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Question Bank

Standard :- 12th (Commerce)

Subject :- MATHEMATICS AND STATISTICS (88)

सूचना

१. फक्त विद्यार्थ्यांना प्रश्नप्रकारांचा सराव करून देण्यासाठीच

२. सदर प्रश्नसंचातील प्रश्न बोर्डाच्या प्रश्नपत्रिकेत येतीलच असे नाही याची नोंद घ्यावी.

State Council Of Educational Research and Training, Maharashtra Pune

QUESTION BANK

STD XII   Commerce Stream

MATHEMATICS AND STATISTICS   (88)

Part-I

1. MATHEMATICAL LOGIC

Q1) Select and write the most appropriate answer from the given alternatives:

i) Which of the following statement is true
   a) 3 + 7 =4 or 3 – 7 = 4
   b) If Pune is in Maharashtra, then Hyderabad is in Kerala
   c) It is false that 12 is not divisible by 3
   d) The square of any odd integer is even.

ii) Which of the following is not a statement
   a) 2+2 =4
   b) 2 is the only even prime number
   c) Come here
   d) Mumbai is not in Maharashtra

iii) If p is any statement then $(p \vee \sim p)$ is a
   a) Contingency
   b) Contradiction
   c) Tautology
   d) None of these

iv) If p and q are two statements , then $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is
   a) Contradiction
   b) Tautology
   c) Neither (i) nor (ii)
   d) None of these

v) Negation of $p \rightarrow (p \vee \sim q)$ is

a) $\sim p \rightarrow (\sim p \vee q)$

b) $p \wedge (\sim p \wedge q)$

c) $\sim p \vee (\sim p \vee \sim q)$

d) $\sim p \rightarrow (\sim p \rightarrow q)$

vi) If p : He is intelligent

q : He is strong

Then, symbolic form of statement "It is wrong that, he is intelligent or strong " is

a) $\sim p \vee \sim q$

b) $\sim(p \wedge q)$

c) $\sim(p \vee q)$

d) $p \vee \sim q$

vii) A biconditional statement is the conjunction of two -------------- statements

a) Negative

b) Compound

c) Connective

d) Conditional

viii) If $p \rightarrow q$ is an implication , then the implication $\sim q \rightarrow \sim p$ is called its

a) Converse

b) Contrapositive

c) Inverse

d) Alternative

ix) The dual of the statement $(p \vee q) \wedge (r \vee s)$

a) $(p \wedge q) \wedge (r \wedge s)$

b) $(p \wedge q) \vee (r \wedge s)$

c) $(p \vee q) \vee (r \vee s)$

d) $(p \text{ v } q) \wedge (r \vee s)$

x) The false statement in the following is

a) $p \wedge (\sim p)$ is contradiction
b) $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a contradiction
c) $\sim (\sim p) \leftrightarrow p$ is a tautology
d) $p \vee (\sim p) \leftrightarrow p$ is a tautology

Q 2) State whether the given statements are true or false

i) The negation of $10 + 20 = 30$ is, it is false that $10 + 20 \neq 30$
ii) $x^2 = 25$ is true statement
iii) $p \rightarrow q$ is equivalent to $p \rightarrow \sim q$
iv) Truth value of $\sqrt{3}$ is not an irrational number is F
v) $(p \vee q) \wedge \sim p$ is a contradiction
vi) $p \leftrightarrow q$ is false when p and q have different truth values
vii) The dual of $(p \wedge q) \vee \sim q$ is $(p \vee q) \wedge \sim q$
viii) Mathematical identities are true statements
ix) $p \vee \sim p \equiv \sim c$
x) The converse of inverse of $\sim p \rightarrow q$ is $q \rightarrow \sim p$

Q3) Fill in the following blanks

i) Conjunction of two statement pattern p and q is symbolically written as -------
ii) Negation of "Some men are animal " is -------------------
iii) The truth value of negation of " London is in England " is -----------
iv) The truth value of the statement " Neither 27 is a prime number nor divisible by 4" is ----------
v) The contrapositive of $p \rightarrow \sim q$ is ------------

Q4) Answer the following questions

i) Write the negation of the statement " An angle is a right angle if and only if it is of measure 90^0"
ii) Write the following statements in symbolic form
   a) Milk is white if and only if the sky is not blue
   b) If Kutab – Minar is in Delhi then Taj- Mahal is in Agra

c) Even though it is not cloudy , it is still raining

iii) Use quantifiers to convert the given open sentence defined on N into a true statement

a) $n^2 \geq 1$

b) $3x - 4 < 9$

c) $Y + 4 > 6$

iv) Examine whether the statement pattern is a tautology, contradiction or contingency

$(p \wedge \sim q) \rightarrow (\sim p \wedge \sim q)$

v) Using truth table prove that $\sim p \wedge q \equiv (p \vee q) \wedge \sim p$

vi) Write the dual of the following

a) 13 is prime number and India is a democratic country

b) $(p \wedge \sim q) \vee (\sim p \wedge q) \equiv (p \vee q) \wedge \sim (p \wedge q)$

vii) Write the converse, inverse and contrapositive of the statement "If it snows, then they do not drive the car"

