JEE-Main-31-08-2021-Shift-2 (Memory Based)

PHYSICS

Question: A pendulum having length L has time period T_0 in air. When it is dipped in water having density $\frac{1}{4}^{th}$ of density of Bob of pendulum & length of pendulum is increased to $\frac{3}{4}$ times initial length. Then new time period T' is

Options:

(a) $T' = T_0$

(b) $T' = \frac{4}{3}T_0$

(c) $T' = \frac{3}{4}T_0$

(d) None

Answer: (a)

Solution:

$$T_0 = 2\pi\sqrt{\frac{L}{g}}$$

When dipped in water

$$T' = 2\pi\sqrt{\frac{L}{g\left(1-\frac{\sigma}{\rho}\right)}}$$

$$\Rightarrow T' = 2\pi\sqrt{\frac{3L}{4g\left(1-\frac{\rho}{4\rho}\right)}} = 2\pi\sqrt{\frac{L}{g}}$$

$$\Rightarrow T' = T_0$$

Question: A particle is moving along x – axis with velocity 40 ms^{-1}. It breaks into two parts in mass ratio 1 : 2. If speed of smaller part is 60 ms^{-1} along x-axis, then find the fractional changes in kinetic energy of system.

Options:

(a) $\frac{1}{8}$

(b) $\frac{3}{7}$

(c) $\frac{1}{6}$

(d) $\frac{2}{5}$

Answer: (a)

Solution:

From momentum conservation

$$m\times 40=\frac{m}{3}\times 60+\frac{2m}{3}\times v'$$

$$40=20+\frac{2v'}{3}$$

$$\Rightarrow v'=30\,m/s$$

$$\Delta KE=\frac{1}{2}\frac{m}{3}(60)^2+\frac{1}{2}\times\frac{2m}{3}\times(30)^2-\frac{1}{2}m(40)^2$$

$$\Delta KE=\frac{1}{2}m(40)^2\left[\frac{1}{3}\left(\frac{60}{40}\right)^2+\frac{2}{3}\left(\frac{30}{40}\right)^2-1\right]$$

$$\frac{\Delta KE}{K}=\frac{6+3-8}{8}=\frac{1}{8}$$

Question: In Young's double slit experiment distance between 4th bright fringe on both sides of center of screen is 2.4 cm. Find frequency of light if distance between slits is 0.3 mm and distance of screen from slits is 1.5 m.

Options:

(a) $6\times10^{15}\,Hz$

(b) $7\times10^{15}\,Hz$

(c) $5\times10^{15}\,Hz$

(d) $5\times10^{17}\,Hz$

Answer: (c)

Solution:

$$2y_4=2.4\times10^{-2}\,m$$

$$y_4=1.2\times10^{-2}\,m$$

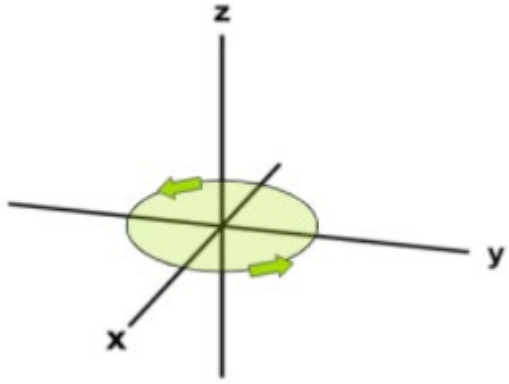
$$y=\frac{n\lambda D}{d}$$

$$y_4=\frac{4\lambda D}{d}$$

$$1.2\times10^{-2}=\frac{4\times3\times10^8\times1.5}{f\times3\times10^{-4}}$$

$$\Rightarrow f=5\times10^{15}\,Hz$$

Question: A loop when viewed from top carries on anti-clockwise current as shown. The direction of magnetic field is



Options:

(a) along positive z-axis

(b) along negative z-axis

(c) along x-axis

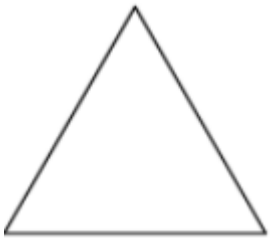
(d) along y-axis

Answer: (a)

Solution:

According to Right Hand thumb rule, when we curl fingers towards the direction of current we will get the direction of magnetic field along positive z-axis.

Question: An equilateral triangle of side 10 cm carries a current of 3 A. Find the magnetic field at centroid of triangle



Options:

(a) $1.2\times10^{-5}T$

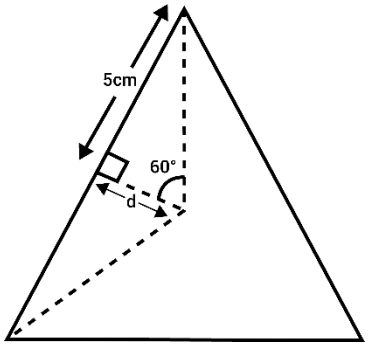
(b) $3.6\times10^{-5}T$

(c) $2.4\times10^{-5}T$

(d) $5.4\times10^{-5}T$

Answer: (d)

Solution:



$$\frac{5}{d} = \tan 60° = \sqrt{3}$$

$$d = \frac{5}{\sqrt{3}} \times 10^{-2} m$$

$$B = 3\left[\frac{\mu_0 i}{4\pi d}\left(\sin 60° + \sin 60°\right)\right] (\text{due to all three section})$$

$$= 3 \times \frac{\mu_0 \times 3}{4\pi \times \frac{5}{\sqrt{3}} \times 10^{-2}}\left(\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}\right)$$

$$= 5.4 \times 10^{-5} T$$

Question: The ratio of gravitational field at depth R from earth surface to height R above surface is

Options:

(a) $\sqrt{2}$

(b) zero

(c) $\frac{1}{\sqrt{2}}$

(d) None

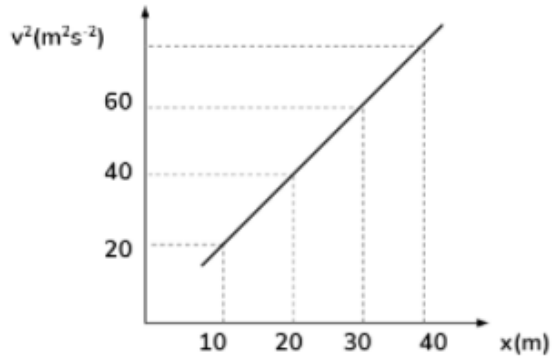
Answer: (b)

Solution:

Gravitational field at depth R from surface i.e., at center of earth is 0, and at height R from the surface $= \frac{GM}{(R+R)^2}$

$$\text{Ratio} = \frac{0}{\frac{GM}{4R^2}} = 0$$

Question: The v^2 – x graph of a particle travelling in straight line is shown below. The acceleration of particle is



Options:

(a) $4\,ms^{-2}$

(b) $2\,ms^{-2}$

(c) $1\,ms^{-1}$

(d) None

Answer: (c)

Solution:

We know $v^2 - u^2 = 2aS$

$\therefore v^2 = u^2 + 2aS$

Which represents a line with slope '2a' and intercept 'u^2'

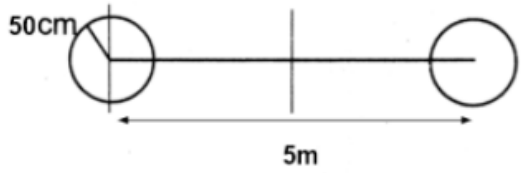
From the graph, slope is

$$m = \frac{60-40}{30-20} = \frac{20}{10} = 2\,m/s^2$$

That means $2a = 2\,m/s^2$

$\Rightarrow a = 1\,m/s^2$

Question: Two spheres are of same mass (20 kg) and same radius (50 cm) connected to two ends of a rod of negligible mass. Find moment of inertia which is perpendicular to rod and passing through the midpoint of rod. Distance between spheres is 5 m.



Options:

(a) 367 kgm^2

(b) 728 kgm^2

(c) 364 kgm^2

(d) 734 kgm^2

Answer: (c)

Solution:

Moment of inertia of a solid sphere about its centre of mass

$$I_{COM} = \frac{2}{5}MR^2$$

$$= \frac{2}{5}(20)\left(\frac{1}{2}\right)^2$$

$$= 2\,kg - m^2$$

Now using parallel axis theorem, we get M.I. of this sphere about given axis is

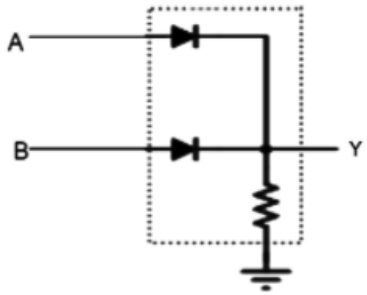
$$I' = I_{COM} + md^2$$

$$I' = 2 + (20)(3)^2 = 182\,kg - m^2$$

Now both spheres are symmetrical about the axis so that M.I. of system

$$I_{total} = 182 \times 2 = 364\,kg - m^2$$

Question: Identify the Gate



Options:

(a) OR Gate

(b) NAND Gate

(c) NOR Gate

(d) AND Gate

Answer: (a)

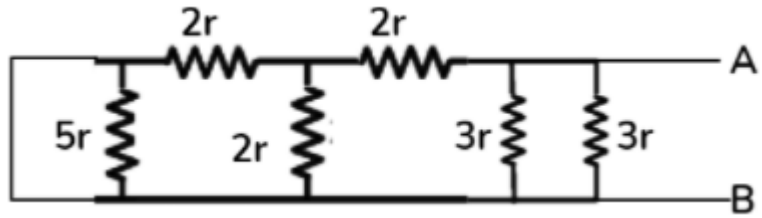
Solution:

Whenever A or B or A and B both are forward biased (i.e. + ive voltage), Y will have same potential as it is connected to junction of A and B. Also current will flow whenever A or B or both are forward biased i.e. output Y will be non-zero. Y will be zero only when A & B both are zero or reverse biased.

A	B	Y
1	0	1
0	1	1
1	1	1
0	0	0

This is a truth table of an OR gate

Question: Find equivalent resistance across the points A and B.



Options:

(a) 1 r

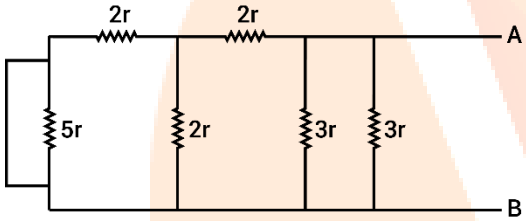
(b) 3 r

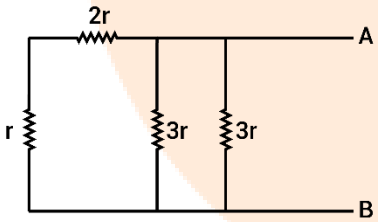
(c) 2 r

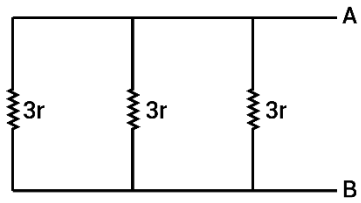
(d) 4 r

Answer: (a)

Solution:







$$R_{eq} = \frac{3r}{3} = r$$

Question: A resistor produces $192\,\text{J}\,\text{s}^{-1}$ heat when 4 A current is passed through it. Find heat produced when 8 A current is passed for 5 seconds.

