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COMPUTER BASED TEST (CBT) Memory Based Questions & Solutions

Date: 27 August, 2021 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. Evaluate $\int_6^{16} \frac{\ln(x^2)}{\ln(x^2) + \ln(484 - 44x + x^2)} dx$

(1) 10 (2) 20 (3) 5 (4) 0

Ans. (3)

Sol. $I = \int_6^{16} \frac{\ln(x^2)}{\ln(x^2) + \ln(484 - 44x + x^2)} dx$

$$I = \int_6^{16} \frac{\ln(x^2)}{\ln(x^2) + \ln(22-x)^2} dx$$

$$= \int_6^{16} \frac{2\ln x}{2\ln x + 2\ln(22-x)} dx$$

$$= \int_6^{16} \frac{\ln x}{\ln x + \ln(22-x)} dx \dots\dots (i)$$

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$$\int_6^{16} \frac{\ln(22-x)}{\ln(22-x) + \ln x} dx \dots\dots (ii)$$

By adding (i) & (ii)

$$2I = \int_6^{16} dx = x \Big|_6^{16} = 10$$

$$I = 5$$

2. The value of $\sum_{k=0}^{20} {}^{20}C_k$ is equal to

(1) ${}^{30}C_{20}$ (2) ${}^{40}C_{20}$ (3) $({}^{20}C_{10})^2$ (4) ${}^{40}C_{15}$

Ans. (2)

Sol. $\sum_{k=0}^{20} ({}^{20}C_k)^2 = \sum_{k=0}^{20} {}^{20}C_k \cdot {}^{20}C_{20-k} = {}^{40}C_{20}$

3. Let two points are A(0,6) and B(2t, 0) and M is the mid point of A & B. If perpendicular bisector of A & B cuts the y-axis at point C then the locus of mid point of MC is

(1) $x^2 = 3(2-y)$ (2) $2x^2 = 3(3-y)$ (3) $2x^2 = 3(2-y)$ (4) $3x^2 = 2(3-y)$

Ans. (2)

Sol. A(0,6) and B(2t,0)
 M is mid point of AB $\Rightarrow M = (t, 3)$
 perpendicular bisector of A and B is

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$y - 3 = \frac{t}{3}(x - t)$

On putting value of t = 2

On y-axis point of intersection $\left(0, 3 - \frac{t^2}{3}\right)$

Let mid point of MC is (h, k)

$$\Rightarrow (h, k) = \left(\frac{t}{2}, 3 - \frac{t^2}{6}\right)$$

$$2h = t \text{ and } \frac{t^2}{6} = 3 - k$$

$$\Rightarrow 4h^2 = 6(3 - k)$$

$$\text{Locus of (h, k) is } 2x^2 = 3(3 - y)$$

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$$(1) \frac{e^2}{4} \quad (2) \frac{e^2}{16} \quad (3) \frac{4}{e} \quad (4) \frac{16}{e^2}$$

Ans. (2)

$$\text{Sol. } y = \lim_{n \rightarrow \infty} \left(1 + \frac{1^2}{n^2}\right)^{\frac{4}{n^2}} \left(1 + \frac{2^2}{n^2}\right)^{\frac{4}{n^2}} \left(1 + \frac{3^2}{n^2}\right)^{\frac{4}{n^2}} \dots$$

$$\ln y = \lim_{n \rightarrow \infty} \sum_{r=1}^n \frac{4}{n^2} \ln \left(1 + \frac{r^2}{n^2}\right)$$

$$\text{Put } \frac{r}{n} = x$$

$$\ln y = \int_0^1 -4x \ln(1+x^2) dx \quad 1+x^2 = t; 2x dx = dt$$

$$= -2 \int_1^2 \ln t dt$$

$$= -2(t \ln t - t) \Big|_1^2$$

$$= -2(2 \ln 2 - 2 + 1)$$

$$= -2(2 \ln 2 - 1)$$

$$\ln y = \ln \frac{1}{16} + 2$$

$$\therefore y = \frac{1}{16} e^2$$

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5. If $(\sin^{-1}x)^2 - (\cos^{-1}x)^2 = a$, then the value of $(2x^2 - 1)$ is:

$$(1) \sin\left(\frac{2a}{\pi}\right) \quad (2) \cos\left(\frac{2a}{\pi}\right) \quad (3) \sin\left(\frac{4a}{\pi}\right) \quad (4) \cos\left(\frac{4a}{\pi}\right)$$