Q5) Answer the following questions

i) Examine whether the statement pattern $[p \rightarrow (\sim q \vee r)] \leftrightarrow \sim [p \rightarrow (q \rightarrow r)]$ is a tautology, contradiction or contingency.

ii) Using truth table prove that $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

iii) Without using truth table show that $(p \vee q) \wedge (\sim p \vee \sim q) \equiv (p \vee \sim q) \wedge (\sim p \vee q)$

iv) With proper justification state the negation of $(p \leftrightarrow q) \vee (\sim q \rightarrow \sim r)$

v) Prepare truth table for $(p \wedge q) \vee \sim r$

vi) If p ↔ q and p → q both are true then find truth values of the following with the help of activity.

a) P ∨ q
b) P ∧ q

p ↔ q and p → q both are true if p and q has truth value ☐ ☐ or

☐ ☐

P v q

i) If both p and q are true then p v q = ☐ v ☐ = ☐
ii) If both p and q are false , then p v q = ☐ v ☐ = ☐

P ^ q

i) If both p and q are true then p ^ q = ☐ ^ ☐ = ☐
ii) If both p and q are false then p ^ q = ☐ ^ ☐ = ☐

vii) Given following statements

P : 9 x 5 = 45
q : Pune is in Maharashtra
r : 3 is the smallest prime number

Write truth values by activity

i) $(p \wedge q) \wedge r = (\square \wedge \square) \wedge \square$

$= \square \wedge \square$

$= \square$

ii) $\sim (p \wedge r) = \sim (\square \wedge \square)$

$= \sim \square$

$= \square$

iii) $p \rightarrow q = \square \rightarrow \square$

$= \square$

viii) Complete the truth table

p	q	r	q→r	r→p	(q→r)v (r→p)
T	T	T	T	$\square$	T
T	T	F	F	$\square$	$\square$
T	F	T	T	$\square$	T
T	F	F	T	$\square$	$\square$
F	T	T	$\square$	F	T
F	T	F	$\square$	T	$\square$
F	F	T	$\square$	F	T
F	F	F	$\square$	T	$\square$

The given statement pattern is $\square$

2. MATRICES

I) Select an appropriate option and write it on blank space along with their alphabet code. [1 marks each]

1) If $A = \begin{bmatrix} 6 & 0 \\ p & q \end{bmatrix}$ is a scalar matrix then the value of p and q are ______.

a) 6 and 0 b) 0 and 6 c) 5 and 6 d) 0 and 1

2) If $B = \begin{bmatrix} 6 & 3 \\ -2 & k \end{bmatrix}$ is singular matrix then the value of k is_____.

a) -1 b)2 c) 0 d) 1

3) If $A = \begin{bmatrix} 1 & \frac{3}{5} & x \\ y & -5 & -7 \\ -4 & -7 & 0 \end{bmatrix}$ is a symmetric matrix then the value of x and y are ________.

a) $\frac{3}{5}$ and 4 b) $\frac{5}{3}$ and -4 c) $\frac{3}{5}$ and -4 d) -4 and $\frac{3}{5}$

4) $\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \\ -1 \end{bmatrix} =$ ____.

a) [2] b) 1 c) [1] d) 2

5) If $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ then $A^2 - 3I$ =_____.

a) O b) I c) A d) 3A

6) For any square matrix B, matrix $B + B^T$ is ____.

a) Null matrix b) Identity matrix c) Symmetric matrix d) Skew symmetric matrix

7) If A and B are two square matrices of order 3, then $(AB)^T =$ _____.

a) $B^T.A^T$ b) $A^T B^T$ c) $(BA)^T$ d) I

8) If $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$ then adj(A) = _____.

a) $\begin{bmatrix} 1 & -2 \\ 2 & -1 \end{bmatrix}$ b) $\begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$ c) $\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}$ d) $\begin{bmatrix} 1 & 2 \\ -2 & -1 \end{bmatrix}$

9) If A is a non singular matrix of order 3 then $|\text{adj(A)}| =$ _____.

a) $|A|^2$ b) $|A|^3$ c) 0 d) 1

10) If $A^2 + 5A + 3I = 0$, $|A| \neq 0$ then A^{-1} = ______.

a) $\frac{-1}{3}(A + 5I)$ b) $\frac{-1}{5}(A + 3I)$ c) $(A + 15I)$ d) $\frac{-1}{3}(I + 5A)$

II) State whether each of the following is True or False. [1 marks each]

1) If A is non singular then $|A| = 0$.

2) Inverse of $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ is $\begin{bmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{3} \end{bmatrix}$.

3) If $\begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ then x= 1 and y= -1.

4) Every square matrix of order n can be expressed as sum of symmetric and skew symmetric matrix.