Options:

(a) 3200 J

(b) 4284 J

(c) 3840 J

(d) None

Answer: (c)

Solution:

$H \propto i^2 t$

$$\frac{H_1}{H_2} = \left(\frac{i_1}{i_2}\right)^2 \left(\frac{t_1}{t_2}\right)$$

$$\Rightarrow H_2 = \left(\frac{i_2}{i_1}\right)^2 H_1 \left(\frac{t_2}{t_1}\right)$$

$$= \left(\frac{8}{4}\right)^2 \times 192 \times \frac{5}{1}$$

$$= 3840\,\text{J}$$

Question: A capacitor is connected to a battery is fully charged, now a dielectric (dielectric constant K) is inserted between the plates without disconnecting the battery, then increase in potential energy is

Options:

(a) K times

(b) K – 1 times

(c) 1/K times

(d) K^2 times

Answer: (b)

Solution:

Initial energy $= \frac{1}{2} CV^2$

Final energy $= \frac{1}{2}(KC)V^2$

Increase in energy $= \frac{1}{2} KCV^2 - \frac{1}{2} CV^2$

$$= \frac{1}{2}(K-1)CV^2$$

$\therefore$ increase in energy is $(K-1)$ times.

Question: If two charges $q_1 = 4\pi \,\&\, q_2 = 2\pi$ are placed in a uniform magnetic field $B = B_0 \frac{\hat{i}+\hat{j}}{\sqrt{2}}$ at $\left(0,0,\frac{\pi}{2}\right)$ and $\left(0,0,\frac{3\pi}{2}\right)$ respectively & both are moving with speed 0.5 C and in the direction perpendicular to magnetic field (where C is speed of light). Find ratio of magnetic force acting on them

Options:

(a) 2:1

(b) 2:5

(c) 1:1

(d) $\sqrt{2}:1$

Answer: (a)

Solution:

$$\left|\vec{F}\right| = \left|q\left(\vec{V}\times\vec{B}\right)\right| = qVB\sin\theta$$

$$\frac{F_1}{F_2} = \frac{q_1}{q_2}\times\frac{V_1}{V_2}\times\frac{B_1}{B_2}\times\frac{\sin\theta_1}{\sin\theta_2}$$

$$= \frac{4\pi}{2\pi}\times\frac{0.5C}{0.5C}\times\frac{B}{B}\times\frac{\sin 90}{\sin 90}$$

$$= \frac{2}{1}$$

JEE-Main-31-08-2021-Shift-2 (Memory Based)

CHEMISTRY

Question: Total number of unpaired electrons in O_2^{2-}

Options:

(a) 0

(b) 1

(c) 2

(d) 3

Answer: (a)

Solution:

$O_2^{2-} \Rightarrow KK(\sigma 2s)^2 (\sigma^* 2s)^2 (\sigma 2p_z)^2 (\pi 2p_x)^2 = (\pi 2py)^2 (\pi^* px)^2 = (\pi^* py)^2$

Question: Which one is not polyester?

Options:

(a) Novolac

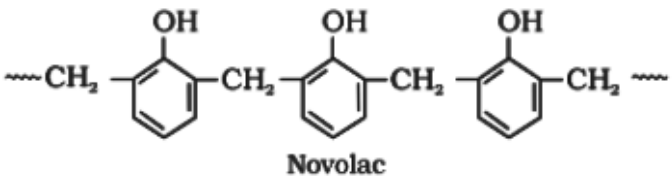
(b) Dacron

(c) Glyptal

(d) PHBV

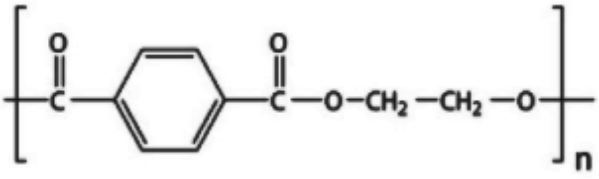
Answer: (a)

Solution:

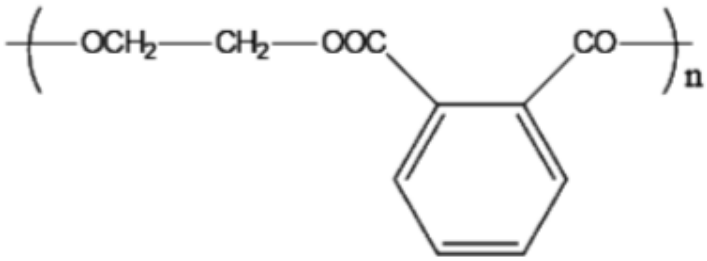


Novolac

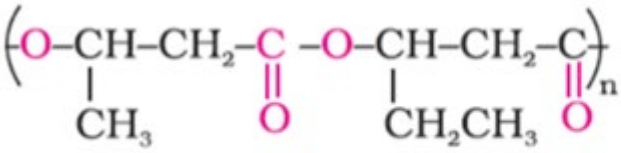
Novolac



Dacron



Glyptal



PHBV

PHBV

Question: Which one is not fibrous protein?

Options:

(a) Albumin

(b) Collagen

(c) Myosin

(d) Fibrin

Answer: (a)

Solution: It is globular protein.

Question: Incorrect statement about hydrogen

Options:

(a) Dihydrogen yields atomic hydrogen when irradiated with UV light

(b) Dihydrogen can be prepared by reacting Zn with HCl and NaOH

(c) Dihydrogen has the highest bond dissociation energy among all diatomic molecules linked with single bonds

(d) At 200 K, only 8.1% of H_2 gets dissociated.

Answer: (d)

Solution: Dihydrogen converts into its atoms is only ~0.081% around 2000 K.

Question: Electronic configuration of Eu^{2+} is

Options:

(a) $4f^6$

(b) $4f^5$

(c) $4f^7$

(d) $4f^8$

Answer: (c)

Solution: $Eu^{2+} \Rightarrow [Xe]4f^7$

Question: Match the column.