Ans. (1)

$$\text{Sol. } (\sin^{-1}x)^2 - (\cos^{-1}x)^2 = a,$$

$$\Rightarrow (\sin^{-1}x + \cos^{-1}x)(\sin^{-1}x - \cos^{-1}x) = a$$

$$\Rightarrow \frac{\pi}{2} (\sin^{-1}x - \cos^{-1}x) = a$$

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$$\Rightarrow 2 \cos^{-1}x = \frac{\pi}{2} - \frac{2a}{\pi}$$

$$\Rightarrow 2 \cos^{-1}x = \frac{\pi}{2} - \frac{2a}{\pi}$$

$$\Rightarrow \cos^{-1}(2x^2 - 1) = \left(\frac{\pi}{2} - \frac{2a}{\pi}\right)$$

$$\Rightarrow 2x^2 - 1 = \cos\left(\frac{\pi}{2} - \frac{2a}{\pi}\right)$$

$$\Rightarrow 2x^2 - 1 = \sin\left(\frac{2a}{\pi}\right)$$

6. If $k = \frac{3}{2}x^2 + \frac{5}{3}x^3 + \frac{7}{4}x^4 + \dots$ then the value of k is

$$(1) \frac{x^2+x}{1-x} + \ln(1-x) \quad (2) \frac{x^2+x}{1+x} + \ln(1+x) \quad (3) 1+x + \ln(1-x) \quad (4) 0$$

Ans. (1)

$$\text{Sol. } k = \left(2 - \frac{1}{2}\right)x^2 + \left(2 - \frac{1}{3}\right)x^3 + \left(2 - \frac{1}{4}\right)x^4 + \dots$$

$$k = 2[x^2 + x^3 + x^4 + \dots] - \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots\right)$$

$$k = \frac{2x^2}{1-x} + x - \left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots\right)$$

$$k = \frac{2x^2+x-x^2}{1-x} + \ln(1-x)$$

7. Let an ellipse $E: \frac{x^2}{b^2} + \frac{y^2}{4a^2} = 1$. If minimum area between the tangents of an ellipse and the co-ordinates axis is kab then the value of K is
- (1) 1 (2) 2 (3) 4 (4) 8
- Ans. (2)

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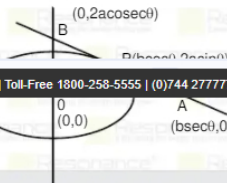
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Sol. Equation of tangent at $P(b\cos\theta, 2a\sin\theta)$ is



$$\frac{x \cos \theta}{b} + \frac{y \sin \theta}{2a} = 1$$

$$A(b \sec \theta, 0), B(0, 2a \csc \theta)$$

$$\text{Area of triangle OAB} = \frac{1}{2} \times b \sec \theta \times 2a \csc \theta$$

$$= \frac{ab}{\cos \theta \sin \theta} = \frac{2ab}{\sin 2\theta}$$

$$\Delta = 2ab (\csc 2\theta)$$

$$\Delta_{\min} = 2ab$$

$$\text{So } k = 2$$

8. If $\frac{2+4+6+8+\dots+2y}{3+6+9+12+\dots+3y} = \frac{4}{\log_{10} x}$ then the value of $y = \log_{10} x + \log_{10} x^{\frac{1}{3}} + \log_{10} x^{\frac{1}{9}} + \dots$ is
- (1) 4 (2) 6 (3) 9 (4) 12
- Ans. (3)

Sol. $\frac{2(1+2+3+\dots+y)}{3(1+2+3+\dots+y)} = \frac{4}{\log_{10} x}$

$$\Rightarrow \log_{10} x = 6 \Rightarrow x = 10^6 \quad \dots (1)$$

Now

$$y = \log_{10} x + \log_{10} x^{\frac{1}{3}} + \log_{10} x^{\frac{1}{9}} + \dots$$

$$= \left(1 + \frac{1}{3} + \frac{1}{9} + \dots\right) \log_{10} x$$

$$= \frac{1}{1 - \frac{1}{3}} \times 6 = 9$$

9. If α and β are the roots of the equation $x^2 + bx + c = 0$, then the value of

$$\lim_{x \rightarrow \beta} \frac{e^{2(x^2+bx+c)} - 1 - 2(x^2+bx+c)}{(x-\beta)^2}$$

$$(1) 4(b^2 - 4c) \quad (2) 2(b^2 - 2c) \quad (3) 2(b^2 - 4c) \quad (4) 2(b^2 + 4c)$$

Ans. (3)