5) If $A = \begin{bmatrix} 1 & 2 & -5 \\ 2 & -3 & 4 \\ -5 & 4 & 9 \end{bmatrix}$ then $A^T = A$.

6) Matrix $\begin{bmatrix} a & b & c \\ p & q & r \\ 2a-p & 2b-q & 2c-r \end{bmatrix}$ is singular.

7) $\begin{bmatrix} 2 & 0 & 0 \\ 3 & -1 & 0 \\ -7 & 3 & 1 \end{bmatrix}$ is a skew symmetric matrix.

8) If $O(A) = m \times n$ and $O(B) = n \times p$ with $m \neq p$ then BA exist but AB does not exist.

9) After applying elementary transformation $R_1 - 3R_2$ on matrix $\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}$ we get $\begin{bmatrix} 0 & -12 \\ 1 & 4 \end{bmatrix}$.

10) If A and B are two square matrices such that AB=BA then $(A-B)^2 = A^2 - 2AB + B^2$.

III) Fill in the blanks.[1 marks each]

1) If $A = \left[a_{ij}\right]_{2\times 2}$ where $a_{ij} = i - j$ then A= ______.

2) In a Skew symmetric matrix, all diagonal elements are ____.

3) If $A = \begin{bmatrix} 1 & -2 \\ 5 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -3 \\ 4 & -7 \end{bmatrix}$ then A-3B= _____.

4) If $A = \begin{bmatrix} 4 & 3 & 2 \\ -1 & 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 1 & -2 \end{bmatrix}$ then $|AB| =$ ___.

5) The value of Cofactor of element a_{21} in matrix $A=\begin{bmatrix}1 & 2\\ 5 & -8\end{bmatrix}$ is ____.

6) The value of Minor of element b_{22} in matrix $B=\begin{bmatrix}2 & -2\\ 4 & 5\end{bmatrix}$ is ____.

7) If matrix form of given equations $3x-y=1$ and $y+4x=6$ is $AX=B$ then $A=\begin{bmatrix}-- & --\\ -- & --\end{bmatrix}$.

8) The suitable elementary row transformation which will reduce the matrix $\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$ into identity matrix is _____.

9) If $A=\begin{bmatrix}2 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$ then $|\text{adj}(A)|=$ ___.

10) If A is a square matrix of order 2 such that $A(\text{adj}A)=\begin{bmatrix}7 & 0\\ 0 & 7\end{bmatrix}$ then $|A|=$ ____.

Solve the following………3 MARKS

1) If $A=\begin{bmatrix}1 & 2\\ -1 & -2\end{bmatrix}$, $B=\begin{bmatrix}2 & a\\ -1 & b\end{bmatrix}$ and $(A+B)^2=A^2+B^2$ then find the value of a and b.

2) Find matrices A and B, if $2A - B = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$ and $A - 2B = \begin{bmatrix} 3 & 2 & 8 \\ -2 & 1 & 7 \end{bmatrix}$.

3) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then prove that $A^2 - 5A + 7I = O$ where I is unit matrix of order 2.

4) Find k, if $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $A^2 = kA - 2I$, where I is identity matrix of order 2.

5) If $A = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 3 & 5 \\ 1 & -7 & 2 \end{bmatrix}$, then verify $(BA)^T = A^T B^T$.

6) Find the inverse of matrix $B = \begin{bmatrix} 3 & 1 & 5 \\ 2 & 7 & 8 \\ 1 & 2 & 5 \end{bmatrix}$ by using adjoint method.

7) If $A = \begin{bmatrix} 3 & 1 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 5 & -2 \end{bmatrix}$ then verify $|AB| = |A||B|$.

8) If $A = \begin{bmatrix} 1 & 3 & 3 \\ 3 & 1 & 3 \\ 3 & 3 & 1 \end{bmatrix}$ then show that $A^2 - 5A$ is a scalar matrix.

4 MARKS

1) Find the inverse of matrix $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$ by using elementary

row transformation.

2) Solve the following equations by using method of inversion.
$4x-3y-2=0, \quad 3x-4y+6=0$.

3) Solve the following equations by method of reduction
$x-3y+z=2,\ 3x+y+z=1,\ 5x+y+3z=3.$

4) The sum of three numbers is 6. If we multiply third number by 3 and add it the second number we get 11. By the adding first and third number we get a number which is double the second number. Use this information and find a system of linear equations. Find the three number using matrices.

5) The total Cost of 3 T.V and 2 V.C.R is ₹ 35000. The shopkeeper wants profit of ₹1000 per T.V and ₹ 500 per V.C.R. He sell 2 T.V. and 1 VCR and he gets total revenue as ₹21500. Find the cost and selling price of T.V and V.C.R.

ACTIVITIES.

1) Complete the following activity toverify A. adj(A)=det(A) I.

