

Gold $= \pm 5\%$

$R = 75 \times 10^2 \pm 5\%$

$= 7500 \pm 5\%\,\Omega$

Question: A shell has mass 100 kg & Radius 50 m. A point mass 50 kg is placed at center of shell. Find potential at a point 25 m from center of shell.



Options:

(a) -G

(b) -3G

(c) -2G

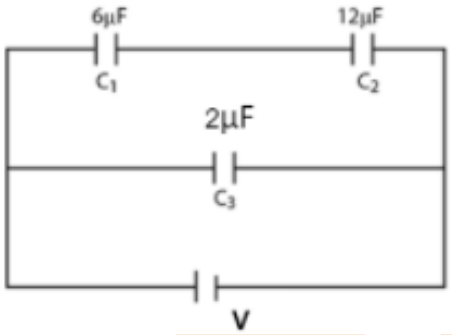
(d) -4G

Answer: (d)

Solution:

$$V = -\frac{GM}{R} + \left(\frac{Gm}{r}\right)$$
$$V = \frac{-G \times 100}{500} - \frac{G \times 50}{25}$$
$$V = -4G$$

Question: For the given network of capacitors find ratio of charge stored on capacitors $2\mu F$ to $6\mu F$ to $12\mu F$.



Options:

(a) 1:1:1

(b) 2:1:1

(c) 1:2:2

(d) 1:3:6

Answer: (c)

Solution:

Equivalent capacitance of C_1 and C_2

$$C_{eq} = \frac{6 \times 12}{6+12} = 4\mu F$$

C_1 and C_2 will have same.

Charge $Q_1 = Q_2 = 4V\ \mu C$

$$Q_3 = 2V\ \mu C$$
$$Q_3 : Q_1 : Q_2 = 2:4:4$$
$$Q_3 : Q_1 : Q_2 = 1:2:2$$

Question: Victoria falls is 68 m high. What is the change in temperature of a drop if it falls from that height? (specific heat capacity is 1 cal/g C)?

[assume that all the gravitational potential energy is converted into heat]

Options:

(a) 0.16°C

(b) 0.10°C

(c) 0.66°C

(d) 1°C

Answer: (c)
Solution:

Loss in P.E. = heat gained

$\Rightarrow mgh = ms\Delta T$

$\Rightarrow \Delta T = \frac{gh}{s} = \frac{9.8\times 68}{\left(1\times 4.2\times 10^3\right)}$

$= 0.16°C$

Question: Two coherent sources of intensities I_0 each, produce minimum intensity of zero, then the maximum intensity they can produce by interference is?
Options:
(a) I_0
(b) $2I_0$
(c) $4I_0$
(d) $8I_0$
Answer: (c)
Solution:

$I_{\min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = 0$

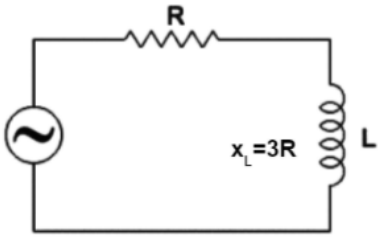
$\Rightarrow I_1 = I_2 = I_0$

$I_{\max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

$= \left(\sqrt{I_0} + \sqrt{I_0}\right)^2$

$= 4I_0$

Question: The power factor of the circuit shown is p_1. Now a capacitor $\left(x_c = 2R\right)$ is also joined in series. Find $\frac{p_1}{p_2}$?



Options:

(a) $\frac{2}{\sqrt{10}}$

(b) $\frac{1}{10}$

(c) $\sqrt{5}$

(d) $\frac{1}{\sqrt{5}}$

Answer: (d)

Solution:

Initially, power factor $\left(P_1\right)=\frac{R}{Z}=\frac{R}{\sqrt{R^2+X_L^2}}$

$=\frac{R}{\sqrt{R^2+9R^2}}=\frac{1}{\sqrt{10}}$

After capacitor is joined

Power factor $\left(P_2\right)=\frac{R}{Z}=\frac{R}{\sqrt{R^2+\left(X_L-X_C\right)^2}}$

$=\frac{R}{\sqrt{R^2+\left(3R-2R\right)^2}}=\frac{1}{\sqrt{2}}$

Ratio $=\frac{P_1}{P_2}=\frac{1}{\sqrt{10}}\times\frac{\sqrt{2}}{1}=\frac{1}{\sqrt{5}}$

Question: Find the height of antenna required if it covers a distance of 44 km on earth (R = 6400 km)

Answer: 151.25 m

Solution:

$X=\sqrt{2hR}$

$\Rightarrow h=\frac{X^2}{2R}$

$\Rightarrow h=\frac{\left(44\times10^3\right)^2}{2\times6400\times10^3}$

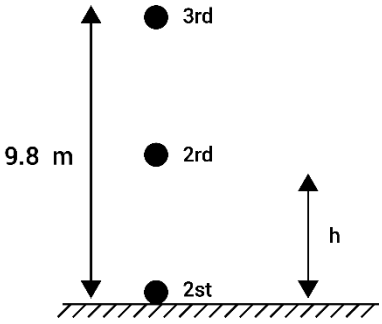
$=\frac{44\times44\times10^6}{2\times64\times10^5}$

$=151.25\,\text{m}$

Question: A drop falling from a shower of height 9.8 m. When it reaches the ground, the third drop falls from shower. What is the height of 2nd drop from ground?