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Sol. $\lim_{x \rightarrow \beta} \frac{e^{2(x^2+bx+c)} - 1 - 2(x^2+bx+c)}{(x-\beta)^2}$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{e^{2(\beta-\alpha+h)h} - 1 - 2(\beta-\alpha+h)h}{h^2}$$

$$\Rightarrow \lim_{h \rightarrow 0} \frac{(1 + 2(\beta-\alpha+h)h + \dots) - 1 - 2h(\beta-\alpha+h)}{h^2} \Rightarrow \lim_{h \rightarrow 0} \frac{(2(\beta-\alpha+h)h)^2 + \dots}{h^2}$$

$$\Rightarrow 2(\beta-\alpha)^2 = 2(b^2 - 4c)$$

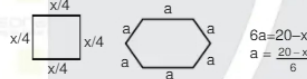
10. A 20 m length wire is cut into 2 pieces. One piece is used to make a square and other piece is used to

make a regular hexagon. The length of side of hexagon so that sum of areas of square and hexagon is minimum will be:

- (1) $\frac{40\sqrt{3}}{3-2\sqrt{3}}$ (2) $\frac{10}{3+2\sqrt{3}}$ (3) $\frac{10}{3-2\sqrt{3}}$ (4) $\frac{40\sqrt{3}}{3+2\sqrt{3}}$

Ans. (2)

Sol. Let 2 pieces are x and $20-x$



$$\text{Area of square } A_1 = \frac{x^2}{16}$$

$$\text{Area of hexagon } A_2 = 6 \times \frac{\sqrt{3}}{4} a^2$$

$$\text{Sum of both area } = A = A_1 + A_2$$

$$A = \frac{x^2}{16} + \frac{\sqrt{3}}{24} (20-x)^2$$

$$\frac{dA}{dx} = \frac{x}{8} - \frac{\sqrt{3}}{12} (20-x) = \frac{3x - 40\sqrt{3} + 2\sqrt{3}x}{24}$$

$$\frac{dA}{dx} = 0 \Rightarrow x = \frac{40\sqrt{3}}{3+2\sqrt{3}} = \frac{40}{\sqrt{3}+2} = 40(2-\sqrt{3})$$

$$\frac{d^2A}{dx^2} = \frac{3+2\sqrt{3}}{24} > 0$$

So area will be minimum, when $x = 40(2-\sqrt{3})$

$$\text{So side of hexagon} = \frac{20 - 40(2-\sqrt{3})}{6}$$

$$= \frac{10 - 40 + 20\sqrt{3}}{3} = \frac{(20\sqrt{3} - 30)}{3} = \frac{10(2\sqrt{3} - 3)}{3} = \frac{10}{2\sqrt{3} + 3}$$

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11. If point (x, y) satisfy the relation $x^2 + 4y^2 - 4x + 3 = 0$, then

(1) $x \in [1, 3]; y \in \left[-\frac{1}{3}, \frac{1}{3}\right]$

(2) $x \in [1, 3]; y \in [1, 3]$

(3) $x \in [1, 3]; y \in \left[-\frac{1}{2}, \frac{1}{2}\right]$

(4) $x \in [-2, 2]; y \in [-1, 1]$

Ans. (3)

Sol. $(x-2)^2 + 4y^2 = 1$

$$(x-2)^2 + \frac{y^2}{1/4} = 1$$

$$\text{Hence } -1 \leq x-2 \leq 1 \text{ and } -1 \leq 2y \leq 1$$

$$1 \leq x \leq 3 \text{ and } -\frac{1}{2} \leq y \leq \frac{1}{2}$$

12. Let 1, 2, 3, ..., n are n variates having variance 14, then value of 'n' is equal to

Ans. 13

Sol. Variance = $\frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n} - \left(\frac{(1+2+3+\dots+n)}{n}\right)^2$

$$14 = \frac{n(n+1)(2n+1)}{6n} - \left(\frac{n+1}{2}\right)^2$$

$$168 = (n+1)(n-1)$$

$$\Rightarrow n^2 = 169$$

$$\Rightarrow n = 13$$

13. If $\frac{z-i}{z+2i} \in \mathbb{R}$ and $z \in \mathbb{C}$, then the correct statement is :

(1) z is straight line in argand plane

(2) z has only 1 value

(3) z has only 2 values

(4) z is circle in argand plane

Ans. (1)

Sol. $\arg\left(\frac{z-i}{z+2i}\right) = 0 \text{ or } \pi$

$$\Rightarrow z \text{ lies on } y\text{-axis}$$



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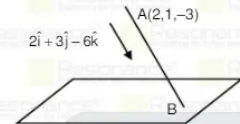
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14. The distance of point $(2, 1, -3)$ parallel to the vector $(2\hat{i} + 3\hat{j} - 6\hat{k})$ from the plane $2x + y + z + 8 = 0$ is :

Ans. 70

Sol.