Given $A=\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ then

$|A| = 2(\_\_\_\_)-0(\_\_\_\_)+(\ \ )(\_\_\_)=6-0-5=\_\_\_\_\neq 0$

Cofactor of all elements of matrix A are

$A_{11}=(-1)^2\begin{vmatrix} (\ ) & (\ ) \\ (\ ) & (\ ) \end{vmatrix}=(\_\_\_\_),$

$$A_{12} = (-1)^3 \begin{vmatrix} 5 & (\ ) \\ (\ ) & 3 \end{vmatrix} = -15,$$

$$A_{13} = (-1)^4 \begin{vmatrix} 5 & (\ ) \\ (\ ) & 1 \end{vmatrix} = 5,$$

$A_{21} =$ ______, $A_{22} =$ ______, $A_{23} =$ ______,

$$A_{31} = (-1)^4 \begin{vmatrix} (\ ) & (\ ) \\ (\ ) & (\ ) \end{vmatrix} = (\underline{\qquad}),$$

$$A_{32} = (-1)^5 \begin{vmatrix} 2 & (\ ) \\ (\ ) & 0 \end{vmatrix} = (\ ),$$

$$A_{33} = (-1)^6 \begin{vmatrix} 2 & (\ ) \\ (\ ) & 1 \end{vmatrix} = 2.$$

Cofactor of matrix A= $\begin{bmatrix} 3 & \_\_ & \_\_ \\ \_\_ & \_\_ & -2 \\ 1 & \_\_ & \_\_ \end{bmatrix}$,

adj(A)= $\begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$

A. adj(A)= $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} . \begin{bmatrix} (\ ) & -1 & 1 \\ -15 & (\ ) & -5 \\ (\ ) & -2 & (\ ) \end{bmatrix} = \begin{bmatrix} 1 & 0 & (\ ) \\ (\ ) & (\ ) & (\ ) \\ 0 & (\ ) & (\ ) \end{bmatrix} = |A| I$

2) Complete the following activity to find inverse of matrix using elementary column transformation and hence verify.

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} B^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_1 + C_3$

$$\begin{bmatrix} (\ ) & 0 & -1 \\ (\ ) & 1 & 0 \\ (\ ) & 1 & 3 \end{bmatrix} B^{-1} = \begin{bmatrix} (\ ) & 0 & 0 \\ (\ ) & 1 & 0 \\ (\ ) & 0 & 1 \end{bmatrix}$$

$C_3 + C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ (\ ) & 1 & (\ ) \\ 3 & 1 & (\ ) \end{bmatrix} B^{-1} = \begin{bmatrix} 1 & 0 & (\ ) \\ 0 & 1 & 0 \\ (\ ) & 0 & (\ ) \end{bmatrix}$$

$$C_1 - 5C_2,\ C_3 - 5C_2$$

$$\begin{bmatrix} 1 & (\ ) & 0 \\ 0 & 1 & 0 \\ (\ ) & 1 & (\ ) \end{bmatrix} B^{-1} = \begin{bmatrix} 1 & 0 & (\ ) \\ (\ ) & 1 & -5 \\ 1 & (\ ) & 2 \end{bmatrix}$$

$$C_1 + 2C_3,\ C_2 - C_3$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} B^{-1} = \begin{bmatrix} 3 & -1 & (\ ) \\ (\ ) & 6 & -5 \\ 5 & (\ ) & (\ ) \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} (\ ) & (\ ) & (\ ) \\ (\ ) & (\ ) & (\ ) \\ (\ ) & (\ ) & (\ ) \end{bmatrix}$$

$$\begin{bmatrix} 2 & (\ ) & -1 \\ (\ ) & 1 & 0 \\ 0 & 1 & (\ ) \end{bmatrix} . \bullet \begin{bmatrix} 3 & (\ ) & (\ ) \\ (\ ) & 6 & (\ ) \\ (\ ) & -2 & (\ ) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3) Complete the following activity.

The cost of 4 kg potato, 3kg wheat and 2kg rice is ₹ 60. The cost of 1 kg potato, 2 kg wheat and 3kg rice is ₹45. The cost of 6 kg potato, 3 kg rice and 2 kg wheat is ₹ 70. Find the per kg cost of each item by matrix method.

Solution: Let the cost of potato, wheat and rice per kg are x, y and z respectively.

Therefore by given condition,

$$4x + (\ )y + 2(\ ) = (\ )$$
$$x + 2y + (\ )(\ ) = (\ )$$
$$(\ )x + 2y + 3z = (\ )$$

Matrix form of above equation is,

$$\begin{bmatrix} (\ ) & 3 & (\ ) \\ 1 & (\ ) & 3 \\ (\ ) & 2 & (\ ) \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (\ ) \\ 45 \\ (\ ) \end{bmatrix}$$

$$R_1 \leftrightarrow R_2$$

$$\begin{bmatrix} 1 & 2 & 3 \\ (\ ) & (\ ) & (\ ) \\ 6 & 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (\ ) \\ 60 \\ (\ ) \end{bmatrix}$$

$$R_2 - 4R_1, \quad R_3 - 6R_1$$

$$\begin{bmatrix} 1 & 2 & 3 \\ (\ ) & -5 & (\ ) \\ 0 & (\ ) & -15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ (\ ) \\ -200 \end{bmatrix}$$

$$\frac{-1}{5} R_2, \quad \frac{-1}{5} R_3$$

$$\begin{bmatrix} (\ ) & 2 & 3 \\ 0 & (\ ) & 2 \\ 0 & 2 & (\ ) \end{bmatrix} \begin{bmatrix} x \\ (\ ) \\ z \end{bmatrix} = \begin{bmatrix} 45 \\ 24 \\ 40 \end{bmatrix}$$

$$R_3 - 2R_2$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (\ ) \\ (\ ) \\ (\ ) \end{bmatrix}$$