Answer: 7.35 m

Solution:



Time taken by 1st drop to reach the ground.

$$s = ut + \frac{1}{2}gt^2$$

$$\sqrt{\frac{2s}{g}} = t$$

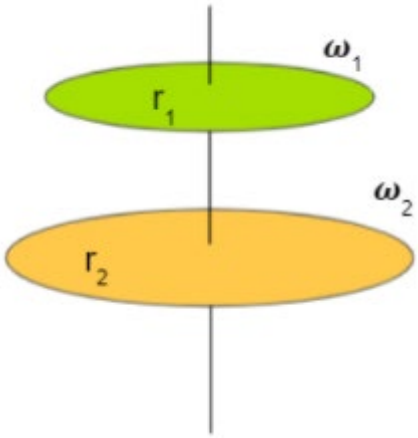
$$t = \sqrt{2}\,\text{sec}$$

$\therefore$ time travel by 2nd drop is $\frac{t}{2} = \frac{1}{\sqrt{2}}$

$$s = 0 + \frac{1}{2}g \times \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{9.8}{4}$$

$$h = 9.8 - \frac{9.8}{4} = 7.35\,\text{m}$$

Question: Two coaxial discs rotating in the same sense, stick to each other and spin with common angular speed. Find the loss in kinetic energy of the system?



Options:

(a) $\frac{I_1 I_2}{I_1 + I_2}\left(\omega_1^2 + \omega_2^2\right)$

(b) $\frac{1}{2}\frac{I_1 I_2}{I_1 + I_2}\left(\omega_1 - \omega_2\right)^2$

(c) $\frac{1}{2}\left(I_1\omega_1^2 - I_2\omega_2^2\right)$

(d) $\frac{1}{2}\left(I_1 + I_2\right)\left(\omega_1 - \omega_2\right)^2$

Answer: (b)

Solution:

Since external torque is zero.

$\therefore$ angular momentum will be conserved.

$$L_i = L_f$$

$$I_1\omega_1 + I_2\omega_2 = \left(I_1 + I_2\right)\omega_f$$

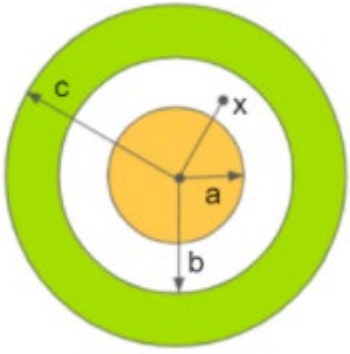
$$\omega_f = \frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}$$

$$KE_i = \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}I_2\omega_2^2$$

$$KE_f = \frac{1}{2}\left(I_1 + I_2\right)\left(\frac{I_1\omega_1 + I_2\omega_2}{I_1 + I_2}\right)^2$$

Loss in KE is $KE_i - KE_f = \frac{1}{2}\frac{I_1 I_2}{I_1 + I_2}\left(\omega_1 - \omega_2\right)^2$

Question: Consider a coaxial cable which consists of an inner wire of radius a carrying current i surrounded by an outer shell of inner radius b and outer radius c also carrying current i but in opposite direction. Find ratio of magnetic field at a distance of x from the axis when x < a and a < x << b.



Options:

(a) $\frac{x}{a}$

(b) $\frac{2x}{a}$

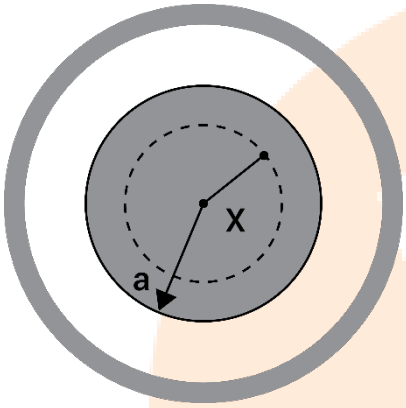
(c) $\frac{x^2}{a^2}$

(d) $\frac{a^2}{x^2}$

Answer: (c)

Solution:

Case I: For (x < a)



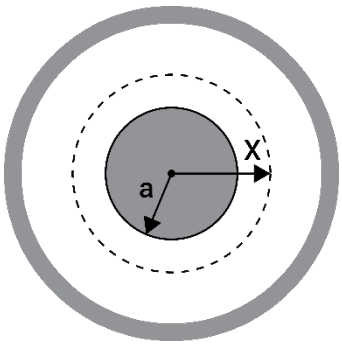
Consider a amperian loop of radius x.

$$\oint \vec{B} \cdot d\vec{l} = i_{enc}\mu_0$$

$$i_{enc} = \frac{i}{\pi a^2} \times \pi x^2 = \frac{ix^2}{a^2}$$

$$B\ 2\pi x = \frac{ix^2\mu_0}{a^2} \Rightarrow B = \frac{\mu_0 ix}{2\pi a^2}$$

Case II:



$\oint B \cdot dl = i_{enc}\mu_0$

$i_{enc} - i$

$B\,2\pi x = i\mu_0$

$B = \dfrac{\mu_0 i}{2\pi x}$

$\therefore \text{ratio} = \dfrac{x^2}{a^2}$

Question: A ring of radius 1 m is placed in X-Y plane with center (-1, 0). It is moving with velocity 1 ms^{-1} along +ve X-axis. For x > 0. A uniform magnetic field 1T exists along Z-axis. Find emf induced in loop at t = 1 sec.