Column I (Physical quantities)	Column II (Units)
I) Molar conductance	i) s/m
II) Conductance	ii) No units
III) Degree of dissociation of electrolyte	iii) S cm^2 mol^{-1}

Options:

(a) I → (iii); II → (ii); III → (i)

(b) I → (iii); II → (i); III → (ii)

(c) I → (ii); II → (iii); III → (i)

(d) I → (i); II → (ii); III → (iii)

Answer: (b)

Solution:

Molar conductance ⇒ S cm^2 mol^{-1}

Conductance ⇒ s/m

Degree of dissociation of electrolyte ⇒ No units

Question: S1: Lithium salts are hydrated.

S2: Li has highest polarising power.

Options:

(a) Both S1 and S2 are correct

(b) Both S1 and S2 are incorrect

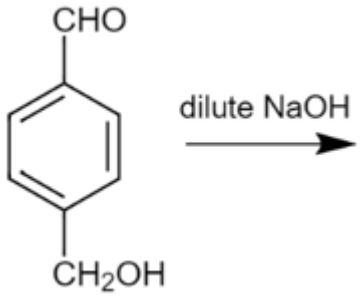
(c) S1 is correct, S2 is incorrect

(d) S1 is incorrect, S2 is correct

Answer: (a)

Solution: Li salts are highly hydrated and has highest polarising power due to small size of Li

Question:



Options:

(a) Monocarboxylic acid

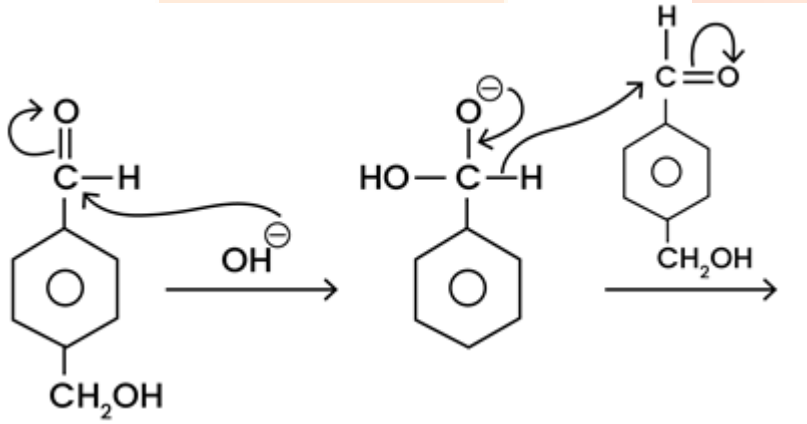
(b) Dicarboxylic acid

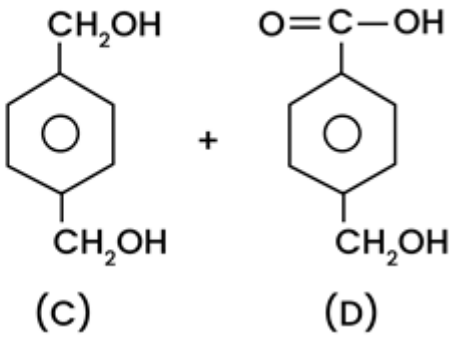
(c) Diol

(d) Acid and alcohol funcitonal group on the same compound

Answer: (d)

Solution:





(C) (D)

Question: Oxide stability of X_2O where X is Halogen?

Cl_2O, Br_2O, I_2O

Options:

(a) $I_2O > Cl_2O > Br_2O$

(b) $Cl_2O > Br_2O > I_2O$

(c) $Br_2O > I_2O > Cl_2O$

(d) $I_2O > Br_2O > Cl_2O$

Answer: (a)

Solution: Higher stability of oxides of iodine is due to greater polarisibility of bond between iodine and oxygen $I_2O > Cl_2O > Br_2O$

Question: Anion and cation they form a ccp lattice. Cation occupies octahedral void. The form is AxB. Find X

Options:

(a) 1

(b) 2

(c) 3

(d) 4

Answer: (a)

Solution:

A = 4

B = 4

AB

X = 1

Question: Number of S=O present in peroxodisulphuric acid, sulphurous acid, pyrosulphuric acid

Options:

(a) 4, 1, 4

(b) 2, 1, 3

(c) 1, 1, 1

(d) 2, 3, 1

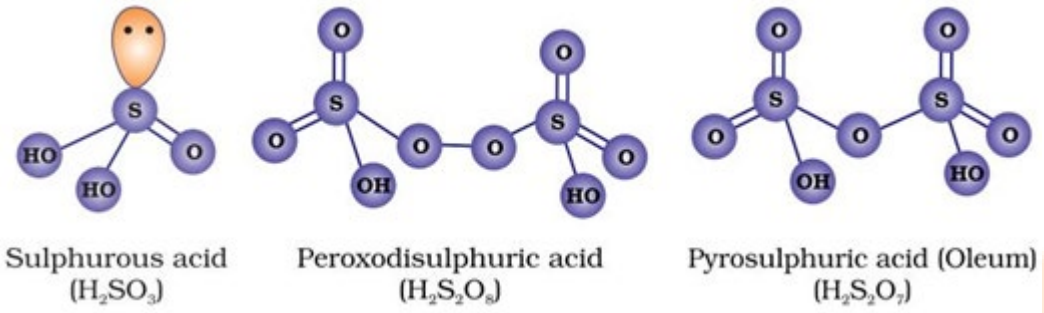
Answer: (a)

Solution:

peroxodisulphuric acid $\Rightarrow$ 4

sulphurous acid $\Rightarrow$ 1

pyrosulphuric acid $\Rightarrow$ 4



Question: The magnetic quantum number of last electron of Zn^+.