By Re multiplying we get,

$x + 2y + (\ )z = (\ ) ....(1) \quad y + 2z = 24 ....(2) \quad -z = (\ ) ......(3)$

From (3), we get, z=( )

From (2), we get, y=( )

From (1), we get, x=( )

Therefore the cost of Potato, Wheat and Rice per kg are ___, ____ and ____.

3. DIFFERENTIATION

I. Choose the correct alternatives :

1. If $y = \frac{1}{\sqrt{3x^2-2x-1}}$ then $\frac{dy}{dx}$ =?

a) $\frac{-2}{3}$ (3x-2) $(3x^2 - 2x - 1)^{\frac{-3}{2}}$ b) $\frac{-3}{2}$ (3x-2) $(3x^2 - 2x - 1)^{\frac{-3}{2}}$

c) (3x-1) $(3x^2 - 2x - 1)^{\frac{-3}{2}}$ d) -(3x-1) $(3x^2 - 2x - 1)^{\frac{-3}{2}}$

2. If $y = \sqrt[3]{(3x^2 + 8x - 6)^5}$ then $\frac{dy}{dx}$ =?

a) $\frac{5}{3}$ $(6x + 8)(3x^2 + 8x - 6)^{\frac{2}{3}}$ b) $\frac{-5}{3}$ $(6x + 8)(3x^2 + 8x - 6)^{\frac{2}{3}}$

c) $\frac{3}{5}$ $(3x + 4)(3x^2 + 8x - 6)^{\frac{2}{3}}$ d) $\frac{-3}{5}$ $(3x + 4)(3x^2 + 8x - 6)^{\frac{2}{3}}$

3. What is the rate of change of demand (x) of a commodity with respect to it's

price (y) if $y = 10+x+25x^3$

a) $\frac{10}{1+75x^2}$ b) $\frac{1}{1+75x^2}$

c) $1+75x^2$ d) $\frac{-1}{1+75x^2}$

4. What is the rate of change of demand (x) of a commodity with respect to it's

price (y) if $y = \frac{3x+7}{2x^2+5}$

a) $\frac{(2x^2+5)^2}{(-6x^2-38x+15)}$ b) $\frac{(2x^2+5)^2}{(-6x^2-28x+15)}$

c) $\frac{(2x^2+5)^2}{(6x^2-28x+15)}$ d) $\frac{(2x^2+5)^2}{(6x^2-38x+15)}$

5. If $y = x^{\sqrt{x}}$ then $\frac{dy}{dx}$ =?

a) $\frac{y}{2\sqrt{x}}$ (logx+2) b) $\frac{y}{\sqrt{x}}$ (logx+2)

c) $\frac{y}{2\sqrt{x}}$ (logx-2)   d) $\frac{y}{\sqrt{x}}$ (logx-2)

6. If y = $(x)^x + (10)^x$ then $\frac{dy}{dx}$ =?

a) $x^x(1 - logx)$ + $10^x log10$ b) $x^x(1 + logx)$ - $10^x log10$

c) $x(1 + logx)$ + $10^x log10$ d) $x^x(1 + logx)$ + $10^x log10$

7. If $x^m . y^n = (x+y)^{(m+n)}$ then $\frac{dy}{dx}$ = ?

a) $\frac{y}{x}$ b) $\frac{-y}{x}$

c) $\frac{x}{y}$ d) $\frac{-x}{y}$

8. If $x^y = 2^{x-y}$ then $\frac{dy}{dx}$ = ?

a) $\frac{xlog2-y}{xlog2x}$ b) $\frac{xlog2+y}{xlog2x}$

c) $\frac{xlog2+x}{ylog2x}$ d) $\frac{ylog2-x}{xlog2x}$

9. If x=2am , y =2am^2 where m be the parameter then $\frac{dy}{dx}$ = ?

a) 2m   b) -2m

c) –am   d) am

10. If x=a$\left(t - \frac{1}{t}\right)$, y = a$\left(t + \frac{1}{t}\right)$, where t be the parameter then $\frac{dy}{dx}$ = ?