Options:

(a) 2V

(b) 1V

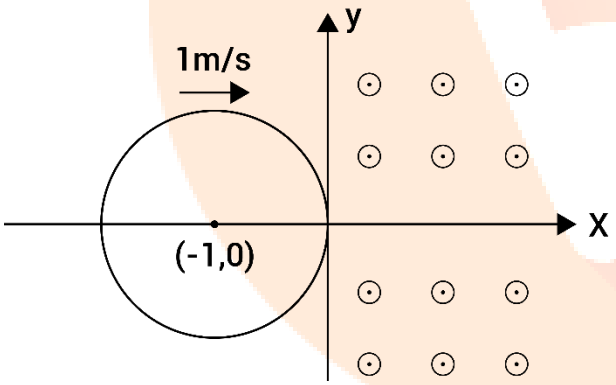
(c) 3V

(d) None

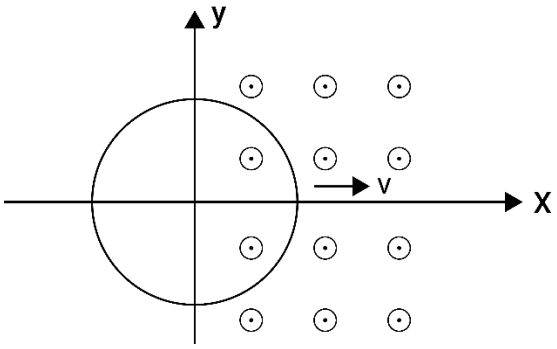
Answer: (a)

Solution:

At t = 0



At t = 1 sec



e.m.f. induced $= B L_{eff} V$

$= 1 \times 1 \times 1 \qquad \left[\because L_{eff} = 2R = 2\text{m}\right]$

$= 2V$

Question: A closed organ pipe is vibrating in Fundamental mode with frequency 450 Hz. Find length of pipe. If velocity of sound is 340 ms^{-1}.

Options:

(a) 14.2 cm

(b) 21.2 cm

(c) 18.8 cm

(d) None

Answer: (c)

Solution:

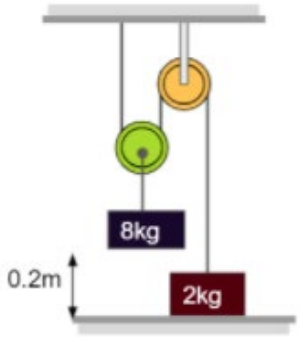
For closed organ pipe fundamental frequency $f = \frac{V}{4L}$

$450 = \frac{340}{4L}$

$L = 0.188\,\text{m}$

= 18.8 cm

Question: If the system is left from rest, find the time taken by the 8 kg block to reach the ground?



Options:

(a) $\frac{\sqrt{3}}{10}s$

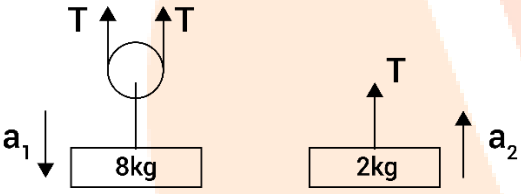
(b) $\frac{\sqrt{3}}{5}s$

(c) $\frac{3}{5}s$

(d) $5\sqrt{3}\,s$

Answer: (b)

Solution:



$8g-2T=8a_1 \quad ...(i)$

$T-2g=2a_2 \quad ...(ii)$

If we move 8 kg block by x, 2kg

Block moves 2x

So we can say acceleration of 2 kg Block is twice of 8 kg block

$a_2=2a_1 \quad ...(iii)$

Solving equations (i), (ii), & (iii)

$T-2g=2[2a_1]$

$T-2g=4a_1$

$\left[\frac{8g-8a_1}{2}\right]-2g=4a_1$

$8g-8a_1-4g=8a_1$

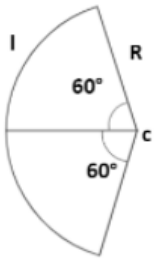
$g=4a_1$

$a_1=\frac{g}{4}$

Using, $s = ut + \frac{1}{2}a_1 t^2$

$0.2 = 0 + \frac{1}{2}\left(\frac{g}{4}\right)t^2 \Rightarrow t = 0.4\,s$

Question: Find the electric field due to an uniformly charged arc, at the center 'c'?



Options:

(a) $\frac{k\lambda}{2R}$

(b) $\frac{k\lambda}{\sqrt{3}R}$

(c) $\frac{k\lambda 2\sqrt{3}}{R}$

(d) $\frac{k\lambda\sqrt{3}}{R}$

Answer: (d)

Solution:

Electric field due to an uniformly charged arc $= \frac{2k\lambda}{R}\sin\frac{\theta}{2}$

Here $\theta = 120°$

$E = \frac{2k\lambda}{R}\sin\left(\frac{120}{2}\right)$

$E = \frac{2k\lambda}{R}\sin 60°$

$\boxed{E = \frac{\sqrt{3}k\lambda}{R}}$

Question: If force F, time T and length L are used as fundamental quantities, then dimensional formula for density will be.

Options:

(a) FT^2L^{-4}

(b) F^2TL^{-3}

(c) FT^2L^4

(d) $F^3T^6L^5$

Answer: (a)

Solution:

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

$$= \frac{\text{Force}}{\text{Acceleration}} \times \text{Volume}$$

$$= \frac{F}{LT^{-2}L^3} = \frac{F}{L^4T^{-2}}$$

$$\text{Density} = FT^2L^{-4}$$

Question: Two particle are performing SHM such that $y_1 = 10\sin[\omega_1 t]$, $y_2 = 5\sin[\omega_2 t] + \sqrt{3}\cos\omega_2 t$. Find ratio of their amplitudes.