Options:

(a) 0

(b) 1

(c) 2

(d) 3

Answer: (a)

Solution: $Zn^+ = 3d^{10}\ 4s^1$

$m_l = 0$

Question: Match (Qualitative Analysis)

Column I (Ions)	Column II (Groups)

A) Mn^{2+}	P) IV
B) Cu^{2+}	Q) IIB
C) Al^{3+}	R) III
D) As^{3+}	S) IIA

Options:

(a) A → S; B → Q; C → R; D → P

(b) A → Q; B → Q; C → P; D → S

(c) A → S; B → P; C → R; D → Q

(d) A → P; B → S; C → R; D → Q

Answer: (d)

Solution:

$Mn^{2+} \Rightarrow$ IV

$Cu^{2+} \Rightarrow$ IIA

$Al^{3+} \Rightarrow$ III

$As^{3+} \Rightarrow$ IIB

Question: pH of 50 mL 1 M HCl and 30 mL 1 M NaOH [log (2.5) = 0.3679]

Options:

(a) 0.6

(b) 0.5

(c) 0.7

(d) 0.3

Answer: (a)

Solution:

$$N_F = \frac{N_1V_1 - N_2V_2}{V_1 + V_2} = \frac{50\times1 - 30\times1}{80} = \frac{20}{80} = \frac{1}{4}$$

$$pH = -\log\left(\frac{1}{4}\right) = 0.6$$

Question: Incorrect relation is:

Options:

(a) $K = e^{-\Delta G_o / RT}$

(b) PV = nRT

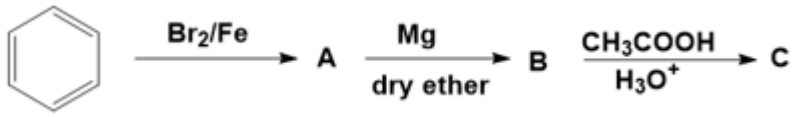
(c) $\Delta G/\Delta S = -T$ (at constant P)

(d) $\Delta G = \Delta H - T\Delta S$

Answer: (c)

Solution: $\Delta G/\Delta S = -T$ (at constant P) is incorrect relation

Question:



Find 'C'

Options:

(a) Bromobenzene

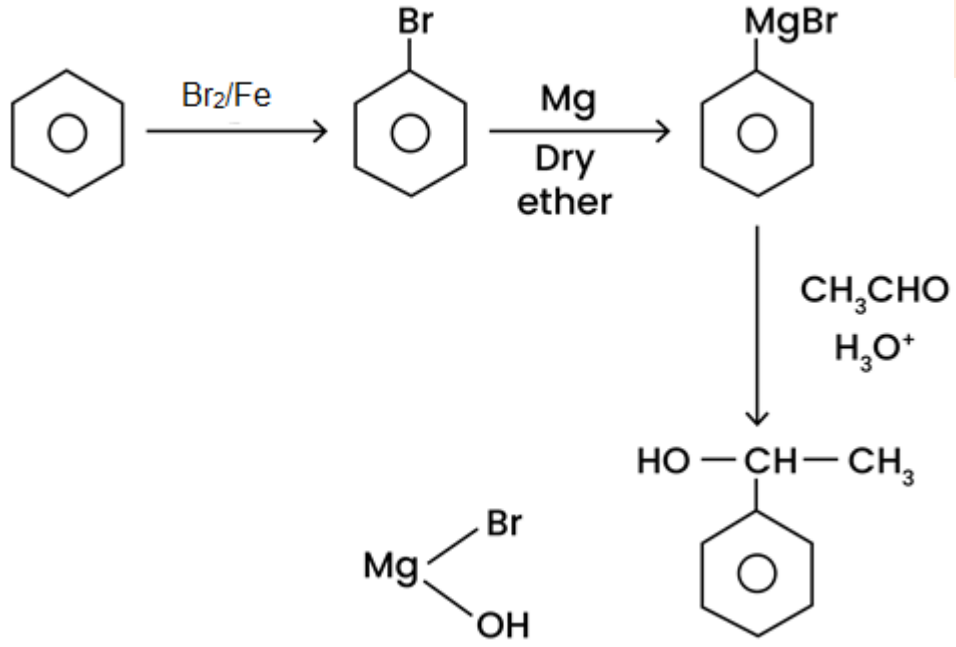
(b) Di-bromobenzene

(c) 1-phenyl ethane 1-ol

(d) Both (a) and (c)

Answer: (c)

Solution:



Question: The magnetic moment of $[Fe(CO)_4(C_2O_4)]^+$ is

Options:

(a) $\sqrt{1.4}$

(b) $\sqrt{3.2}$

(c) $\sqrt{5}$

(d) $\sqrt{3}$

Answer: (d)

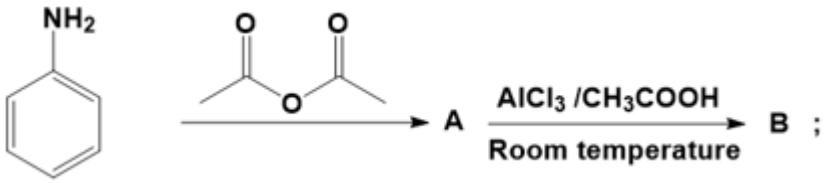
Solution:

$Fe^{+3} = 3d^5 4s^0$

n = 1

$\sqrt{n(n+2)} = \sqrt{3}$

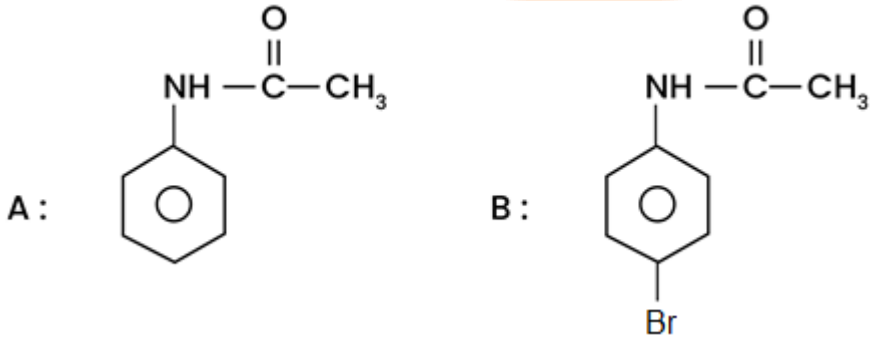
Question:



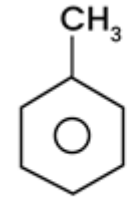
Find A and B

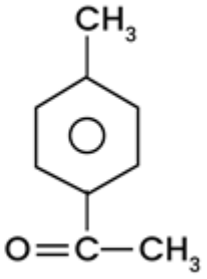
Options:

(a)

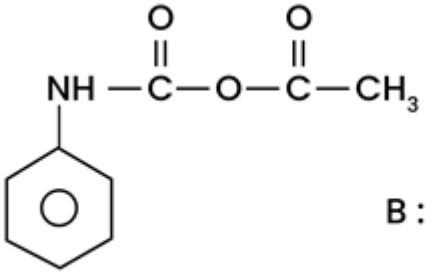


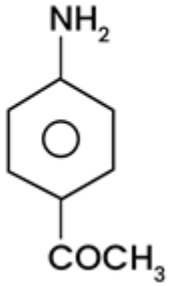
(b)

A : 

B : 

(c)

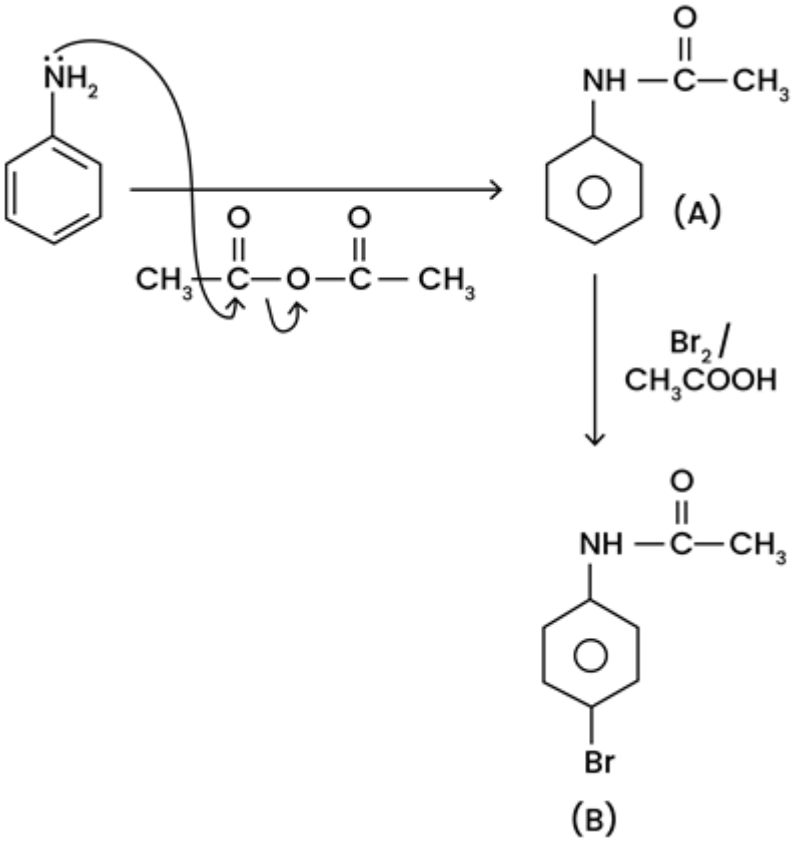
A : 

B : 

(d) None of these

Answer: (a)

Solution:



Question: When nitrobenzene reacts with Cl_2 in presence of $AlCl_3$ gives P, when P reacts with Fe/HCl gives Q. Find Q

Options:

(a) 3-chloronitrobenzene

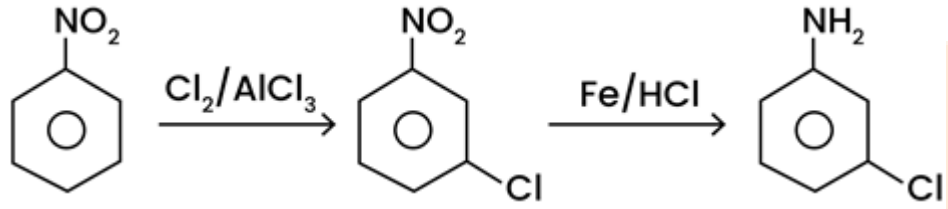
(b) Chlorobenzene

(c) Aniline

(d) 3-chloroaniline

Answer: (d)

Solution:



Question: Total number of main impurities in electrolysis of blister copper:

Au, Ag, Pt, Ru, Th, Te, Sb, Se

Answer: 3.00

Solution: Au, Ag and Pt

Question: Of the given species, how many will show disproportionation reaction:

Mn^{3+}, Cl_2, F_2, ClO_3^-, $Cr_2O_7^{2-}$, MnO_4^-

Answer: 3.00

Solution: Mn^{3+}, Cl_2 and ClO_3^-,

JEE-Main-31-08-2021-Shift-2 (Memory Based)

MATHEMATICS

Question: Number of 4 digit numbers which are divisible by neither 7 or 3
Answer: 5143.00
Solution:

Number of 4 digit number = 9000

Number divisible by 3 = $\left[\frac{9000}{3}\right] = 3000$

Number divisible by 7 = $\left[\frac{9000}{7}\right] = 1285$

Number divisible by both = $\left[\frac{9000}{3\times 7}\right] = 428$

So, numbers not divisible by 3 or 7 = $9000 - 3000 - 1285 + 428$

$= 5143$

Question: $\lim\limits_{x\to\frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(\frac{\pi}{4}+x\right)} = \alpha$ and $\lim\limits_{x\to 0}(\cos x)^{\cot x} = \beta$ if α and β are root of equation $ax^2 + bx - 4$ then ordered pair (a, b) is

Answer: $a = 1, b = 3$
Solution:

$$\lim_{x\to\frac{\pi}{4}} \frac{\tan x\left(\tan^2 x - 1\right)\sqrt{2}}{\left(\cos x - \sin x\right)} = \alpha$$

$$\alpha = \lim_{x\to\frac{\pi}{4}} \frac{\left(\tan x + 1\right)\sqrt{2}}{-\cos x} = -4$$

$$\beta = \lim_{x\to 0}(\cos x)^{\cot x}$$

$$= e^{\lim\limits_{x\to 0}\frac{(\cos x - 1)}{\tan x}}$$

$$\beta = e^{\lim\limits_{x\to 0}\frac{-\sin x}{\sec^2 x}} = 1$$

$\therefore \alpha = -4, \beta = 1$

$ax^2 + bx - 4 = 0$

$\alpha\beta = -4 = \dfrac{-4}{a} \Rightarrow a = 1$

$\alpha + \beta = \dfrac{-b}{a} = -3; b = -3$

Question: If $[x]$ denotes GIF, then value of $\pi^2 \int_0^2 \sin\left(\dfrac{\pi}{2}x\right)(x-[x])^{[x]}\,dx$

Answer: $4\pi - 4$

Solution:

$$I = \pi^2\left[\int_0^1 \sin\left(\frac{\pi x}{2}\right)dx + \int_1^2 (x-1)\sin\left(\frac{\pi x}{2}\right)dx\right]$$

$$= \pi^2\left[\left\{\frac{-\cos\left(\frac{\pi x}{2}\right)}{\frac{\pi}{2}}\right\}_0^1 + \left\{\frac{x\left(-\cos\frac{\pi x}{2}\right)}{\frac{\pi}{2}} + \frac{\sin\frac{\pi x}{2}}{\frac{\pi^2}{4}} + \frac{\cos\frac{\pi x}{2}}{\frac{\pi}{2}}\right\}_1^2\right]$$

$$\Rightarrow I = \pi^2\left[\frac{2}{\pi} + \frac{4}{\pi} - \frac{4}{\pi^2} - \frac{1}{\pi}\right]$$

$$= 4(\pi - 1)$$

Question: If $\dfrac{z-1}{z-i}$ is purely imaginary then find minimum value of $\left|z-(3+3i)\right|$

Answer: $2\sqrt{2}$

Solution:

Let $z = x + iy$

$$\Rightarrow \left[\frac{(x-1)+iy}{x+(y-1)i}\right]\left[\frac{x-i(y-1)}{x-i(y-1)}\right]$$

$$\Rightarrow x^2 - x + y^2 - y = 0$$

$$\Rightarrow \left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{2}\right)^2=\frac{1}{2}$$

$$\therefore \text{ Minimum value} = \left(\sqrt{\left(3-\frac{1}{2}\right)^2+\left(3-\frac{1}{2}\right)^2}-\frac{1}{\sqrt{2}}\right)$$

$$=\frac{5}{\sqrt{2}}-\frac{1}{\sqrt{2}}=\frac{4}{\sqrt{2}}=2\sqrt{2}$$

Question: $32^{\tan^2 x}+32^{\sec^2 x}=81$ find number of solutions if $0 \le x \le 2\pi$.

Answer: 4.00

Solution:

$$32^{\tan^2 x}+32^{\sec^2 x}=81$$

$$32^{\tan^2 x}+32^{1+\tan^2 x}=81$$

$$t+32t=81$$

$$33t=81$$

$$t=\frac{81}{33}=\frac{27}{11}$$

$$32^{\tan^2 x}=\frac{22}{11}$$

$\tan^2 x$ have same value less than 1

$\Rightarrow$ 4 solutions possible

Question: A function defined $g \to g$ from set 1, 2 3, 4, 5, 6, …. And it is onto then find probability that $g(3)=2g(1)$

Answer: $\frac{1}{10}$

Solution:

Total number of onto function $=6!$

For $g(3)=2\ g(1)$ we ca have

$g(1)=1$ and $g(3)=2$, $g(1)=2$ and $g(3)=4$

$g(1)=3$ and $g(3)=6$

$\Rightarrow 3$ favourable case

Required probability $=\frac{3}{6!}$

Question: The coefficient of a^7a^8 in $(a+2b+4ab)^{10} = k2^{16}$. Find k.

Answer: 315.00

Solution:

$a^7b^8 \to (a+2b+4ab)^{10}$

$\because (a+2b+4ab)^{10} = \sum \frac{10!}{x!y!z!}(a)^x(2b)^y(4ab)^z$

$= \sum \frac{10!}{x!y!z!}(a)^{x+z}(b)^{y+z}(z)^{y+2z}$

Where, $x+y+z=10$

$x+z=7, y+z=8$

$x=2, y=3, z=5$

$\therefore$ coefficient of $a^7b^8 = \frac{10!2^{13}}{2!3!5!}$

$= \frac{10\times9\times8\times7\times6}{6\times2}\times2^{13}$

$= 315\times2^{16}$

$\Rightarrow k = 315$

Question: If $\frac{a_1+a_2\ldots a_{10}}{a_1+a_2\ldots(ap)^2} = \frac{100}{p^2}$ find value of $\frac{a_{10}}{a_{11}}$

Answer: $\frac{19}{21}$

Solution:

$\frac{a_1+a_2\ldots a_{10}}{a_1+a_2\ldots ap} = \frac{10^2}{p^2}$

We know $\sum_{r=1}^{n}(2r-1) = n^2$

$\Rightarrow a_r = 2r-1$

$\Rightarrow \frac{a_{10}}{a_{11}} = \frac{19}{21}$

Question: Line $\frac{x-2}{\alpha}=\frac{y-2}{-3}=\frac{z+2}{2}$ lies in $x+3y-2z+\beta=0$ then $\alpha+\beta=?$

Answer: 1.00

Solution:

$\frac{x-2}{\alpha}=\frac{y-2}{-3}=\frac{z+2}{2}$ lies in $x+3y-2z+\beta=0$

$\Rightarrow 2+3(2)-2(-2)+\beta=0$

$\Rightarrow 2+6+4+\beta=0$

$\Rightarrow \beta=-12$

Also $\alpha+(-3)(3)+(2)(-2)=0$

$\Rightarrow \alpha-9-4=0$

$\Rightarrow \alpha=13$

$\alpha+\beta=13-12=1$

Question: Let f be a function such that $f(m+n)=f(m).f(n)$ and $f(6)=18,\ f(1)=3.$

Then find the value of $f(2).f(3)$

Answer: 6.00

Solution:

$f(m+n)=f(m)\cdot f(n)$

$f(1+5)=f(1)\cdot f(5)$

$18=3f(5)$

$\therefore f(5)=6$

$f(2+3)=f(2)\cdot f(3)=6$

Question: Find number of elements in

$\left\{A=\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}; a,b,d \in \{-1,0,1\} \text{ such that } (I-A)^3 = I - A^3\right\}$; I is 2×2 identity matrix

Answer: 8.00

Solution:

$(I-A)^3 = I - A^3 - 3A + 3A^2$

$\therefore 3A^2 - 3A = 0$

$A^2 = A$

$A = I$ or $A = 0$

$a = d = 1, b = 0$

$a = d = b = 0$

$\therefore$ Number of elements $= 2^3 = 8$

Question: The negation of the statement $(p \vee q) \Rightarrow (p \vee r)$

Options:

(a) $p \wedge q \wedge r$

(b) $p \wedge q \wedge \sim r$

(c) $p \wedge \sim q \wedge \sim r$

(d) $\sim p \wedge q \wedge r$

Answer: (c)

Solution:

$\sim[(p \vee q)(q \vee r)]$

$(p \vee q) \wedge \sim (q \vee r)$

$(p \vee q) \wedge (\sim q \wedge \sim r)$

$(\sim q \wedge \sim r) \wedge (p \vee q)$

$(p \wedge \sim q \wedge \sim r) \vee (q \wedge \sim q \wedge \sim r)$

$(p \wedge \sim q \wedge \sim r)$

Question: Angle between the curves $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and $x^2+y^2=ab(a>b)$.

Answer: $\tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$

Solution:

Curves $\frac{x^2}{a^2}+\frac{y^2}{b^2}$ $....(1)$

& $x^2+y^2=ab$ $...(2)$

Second curve is $x^2+y^2=ab$

$y^2=ab-x^2$

Substituting this in equation (1)

$\Rightarrow \frac{x^2}{a^2}+\frac{ab-x^2}{b^2}=1$

$\Rightarrow x=\pm\sqrt{\frac{a^2b}{(b+a)}}$ $....(3)$

Since, $y^2=ab-x^2$

$\Rightarrow y=\pm\sqrt{\frac{ab^2}{(b+a)}}$ $...(4)$

Since, curves are $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ & $x^2+y^2=ab$

Differentiating above w.r.t. x

$\Rightarrow \frac{2x}{a^2}+\frac{2y}{b^2}\cdot\frac{dy}{dx}=0$

$\Rightarrow \frac{dy}{dx}=\frac{-b^2x}{a^2y}$

$\Rightarrow m_1=\frac{dy}{dx}=\frac{-b^2x}{a^2y}$ $....(5)$

Second curve is $x^2+y^2=ab$

$\Rightarrow 2x+2y\cdot\frac{dy}{dx}=0$

$$\Rightarrow m_2 = \frac{dy}{dx} = \frac{-x}{y} \quad ....(6)$$

Substituting (3) in (4), above values for m_1 & m_2, we get

At $\left(\sqrt{\frac{a^2b}{(b+a)}}, \sqrt{\frac{ab^2}{(b+a)}}\right)$ in equation (5), we get

$$\Rightarrow \frac{dy}{dx} = \frac{-b^2 \times \sqrt{\frac{a^2b}{(b+a)}}}{a^2 \times \sqrt{\frac{ab^2}{(b+a)}}}$$

$$\Rightarrow m_1 = \frac{dy}{dx} = \frac{-b\sqrt{b}}{a\sqrt{a}}$$

At $\left(\sqrt{\frac{a^2b}{(b+a)}}, \sqrt{\frac{ab^2}{(b+a)}}\right)$ in equation (6), we get

$$\Rightarrow \frac{dy}{dx} = \frac{-\sqrt{\frac{a^2b}{(b+a)}}}{\sqrt{\frac{ab^2}{(b+a)}}}$$

$$\Rightarrow m_2 = \frac{dy}{dx} = -\sqrt{\frac{a}{b}}$$

When $m_1 = \frac{-b\sqrt{b}}{a\sqrt{a}}$ & $m_2 = -\sqrt{\frac{a}{b}}$

$$\Rightarrow \tan\theta = \left|\frac{\frac{-b\sqrt{b}}{a\sqrt{a}} - \sqrt{\frac{a}{b}}}{1 + \frac{-b\sqrt{b}}{a\sqrt{a}} \times -\sqrt{\frac{a}{b}}}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{(a-b)}{\sqrt{ab}}\right|$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{(a-b)}{\sqrt{ab}}\right)$$