a) $\frac{x}{y}$ b) $\frac{-x}{y}$

c) $\frac{y}{x}$ d) $\frac{-y}{x}$

11. If x= at^2 , y =2at then $\frac{d^2y}{dx^2}$ =?

a) $\frac{1}{at^3}$ b) $\frac{-1}{at^3}$

c) $\frac{-1}{at^2}$ d) $\frac{1}{at^2}$

II. Fill in the blanks :

1. If y = $(5x^3 - 4x^2 - 8x)^9$ then $\frac{dy}{dx}$ is…………

2. If y = $a^{(1+\log x)}$ then $\frac{dy}{dx}$ is…………

3. The rate of change of demand (x) of a commodity with respect to it's price (y)

   is……..…… if y = $5+x^2e^{-x}+2x$

4. The rate of change of demand (x) of a commodity with respect to it's price (y)

   is……….….. if y = $xe^{-x}+7$

5. If y = x^{10} then $\frac{dy}{dx}$ is…………

6. If y = $(e)^{(2x+5)}$ then $\frac{dy}{dx}$ is…………

7.If $\sqrt{x} + \sqrt{y} = \sqrt{a}$ then $\frac{dy}{dx}$ is…………

8. If differentiating 5^x with respect to logx then $\frac{dy}{dx}$ is…………

9. If differentiate e^x with respect to $\log_e x$ then $\frac{dy}{dx}$ is…………

10. If y = x^2 then $\frac{d^2y}{dx^2}$ is…………

III. State whether each of the following is True or False :

1. If y = log(logx) then $\frac{dy}{dx}$ = logx

2. If y = 10^x+1 then $\frac{dy}{dx}$ = 10^x.log10

3. If y = x^2 then the rate of change of demand (x) of a commodity with respect to it's

price (y) is $\frac{1}{2x}$

4. If y = 7x+1 then the rate of change of demand (x) of a commodity with respect to

it's price (y) is 7

5. If y = e^x then $\frac{dy}{dx} = e^x$

6. If y = 4^x then $\frac{dy}{dx} = 4^x$

7. If $\sqrt{x} + \sqrt{y} = \sqrt{a}$ then $\frac{dy}{dx} = \frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} = \frac{1}{2\sqrt{a}}$

8. If $x^2+y^2 = a^2$ then $\frac{dy}{dx}$ = 2x+2y =2a

9. If x= 2at , y= 2a , where t is parameter, then $\frac{dy}{dx} = \frac{1}{t}$

10. If x= 5m, y= m , where m is parameter, then $\frac{dy}{dx} = \frac{1}{5}$

11. If y= e^x then $\frac{d^2y}{dx^2}$ = e^x

IV. Solve the following : (3 Marks)

1. Find $\frac{dy}{dx}$, if y = $[log(log(logx))]^2$

2. Find $\frac{dy}{dx}$, if y = $(6x^3-3x^2-9x)^{10}$

3. Find the rate of change of demand (x) of a commodity with respect to it's price (y)

if $y = \frac{5x+7}{2x-13}$

4. Find the rate of change of demand (x) of a commodity with respect to it's price (y)

if $y = 5+x^2e^{-x}+2x$

5. Find $\frac{dy}{dx}$, if $x^y = y^x$

6. Find $\frac{dy}{dx}$, if $xy = \log(xy)$

7. Find $\frac{dy}{dx}$, if $x = \sqrt{1+u^2}$, $y = \log(1+u^2)$

8. If $x = t.\log t$, $y = t^t$ then, show that $\frac{dy}{dx} = t^t$

9. Find $\frac{d^2y}{dx^2}$ if , $y = e^{(2x+1)}$

V. Solve the following : (4 Marks)

1. Find $\frac{dy}{dx}$, if $y = (\log x)^x + (x)^{\log x}$

2. Find $\frac{dy}{dx}$, if $y = \sqrt[5]{(3x^2+8x+5)^4}$

3. Find $\frac{dy}{dx}$, if $y = x^x + (7x-1)^x$

4. Find rate of change of demand (x) of a commodity with respect to it's price (y) if

$y = \frac{3x+7}{2x^2+5}$

5. Find $\frac{dy}{dx}$, if $y = x^{x^x}$

6. Find $\frac{dy}{dx}$, if $y = \sqrt[3]{\frac{(3x-1)}{(2x+3)(5-x)^2}}$

7. If $x^5.y^7 = (x+y)^{12}$ then show that, $\frac{dy}{dx} = \frac{y}{x}$

8. If $x^a.y^b = (x+y)^{(a+b)}$ then Show that $\frac{dy}{dx} = \frac{y}{x}$

9. If $x = \frac{4t}{1+t^2}$, $y = 3\left(\frac{1-t^2}{1+t^2}\right)$ then, show that $\frac{dy}{dx} = \frac{-9x}{4y}$

10. If $x^2+6xy+y^2 = 10$ then show that $\frac{d^2y}{dx^2} = \frac{80}{(3x+y)^3}$

VI. Activity :

1. $y = (6x^4 - 5x^3 + 2x + 3)^6$ find $\frac{dy}{dx}$

Solution :- Given

$$y = (6x^4 - 5x^3 + 2x + 3)^6$$

Let $u = [6x^4 - 5x^3 + \square + 3]$

$\therefore y = u^{\square}$

$\therefore \frac{dy}{du} = 6u^{6-1}$

$\therefore \frac{dy}{du} = 6(\quad)^5$

And $\frac{du}{dx} = 24x^3 - 15(\quad)^{\square} + 2$

By chain rule

$\frac{dy}{dx} = \frac{dy}{\dots} \times \frac{\square}{dx}$

$\therefore \frac{dy}{dx} = 6(6x^4 - 5x^3 + 2x + 3)^{\square} \times (24x^3 - 15x^2 + \quad)$