Options:

(a) $\frac{1}{2}$

(b) $\frac{5}{\sqrt{7}}$

(c) $\frac{1}{\sqrt{3}}$

(d) None

Answer: (a)

Solution:

$$Y_1 = 10\sin(\omega_1 t)$$

$$Y_2 = 5\sin(\omega_2 t) + 5\sqrt{3}\cos(\omega_2 t)$$

$$Y_2 = 5(\sin(\omega_2 t)) + \sqrt{3}(\cos(\omega_2 t))\frac{2}{2}$$

$$Y_2 = 10\left[\frac{1}{2}\sin(\omega_2 t) + \frac{\sqrt{3}}{2}\cos(\omega_2 t)\right]$$

$$Y_2 = 10\left[\sin\left(\omega_2 t + \frac{\pi}{3}\right)\right]$$

$$\frac{A_1}{A_2} = \frac{10}{10} = 1$$

JEE-Main-27-08-2021-Shift-2 (Memory Based)

CHEMISTRY

Question: Ozone layer is depleted by which of the following rays?

Options:

(a) UV rays

(b) Gamma rays

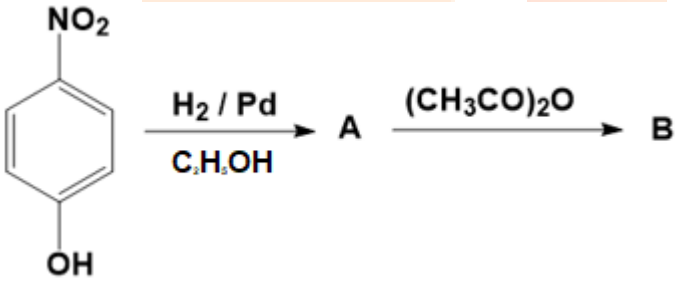
(c) X-rays

(d) Visible rays

Answer: (a)

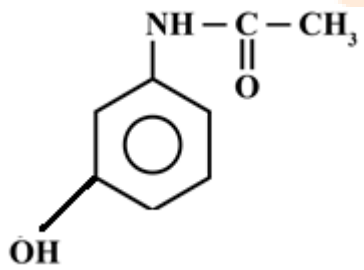
Solution: UV rays releases chlorine free radicals by reacting with CFC. These Cl atoms then react with ozone to break it down into O_2

Question:

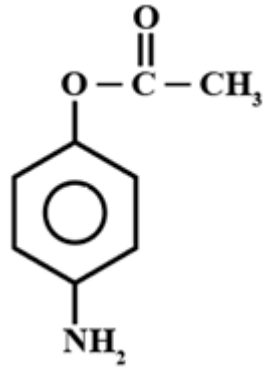


Options:

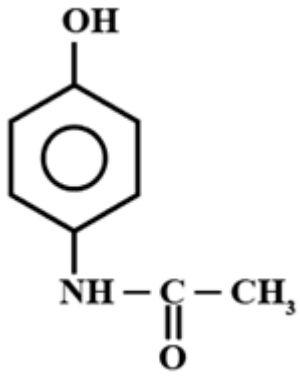
(a)



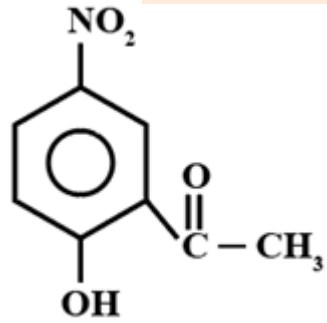
(b)



(c)

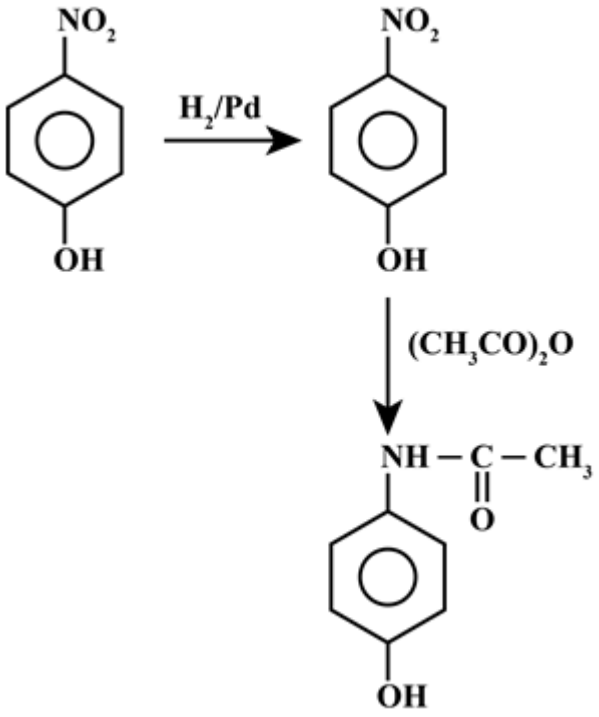


(d)



Answer: (c)

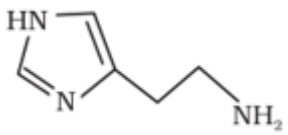
Solution:



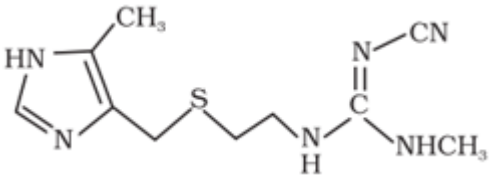
Question: Structure of compound which is responsible for acidity in stomach and stomach disorder