Equation of line AB

$$\frac{x-2}{2} = \frac{y-1}{3} = \frac{z+3}{-6} = \lambda$$

Let B $(2\lambda + 2, 3\lambda + 1, -6\lambda - 3)$

B satisfy plane $2x + y + z + 8 = 0$

$$2(2\lambda + 2) + (3\lambda + 1) - 6\lambda - 3 + 8 = 0$$

$$\lambda = -10$$

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15. If $\int \frac{1}{(x^2 + x + 1)^2} dx = a \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + b \left(\frac{-2x+1}{x^2 + x + 1}\right) + C$ then value of $9\left(\frac{a}{\sqrt{3}} + b\right)$ is equal to

Ans. 07.00

Sol.

$$\int \frac{1}{\left(x + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} dx$$

$$x + \frac{1}{2} = \frac{\sqrt{3}}{2} \tan \theta$$

$$dx = \frac{\sqrt{3}}{2} \sec^2 \theta d\theta$$

$$\int \frac{\frac{\sqrt{3}}{2} \sec^2 \theta}{\left(\frac{3}{4} \sec^2 \theta\right)^2} d\theta$$

$$\frac{8}{3\sqrt{3}} \int \cos^2 \theta d\theta = \frac{8}{3\sqrt{3}} \int \frac{1 + \cos 2\theta}{2} d\theta$$

$$\frac{4}{3\sqrt{3}} \left[\theta + \frac{\sin 2\theta}{2} \right] + C$$

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$$\frac{4}{3\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{4}{3\sqrt{3}} \cdot \frac{\frac{2x+1}{\sqrt{3}}}{1 + \left(\frac{2x+1}{\sqrt{3}}\right)^2} + C$$

$$\frac{4}{3\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right) + \frac{1}{3} \cdot \frac{2x+1}{(x^2 + x + 1)} + C$$

$$\text{So, } a = \frac{4}{3\sqrt{3}}, b = \frac{1}{3}$$

$$9\left(\frac{a}{\sqrt{3}} + b\right) = 9\left(\frac{4}{9} + \frac{1}{3}\right) = 7$$

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(1) $q \rightarrow \sim r$ (2) $p \rightarrow \sim r$ (3) fallacy (4) tautology

Ans. (4)

$$\text{Sol. } \{(p \wedge (p \rightarrow q)) \wedge (q \rightarrow r)\} \rightarrow r$$

$$= \{(p \wedge (\sim p \vee q)) \wedge (q \rightarrow r)\} \rightarrow r$$

$$= \{(p \wedge \sim p) \vee (p \wedge q)\} \wedge (q \rightarrow r) \rightarrow r$$

$$\begin{aligned}
 &= \{(f \vee (p \wedge q) \wedge (q \rightarrow r)) \rightarrow r\} \\
 &= \{(p \wedge q) \wedge (\sim q \vee r)\} \rightarrow r \\
 &= \{((p \wedge q) \wedge \sim q) \vee ((p \wedge q) \wedge r)\} \rightarrow r \\
 &= \{(f \vee ((p \wedge q) \wedge r))\} \rightarrow r \\
 &= \{(p \wedge q) \wedge r\} \rightarrow r \\
 &= \sim((p \wedge q) \wedge r) \vee r \\
 &= (\sim(p \wedge q) \vee \sim r) \vee r \equiv t
 \end{aligned}$$

17. In a ΔABC if $\frac{\sin A}{\sin B} = \frac{\sin(C-A)}{\sin(B-C)}$, then

- (1) b^2, a^2, c^2 are in A.P. (2) a^2, c^2, b^2 are in A.P.
 (3) $a^2 - b^2, b^2 - c^2, c^2 - a^2$ are in G.P. (4) a^2, b^2, c^2 are in G.P.

Ans. (2)

Sol. $\frac{\sin A}{\sin B} = \frac{\sin(C-A)}{\sin(B-C)} \Rightarrow \frac{\sin(B+C)}{\sin(A+C)} = \frac{\sin(C-A)}{\sin(B-C)} \Rightarrow \sin^2 B - \sin^2 C = \sin^2 C - \sin^2 A$
 $\Rightarrow \sin^2 A, \sin^2 C, \sin^2 B$ are in A.P. $\Rightarrow a^2, c^2, b^2$ are in A.P.