2. The rate of change of demand (x) of a commodity with respect to its price (y), if $y = 20 + 15x + x^3$

Solution :- Let $y = 20 + 15x + x^3$

Diff. w.r.to x , we get

$\therefore \frac{dy}{dx} = \square \quad \square + \square +$

$\therefore \frac{dy}{dx} = 15 + 3x^2$

$\therefore$ By derivation of the inverse function

$\frac{dx}{dy} = \square \frac{1}{\ } , \frac{dy}{dx} \neq 0$

$\therefore$ Rate of change of demand with respe□t □o price $= \frac{1}{+}$

3. Find $\frac{dy}{dx}, if\ y = x^{(x)} + 20^{(x)}$

Solution :- let $y = x^{(x)} + 20^{(x)}$

Let $u = \square x \quad and \square v = \quad x$

$\therefore y = u + v$

Diff. w.r.to x , we get

$\therefore \frac{dy}{dx} = \frac{\square}{dx} \square + \frac{dv}{\ }$ -----------(I)

Now , $u = x^x$

Taking log on both sides , we get

$\therefore \log u = \quad x \times \log x$

Diff. w.r.to x

$\therefore \frac{1}{u}\frac{du}{dx} = x \times \frac{1}{\ } + log \square x \times$

$\therefore \frac{du}{dx} = u[1 + \log x]$

$\therefore \frac{du}{dx} = x^x[1 \square + \quad]$---------(II)

Now , v=20^{x}

Diff.w.r.to x, we get

$\therefore\frac{dv}{dt} = 20^{\square}$.   --------(III)

Substituting equation (II) & (III) in equation (I) , we get

$\therefore\frac{dy}{dx} = x^{x}[1 + \log x] + 20^{x}.\log(20)$

4. Find $\frac{dy}{dx}, if x = e^{m}, y = e^{\sqrt{m}}$

Solution :- given, $x = e^{m}$ and $y = e^{\sqrt{m}}$

Now, $y = e^{\sqrt{m}}$

Diff.w.r.to m

$\therefore\frac{dy}{dm} = e^{\sqrt{m}}\square\frac{d}{dm}$

$\therefore\frac{dy}{dm} = e^{\sqrt{m}}.\frac{1}{2\sqrt{m}}$ -----------(I)

Now , x=e^{m}

Diff.w.r.to m

$\therefore\frac{dx}{dm} = \square$ --------------(II)

Now, $\frac{dy}{dx}\ \square\ \frac{dy/dm}{}$

$\therefore\qquad = \frac{\frac{e^{\sqrt{m}}}{\square}}{e^{m}}$

$\therefore\qquad = \frac{e^{\sqrt{m}}}{2\sqrt{m}.e^{m}}$

4. APPLICATIONS OF DERIVATIVE

I) Choose the correct alternative.

1. The slope of the tangent to the curve $y = x^3 - x^2 - 1$ at the point whose abscissa is -2, is

(a) - 8 (b) 8 (c) 16 (d) - 16

2. Slope of the normal to the curve $2x^2 + 3y^2 = 5$ at the point (1,1) on it is

(a) $-\frac{2}{3}$ (b) $\frac{2}{3}$ (c) $\frac{3}{2}$ (d) $-\frac{3}{2}$

3. The function $f(x) = x^3 - 3x^2 + 3x - 100, x \in R$ is

(a) increasing for all $x \in R$, $x \neq 1$ (b) decreasing

(c) Neither increasing nor decreasing (d) Decreasing for all $x \in R$, $x \neq 1$

4. If the marginal revenue is 28 and elasticity of demand is 3 then the price is

(a) 24 (b) 32 (c) 36 (d) 42

5. The price P for the demand D is given as $\mathrm{P} = 183 + 120\mathrm{D} - 3\mathrm{D}^2$; then the value of D for

which price is increasing, is

(a) $D < 60$ (b) $D > 60$ (c) $D < 20$ (d) $D > 20$

6. If the elasticity of the demand $\eta = 1$ then demand is

(a) constant (b) in elastic (c) unitary elastic (d) elastic

7. If $0 < \eta < 1$, then the demand is

(a) constant (b) in elastic (c) unitary elastic (d) elastic

II) Fill in the blanks.