Options:

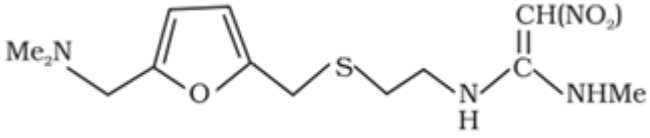
(a)



(b)



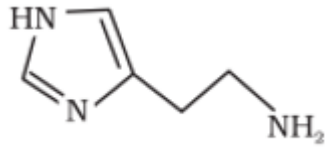
(c)



(d) All of these

Answer: (a)

Solution:



Histamine

Question: Hydrolysis of sucrose gives:

Options:

(a) α-glucose, β -fructose

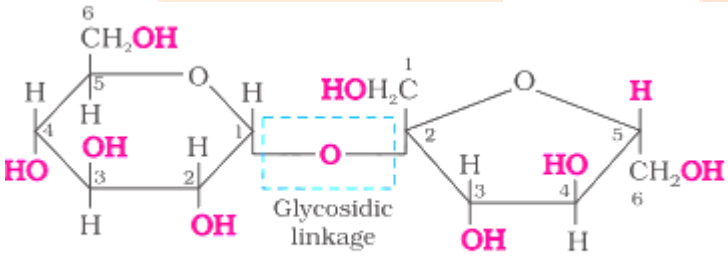
(b) α-glucose, α-fructose

(c) β-glucose, β-fructose

(d) β-glucose, α-fructose

Answer: (a)

Solution: Hydrolysis of sucrose gives α-glucose and β-fructose



α – D – Glucose β – D – Fructose

Question: Plutonium from nuclear fuel is stabilized by which of the following:

Options:

(a) O_2F_2

(b) I_2O_5

(c) ClF_3

(d) BrF_5

Answer: (a)

Solution: $Pu + 3O_2F_2 \longrightarrow PuF_6 + 3O_2$; unreacted Pu is separated by fluorination

Question: Which of the following will show pyramidal geometry?

Options:

(a) CO_3^{2-}

(b) SO_3^{2-}

(c) NO_3^-

(d) NH_4^+

Answer: (b)

Solution: S in $SO_3^{2-}$is sp^3 hybridised with one lone pair which converts tetrahedral geometry into pyramidal shape

Question: Mass of glucose is 40 g (molar mass =180), volume of water is 200 ml. Density of water is 1 g/cm^3 [$K_b = 1.86$]. Find the freezing point of this mixture given that freezing point of water is 273.15 K.

Options:

(a) 271.428 K

(b) 273.428 K

(c) 269.428 K

(d) 268.428 K

Answer: (a)

Solution: $\Delta T_f = k_f m$

$$= 1.86 \times \frac{40}{180 \times 0.24} = 1.722$$

$\Delta T_f = T_2 - T_1$

$T_2 = 273.15 - 1.722 = 271.428$ K

Question: Which of the following will gives H_2O_2 by the addition of H_2O?

Options:

(a) $CO(NH_2)_2.\ H_2O_2$

(b) Na_2HPO_4. H_2O_2

(c) $(NH_4)_2SO_4$. H_2O_2

(d) All of these

Answer: (d)

Solution: All gives H_2O_2 on the addition of water.

Question: What is the bond order and magnetic nature of O_2^-?

Options:

(a) 1.5 and paramagnetic

(b) 2.5 and paramagnetic

(c) 1 and diamagnetic

(d) 2 and diamagnetic

Answer: (a)

Solution: Bond order of $O_2^- = \frac{N_{BO} - N_{A.BO}}{2} = \frac{9-6}{2} = 1.5$

It is paramagnetic in nature

Question: Arrange in increasing order _____

Options:

(a) $Ca^{2+} < K^+ < Cl^- < S^{2-} < P^{3-}$

(b) $K^+ < Cl^- < Ca^{2+} < S^{2-} < P^{3-}$

(c) $K^+ < Ca^{2+} < Cl^- < S^{2-} < P^{3-}$

(d) $P^{3-} < S^{2-} < Cl^- < K^+ < Ca^{2+}$

Answer: (a)

Solution: For isoelectronic species, size of ion decreases with increases in nuclear charge.

Question: In which of the following reagents, copper is not present?

Options:

(a) Barfoed test

(b) Seliwanoff test

(c) Biuret Test

(d) Benedict's test

Answer: (b)

Solution: Barfoed's test used for detection of mono saccharides. It consist of 0.33 molar solution of copper (II) acetate in 1% acetic acid solution

Biuret test is to detect peptide bond and contains copper

Seliwanoff's is to detect ketose and aldose and contains resorcinol and HCl

Benedict's reagent, also known as Benedict's solution, is a chemical reagent which is made up of a complex mixture of sodium citrate, sodium carbonate, and the pentahydrate of copper(II) sulfate

Question: Why are lyophilic colloids stable?

Options:

(a) They are solvated

(b) They have strong intermolecular repulsion

(c) They have negative charge

(d) They have no charge

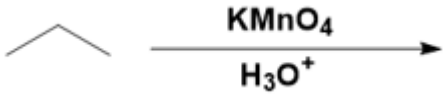
Answer: (a)

Solution: Factual

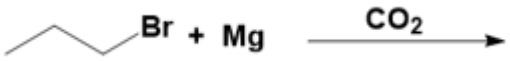
Question: Which of the following will not give propionic acid?