18. Equation of curve passes through point (2, -2) satisfying differential equation $y + x \frac{dy}{dx} = x^2$ is:

- (1) $x^3 - 3xy = -20$ (2) $x^3 + 3xy = 20$ (3) $3xy - x^3 = 10$ (4) $x^3 - 3xy = 20$

Ans. (4)

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$$\frac{dy}{dx} + \frac{y}{x} = x$$

$$\text{If } y = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

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So complete solution

$$yx = \int x^2 dx$$

$$xy = \frac{x^3}{3} + c$$

Passes through point (2, -2)

$$-4 = \frac{8}{3} + c$$

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$$\therefore xy = \frac{x^3}{3} - \frac{20}{3}$$

$$\text{So } 3xy = x^3 - 20$$

$$x^3 - 3xy = 20$$

19. Given matrix $A = \begin{bmatrix} 0 & 2 \\ x & -1 \end{bmatrix}$. If $A(A^3 + 3I) = 2I$ (where I be a unit matrix), then the possible value of x is:

- (1) $-\frac{1}{2}$ (2) $\frac{1}{2}$ (3) 1 (4) 2

Ans. (2)

Sol. $A^4 + 3AI = 2I$

$$A^4 = 2I - 3A$$

$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 0-\lambda & 2 \\ x & -1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda + \lambda^2 - 2x = 0$$

$$A^2 + A - 2xI = 0$$

$$A^2 = 2xI - A$$

$$A^4 = A^2 + 4x^2I - 2(2xA)$$

$$A^4 = 2xI - A + 4x^2I - 4xA$$

$$= (2x + 4x^2)I - A(1+4x)$$

$$4x + 1 = 3$$

$$4x = 2$$

$$x = \frac{1}{2}$$

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face outcome is $\frac{1}{6} - x$ and the probability of each remaining outcome is $\frac{1}{6}$. The die is thrown again for 2

times and the probability of getting the sum 7 is $\frac{13}{96}$. Then value of x is:-

- (1) $\frac{1}{8}$ (2) $\frac{1}{4}$ (3) $\frac{1}{6}$ (4) $\frac{1}{2}$

Ans. (1)

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Sol. $P_1 = \frac{1}{6} + x, P_6 = \frac{1}{6} - x,$

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$P(\text{sum} = 7) = 2P(6,1) + 2P(5,2) + 2P(4,3)$

$\Rightarrow 2\left(\frac{1}{6} + x\right)\left(\frac{1}{6} - x\right) + 2\left(\frac{1}{6}\right)^2 + 2\left(\frac{1}{6}\right)^2 = \frac{13}{96} \Rightarrow 2\left(\frac{1}{36} - x^2 + \frac{1}{36} + \frac{1}{36}\right) = \frac{13}{96}$

$\Rightarrow 2\left(\frac{1}{12} - x^2\right) = \frac{13}{96} \Rightarrow 2x^2 = \frac{1}{6} - \frac{13}{96} = \frac{3}{96} = \frac{1}{32}$

$x^2 = \frac{1}{64} \Rightarrow x = \frac{1}{8}$

21. Let $\vec{A} = \hat{i} + 5\hat{j} + \alpha\hat{k}$, $\vec{B} = \hat{i} + 3\hat{j} + \beta\hat{k}$, $\vec{C} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $|\vec{B} \times \vec{C}| = 5\sqrt{3}$. If $\vec{A}$ is perpendicular to $\vec{B}$ then the maximum value of $|\vec{A}|^2$ is

Ans. 90

Sol. $\vec{A} \cdot \vec{B} = 0$

$1 + 15 + \alpha\beta = 0$

$\alpha\beta = -16$ (1)

$\vec{B} \times \vec{C} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & \beta \\ 1 & -2 & 3 \end{vmatrix}$

$= \hat{i}(9 + 2\beta) - \hat{j}(3 - \beta) + \hat{k}(-2 - 3)$

$|\vec{B} \times \vec{C}|^2 = 75$

$\therefore (9 + 2\beta)^2 + (3 - \beta)^2 + (-5)^2 = 75$

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$\Rightarrow \beta = -2, -4$

for $\beta = -2$, $\alpha = 8$ and for $\beta = -4$, $\alpha = 4$

for maximum value of $|\vec{A}|^2$, $\alpha = 8$

Therefore maximum value of $|\vec{A}|^2$ is $1 + 25 + 64 = 90$.