Options:

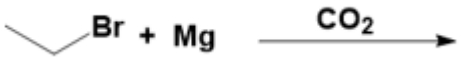
(a)



(b)



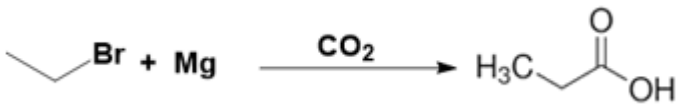
(c)



(d) Both (a) and (b)

Answer: (d)

Solution:



Question: S1: Ethyl-pent-yn-oate reacts with CH_3, MgBr to give 3° alcohol.

S2: 2 moles of CH_3MgBr are used to convert 1 mole of Ethyl-pent-yn-oate

Options:

(a) Both S1 and S2 are correct

(b) Both S1 and S2 are incorrect

(c) S1 is correct, S2 is incorrect

(d) S1 is incorrect, S2 is correct

Answer: (c)

Solution: Esters react with Grignard reagent to get 3° alcohol, in this process 1 molecule of ester uses 3 molecules of Grignard reagent.

Question: 100 gm propane reacts with 1000 gm O_2. Find the mole fraction of CO_2

Options:

(a) 0.59

(b) 0.29

(c) 0.49

(d) 0.19

Answer: (d)

Solution: 100 g of propane (C_3H_8) = $\frac{100}{44}$ = 2.72 mol

$$1000 \text{ g } O_2 = \frac{1000}{32} = 31.25 \text{ mol}$$

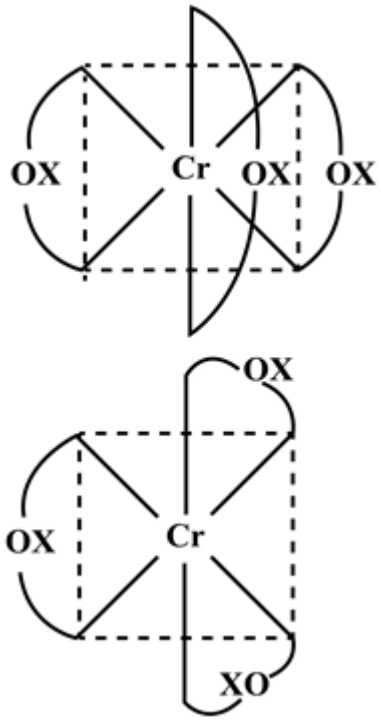
$$\underset{\substack{2.27\,mol\\ o}}{C_3H_8} + \underset{\substack{31.25\,mol\\ 19.89\,mol}}{5O_2} \rightarrow \underset{\substack{o\\ 6.81\,mol}}{3CO_2} + \underset{\substack{o\\ 9.08\,mol}}{4H_2O}$$

$$\text{Mole fraction of } CO_2 = \frac{6.81}{19.89 + 6.81 + 9.08} = 0.19$$

Question: Number of optical isomers of $[Cr(C_2O_4)_3]^{3-}$

Answer: 2.00

Solution:



Question: In Hydrogen like atom , the principal quantum number (excited) is 6, then find number of spectral lines.

Answer: 15.00

Solution: Number of spectral line $= \frac{n(n-1)}{2} = \frac{6 \times 5}{2} = 15$

JEE-Main-27-08-2021-Shift-2 (Memory Based)

MATHEMATICS

Question: Tangent to $(y-2)^2=(x-1)$ is drawn at P, whose ordinate is 3. Find area of region hold by curve, tangent at P & y-axis

Answer: 9

Solution:

$(y-2)^2=x-1$

$y=3$

$\Rightarrow 1=x-1$

$\Rightarrow x=2$

$2(y-2)y'=1$

$\Rightarrow y'=\frac{1}{2(y-2)}=\frac{1}{2(3-2)}=\frac{1}{2}$

Equation of tangent

$\Rightarrow y-3=\frac{1}{2}(x-2)$

$\Rightarrow 2y-6=x-2$

$\Rightarrow x=2y-4$

Area $=\int_0^3\left(\left((y-2)^2+1\right)-(2y-4)\right)dy$

$=\int_0^3\left((y-2)^2-2y+5\right)dy$

$=\frac{(y-2)^3}{3}-y^2+5y\Big|_0^3$

$=\left(\frac{1}{3}-9+15\right)-\left(\frac{8}{3}\right)$

$=6-\frac{7}{3}=\frac{11}{3}$

Question: Two circles are touching externally at A(1, 2) & common tangent is $4x+3y=10$. If $C_1(\alpha,\beta)$ & $C_2(\gamma,\delta)$ are centres of circles then find $\left|(\alpha+\beta)(\gamma+\delta)\right|$

Answer: 40

Solution:

Centre of the circle are given by

$\frac{x-1}{\cos\theta}=\frac{y-2}{\sin\theta}=5$

Where $\tan\theta=\frac{3}{4},\cos\theta=\frac{4}{5},\sin\theta=\frac{3}{5}$ or $\cos\theta=\frac{-4}{5},\sin\theta=\frac{-3}{5}$

$\Rightarrow$ centres are (5, 5) and (-3, -1)

$\therefore \left|(\alpha+\beta)(\gamma+\delta)\right|=\left|10\times(-4)\right|=40$

Question: In class of 50 students there are 20 boys & 30 girls. Average marks of boys is 12, variance of boys marks is 2, variance of girls marks is 2, average marks of boys + girls is 15. If average marks of girls is μ & variance of marks of boys & girls together is σ^2. Then find $\mu+\sigma^2$.

Answer: 25

Solution:

$n_B=20,n_G=30$

$\overline{x}_B=12,{\sigma_B}^2=2,{\sigma_G}^2=2$

$\overline{x}_{B+G}=15,\overline{x}_G=\mu,{\sigma_{B+G}}^2=\sigma^2$

Total marks by boys = $12\times20=240$

Total marks by both = $15\times50=750$

$\therefore$ Total marks by girls = 510

$\therefore \overline{x}_G=\mu=\frac{510}{30}=17$

Also,

$\sigma^2=\frac{1}{50}\left[20\times2+30\times2+\frac{20\times30}{20+30}\times25\right]$

$=\frac{1}{50}[100+300]=8$

$\therefore \mu+\sigma^2=17+8=25$

Question: $\left(3x^2+4x+3\right)^2+(k+1)\left(3x^2+4x+2\right)\left(3x^2+4x+3\right)+k\left(3x^2+4x+2\right)=0$. Find 'k' for which, equation has real roots.

Answer: $\left[\frac{-5}{2},-1\right)$

Solution:

$\left(3x^2+4x+3\right)^2+(k+1)\left(3x^2+4x+2\right)\left(3x^2+4x+3\right)+k\left(3x^2+4x+2\right)=0$

Let $3x^2+4x+3=p$

$3x^2+4x+2=q$

$\Rightarrow p^2+(k+1)pq+kq^2=0$

$\Rightarrow p^2+kpq+pq+kq^2=0$

$\Rightarrow p(p+kq)+q(p+kq)=0$

$\Rightarrow (p+kq)(p+q)=0$

$\Rightarrow \left(3x^2+4x+3+k\left(3x^2+4x+2\right)\right)\left(3x^2+4x+3+3x^2+4x+2\right)=0$

$\Rightarrow \left(x^2(3+3k)+x(4+4k)+3+2k\right)\left(6x^2+8x+5\right)=0$

For real roots

$(4+4k)^2-4(3+3k)(3+2k)=0$

$4(1+k)^2-\left(9+6k+9k+6k^2\right)=0$

$\Rightarrow 4+4k^2+8k-9-15k-6k^2=0$

$\Rightarrow -2k^2-7k-5=0$

$\Rightarrow 2k^2+7k+5=0$

$\Rightarrow 2k^2+2k+5k+5=0$

$\Rightarrow 2k(k+1)+5(k+1)=0$

$\Rightarrow (k+1)(2k+5)=0$

$\Rightarrow k=-1,\frac{-5}{2}$

But $k=-1$ does not satisfy

$\Rightarrow k=\frac{-5}{2}$

Question: $\lim_{x\to\infty}\sqrt{x^2-x+1}-ax+b$. Find a,b

Options:

(a) $1,\frac{-1}{2}$

(b)

(c)

(d)

Answer: ()

Solution:

$\lim_{x\to\infty}\frac{\left(1-a^2\right)x^2-x+1}{\sqrt{x^2-x+1}+ax}$

For limit to exist, $a=1$

$\therefore \lim_{x\to\infty}\frac{1-x}{\sqrt{x^2-x+1}+x}$

$=\frac{-1}{2}=b$

$\therefore a=1, b=\dfrac{-1}{2}$

Question: A & B tosses 3 coins individually. Find probability of both getting same number of heads.

Answer: $\dfrac{5}{16}$

Solution:

Let X be the number of heads obtained by A and Y be the number of heads obtained by B

Note that both X and Y are binomial variate with parameters $n=3$ and $p=\dfrac{1}{2}$

Probability that both A and B obtain the same number of heads is

$=P(X=0)P(Y=0)+P(X=1)P(Y=1)+P(X=2)P(Y=2)+P(X=3)P(Y=3)$

$=\left[{}^3C_0\left(\dfrac{1}{2}\right)^3\right]^2+\left[{}^3C_1\left(\dfrac{1}{2}\right)^3\right]^2+\left[{}^3C_2\left(\dfrac{1}{2}\right)^3\right]^2+\left[{}^3C_3\left(\dfrac{1}{2}\right)^3\right]^2$

$=\left(\dfrac{1}{2}\right)^6[1+9+9+1]=\dfrac{20}{64}=\dfrac{5}{16}$

Question: Consider $C_1: x^2+y^2\le 4, C_2:(x-2)^2+y^2\le 4, C_3:(x-2)^2+(y-2)^2\le 4$. If A & B represents, sets containing integral points common to C_1 & C_2, C_2 & C_3 respectively & 2^P is number of possible relations from A to B then find P.

Answer: 25

Solution:

On plotting C_1, C_2, C_3

$A=\{(0,0),(1,0),(2,0),(1,1),(1,-1)\}$

$B=\{(2,0),(2,2),(2,1),(1,1),(3,1)\}$

$\therefore$ Number of possible relation $=2^{5\times 5}=2^{25}$

$\Rightarrow P=25$

Question: If solution of $2x\,dy+10y^3dy=y\,dx$ satisfies (0, 1) & $(1,\beta)$ then β satisfies:

Options:

(a) $y^5-y^2-1=0$

(b)

(c)

(d)

Answer: ()

Solution:

$2x\,dy+10y^3dy=y\,dx$

$\frac{dx}{dy}-\frac{2x}{y}=10y^2$

IF $=e^{-\int\frac{dy}{y}}=\frac{1}{y}$

$\therefore \frac{x}{y}=5y^2+C$

Passes through (0, 1) $\Rightarrow C=-5$

$\Rightarrow 5y^3-5y=x$

$\therefore \beta$ satisfy $5y^3-5y=1$

Question: Find number of solutions in $[0,4x]$ of $\sin^4\theta+\cos^4\theta=$

Options:

(a)

(b)

(c)

(d)

Answer: ()

Solution:

$\sin^4\theta+\cos^4\theta=1$ is only possible when $\sin\theta=0$ and $\cos\theta=\pm1$ or $\sin\theta=\pm1$ and $\cos\theta=0$

$\Rightarrow \theta=0,\frac{\pi}{2},\pi,\frac{3\pi}{2},2\pi,\frac{5\pi}{2},3\pi,\frac{7\pi}{2},4\pi$

9 solutions

Question: $y=\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+\ldots$ Find e^{1+y} at $x=1$.