22. A tangent and normal are drawn at the point $P(2, -4)$ of parabola $y^2 = 8x$ which cut the directrix at points A and B respectively. A given point Q (a, b) is used to form a square AQBP. Then find the value of $|2a + b|$.

Ans. 16

Sol. Tangent at (2, -4) is $-4y = 4(x + 2)$

$\Rightarrow x + y + 2 = 0$ (1)

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Normal to the parabola at (2, -4) is

$\Rightarrow x - y = 6$ (2)

Equation of directrix of parabola is $x = -2$ (3)

$\Rightarrow A \equiv (-2, 0)$ and $B \equiv (-2, -8)$

Coordinates of Q(a,b) and P(2,-4) are given and AQBP is a square

$\Rightarrow$ mid point of AB = mid point of PQ

$\Rightarrow (-2, -4) = \left(\frac{a+2}{2}, \frac{b-4}{2}\right)$

$\Rightarrow a = -6, b = -4$

$\therefore 2a + b = -16$

23. Let $A = \{x : |x - 2| > 1\}$; $B = \{x : \sqrt{x^2 - 3} > 1\}$ and $C = \{x : |x - 4| \geq 2\}$. If the number of integral values in $(A \cap B \cap C)' \cap Z$ (where Z is set of integers) is k, then the value of k is

Ans. (2)

Sol. $A = \{x : |x - 2| > 1\}$

$A = x \in (-\infty, 1) \cup (3, \infty)$

set $B = \sqrt{x^2 - 3} > 1$

$B = x \in (-\infty, -2) \cup (2, \infty)$

set $C = \{x : |x - 4| \geq 2\}$

$C = x \in (-\infty, 2] \cup [6, \infty)$

$A \cap B \cap C = (-\infty, -2) \cup [6, \infty)$

$(A \cap B \cap C)' = [-2, 6]$

$(A \cap B \cap C)' \cap Z = \{-2, -1, 0, 1, 2, 3, 4, 5, 6\}$

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24. A 5 digit number is formed by using the digits {1,2,3,4,5,6}. Find the total number of such numbers which are divisible by 55.

- (1) 9 (2) 10 (3) 11 (4) 12

Ans. (4)

Sol. A five digit number abcde is divisible by 5 $\Rightarrow e = 5$
Now number abcd5 is divisible by 11 iff.
 $|(a + c + 5) - (b + d)|$ is divisible by 11

Case- I (a, c) $\equiv$ (6, 4) and (b, d) $\equiv$ (1, 3) $\Rightarrow$ 4 ways

Case- II (a, c) $\equiv$ (6, 3) and (b, d) $\equiv$ (1, 2) $\Rightarrow$ 4 ways

Case- III (a, c) $\equiv$ (2, 3) and (b, d) $\equiv$ (6, 4) $\Rightarrow$ 4 ways

$\therefore$ Total possible numbers are 12

25. If $y^{\frac{1}{4}} + y^{-\frac{1}{4}} = 2x$ be a curve satisfying the differential equation $\frac{d^2y}{dx^2}(x^2 - 1) + \beta x \frac{dy}{dx} + \alpha y = 0$ then

ordered pair (α, β) is :

- (1) (16, -1) (2) (-16, 1) (3) (4, -1) (4) (4, 1)

Ans. (2)

Sol. $y^{\frac{1}{4}} + y^{-\frac{1}{4}} = 2x$

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$$\text{Diff. } \frac{1}{4y} \left(y^{\frac{1}{4}} - y^{-\frac{1}{4}} \right) \frac{dy}{dx} = 2 \Rightarrow \left(y^{\frac{1}{4}} - y^{-\frac{1}{4}} \right) \frac{dy}{dx} = 8y$$

$$\text{Now, } y^{\frac{1}{4}} - y^{-\frac{1}{4}} = \sqrt{\left(y^{\frac{1}{4}} + y^{-\frac{1}{4}} \right)^2 - 4} = 2\sqrt{x^2 - 1}$$

$$\Rightarrow \sqrt{x^2 - 1} \frac{dy}{dx} = 4y$$

$$\text{Again Diff. } \sqrt{x^2 - 1} \frac{d^2y}{dx^2} + \frac{x}{\sqrt{x^2 - 1}} \frac{dy}{dx} = 4 \frac{dy}{dx}$$

$$(x^2 - 1) \frac{d^2y}{dx^2} + x \frac{dy}{dx} = 4\sqrt{x^2 - 1} \frac{dy}{dx} = 16y$$

$$\alpha = -16, \beta = 1$$

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