Options:

(a)

(b)

(c)

(d)

Answer: ()

Solution:

$y=\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+\ldots$

$$=\left(1-\frac{1}{2}\right)x^2+\left(1-\frac{1}{3}\right)x^3+\left(1-\frac{1}{4}\right)x^4+\ldots$$

$$=\left(x^2+x^3+x^4+\ldots.\right)-\left(\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\ldots\right)$$

$$=\frac{x^2}{1-x}+x+\log\left(1-x\right)$$

$$y=\frac{x}{1-x}+\log\left(1-x\right)$$

At $x=\frac{1}{2}$

$$y=1-\ln 2$$

$$\therefore e^{1+y}=e^{2-\ln 2}=\frac{e^2}{2}$$

Question: $f\left(x\right)=\tan^{-1}\left(\sin x+\cos x\right)$. If M and m are max and min value respectively. Find $\tan^{-1}\left(M-m\right)$ of f^n

Options:

(a) $3-2\sqrt{2}$

(b) $3-\sqrt{2}$

(c) $2-\sqrt{3}$

(d) $2+\sqrt{3}$

Answer: ()

Solution:

$$f\left(x\right)=\tan^{-1}\left(\sin x+\cos x\right)$$

$$=\tan^{-1}\left(\sqrt{2}\left(\sin\frac{\pi}{4}+x\right)\right)$$

$$f_{\max}=\tan^{-1}\left(\sqrt{2}\right)=M$$

$$f_{\min}=-\tan^{-1}\sqrt{2}=m$$

$$\tan\left(M-m\right)=\tan\left(2\tan^{-1}\sqrt{2}\right)$$

$$=\frac{2\tan\left(\tan^{-1}\sqrt{2}\right)}{1-\tan^2\left(\tan^{-1}\sqrt{2}\right)}$$

$$=\frac{2\cdot\sqrt{2}}{1-2}=-2\sqrt{2}$$

Question: $\begin{vmatrix}[x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4]\end{vmatrix}=192$, $[x]=$ greatest integer. Find value range for x.

Answer: $[62,63)$

Solution:

$$\begin{vmatrix}[x+1] & [x+2] & [x+3] \\ [x] & [x+3] & [x+3] \\ [x] & [x+2] & [x+4]\end{vmatrix}=192$$

$$\begin{vmatrix}[x]+1 & [x]+2 & [x]+3 \\ [x] & [x]+3 & [x]+3 \\ [x] & [x]+2 & [x]+4\end{vmatrix}=192$$

$C_3 \to C_3 - C_1$

$C_2 \to C_2 - C_1$

$$\begin{vmatrix}[x]+1 & 1 & 2 \\ [x] & 3 & 3 \\ [x] & 2 & 4\end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\begin{vmatrix}[x]+1 & 1 & 2 \\ -1 & 2 & 1 \\ -1 & 1 & 2\end{vmatrix}=192$$

$$([x]+1)(4-1)-1(-2+1)+2(-1+2)=192$$

$$3[x]+3+1+2=192$$

$$3[x]=186$$

$$[x]=62$$

$$x\in[62,63)$$

Question: From a point perpendicular tangents are drawn to $y^2=16x-3$. Find locus of point.

Answer: $x=\dfrac{-61}{16}$

Solution:

Equation of tangent to $y^2=16\left(x-\dfrac{3}{16}\right)$ is

$$y=m\left(x-\frac{3}{16}\right)+\frac{4}{m}$$

Let it passes through (h, k)

$$\therefore m^2\left(h-\frac{3}{16}\right)-mk+4=0$$

$$\because m_1\cdot m_2=-1\Rightarrow h=\frac{3}{10}-4$$

$$x=\frac{-37}{10}$$

Question: $y=\cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$ where $x\in\left(\dfrac{\pi}{2},\pi\right)$, $\left(\dfrac{dy}{dx}\right)_{x=\frac{5\pi}{6}}=?$

Answer: $\dfrac{-1}{2}$

Solution:

$$y=\cot^{-1}\left(\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$$

$$=\cot^{-1}\left(\frac{\left|\sin\frac{x}{2}+\cos\frac{x}{2}\right|+\left|\sin\frac{x}{2}-\cos\frac{x}{2}\right|}{\left|\sin\frac{x}{2}+\cos\frac{x}{2}\right|-\left|\sin\frac{x}{2}-\cos\frac{x}{2}\right|}\right)$$

$$=\cot^{-1}\left(\frac{\sin\frac{x}{2}+\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}}{\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}+\cos\frac{x}{2}}\right)$$

$$=\cot^{-1}\left(\tan\frac{x}{2}\right)$$

$$=\frac{\pi}{2}-\tan^{-1}\tan\frac{x}{2}$$

$$=\frac{\pi}{2}-\frac{x}{2}$$

$$y'=\frac{-1}{2}$$