1. The slope of tangent at any point (a,b) is also called as ....
2. If the function $f(x) = \frac{7}{x} - 3$, $x \in R$, $x \neq 0$ is decreasing function then $x \in$ ....
3. The slope of the tangent to the curve $x = \frac{1}{t}, y = t - \frac{1}{t}$, at $t = 2$ is............
4. If the average revenue is 45 and elasticity of demand is 5 then marginal revenues is.......
5. The total cost function for production of articles is given as $C = 100 + 600x - 3x^2$

then the values of x for which the total cost is decreasing is

…………

III) State whether each of the following is True or False.

1. An absolute maximum or minimum must occur at a critical point or at an end point

2. The function $(x) = \frac{3}{x} + 10$, $x \neq 0$ is decreasing

3. The function $(x) = x - \frac{1}{x}$, $x \in$ R , $x \neq 0$ is increasing

4. The equation of tangent to the curve $y = x^2 + 4x + 1$ at (-1,-2) is 2x – y = 0.

5. If the function $f(x) = x^2 + 2x - 5$ is increasing function then $x < -1$.

6. If the marginal revenue is 50 and the price is Rs 75 then elasticity of demand is 4.

Solve the following

3 Marks

1. Find the equation of tangent and normal to the curve $y = 3x^2 - x + 1$ at the point (1,3) on it.

2. Find the values of x , such that $f(x) = 2x^3 - 15x^2 + 36x + 1$ is increasing function.

3. Find the values of x such that $f(x) = 2x^3 - 15x^2 - 144x - 7$ is decreasing function.

4. Show that the function $(x) = \frac{x-2}{x+1}$, $x \neq -1$ is increasing.

5. Divide the number 20 into two parts such that their product is maximum.

6. If the demand function is $\mathrm{D} = 50 - 3p - p^2$. Find the elasticity of demand at

(i) $p = 5$ (ii) $p = 2$. Comment on the result.

7. If the demand function is $\mathrm{D} = \left(\frac{p+6}{p-3}\right)$. Find the elasticity of demand at $p = 4$.

8. The total cost of manufacturing x articles $\mathrm{C} = 47x + 300x^2 - x^4$. Find x for which average

cost is (i) increasing (ii) decreasing .

4 Marks

1. Determine the maximum and minimum values of the function $f(x) = 2x^3 - 21x^2 + 36x - 20$

2. A rod of 108m long bent to form a rectangle. Find it's dimensions when it's area is maximum.

3. Find MPC, MPS, APC and APS , if the expenditure $\mathrm{E_c}$ of a person with income I is given as

$$\mathrm{E_c} = (0.0003)\mathrm{I}^2 + (0.075)\mathrm{I} \text{ when } \mathrm{I} = 1000$$

4. The manufacturing company produces x items at the total cost of Rs 180 + 4x . The demand

function for this product is $\mathrm{P} = (240 - x)$. Find x for which

(i) revenue is increasing (ii) profit is increasing.

5. If $x + y = 3$ then show that the maximum value of x^2y is 4

6. Find the equation of tangent to the curve $x^2 + y^2 = 5$ where the tangent is parallel to the line

2x-y+1=0 .

7. Find the equation of normal to the curve $y = \sqrt{x - 3}$ which is perpendicular to the line

6x+3y-4=0

8. Find the equation of tangent of the curve $y = x^2 + 4x$ at the point whose ordinate is -3

Activity (4 Marks)

1. A metal wire of 36 cm long is bent to form a rectangle. By completing the following activity, find it's dimensions when it's area is maximum.

Solution: Let the dimension of the rectangle be *x* cm and *y* cm.

$\therefore 2x + 2y = 36$

Let $f(x)$ be the area of rectangle in terms of x, then

$f(x) = \boxed{}$

$\therefore\ f'(x) = \boxed{}$

$\therefore\ f''(x) = \boxed{}$

For extreme values of , $f'(x) = 0$,we get

$x = \boxed{}$

$\therefore\ f''(\boxed{}) = -2 < 0$

$\therefore$ Area is maximum when $x = \boxed{}$, $y = \boxed{}$

$\therefore$ Dimension of rectangle are $\boxed{}$

2. By completing the following activity, examine the function $f(x) = x^3 - 9x^2 + 24x$

for maxima and minima.

Solution:

$f(x) = x^3 - 9x^2 + 24x$

$\therefore\ f'(x) = \boxed{\qquad\qquad}$

$\therefore\ f''(x) = \boxed{\qquad\qquad}$

For extreme values of , $f'(x) = 0$,we get

$x = \boxed{\qquad}$ or $\boxed{\qquad}$

$\therefore\ f''(\boxed{\quad}) = -6 < 0$

$\therefore f(x)$ is maximum at $x = 2$

$\therefore$ Maximum value = $\boxed{\qquad}$

$\therefore\ f''(\boxed{\quad}) = 6 > 0$

$\therefore f(x)$ is maximum at $x = 4$

$\therefore$ Minimum value = $\boxed{\qquad}$

3. By completing the following activity, find the values of x such that $f(x) = 2x^3 - 15x^2 - 84x - 7$ is

decreasing function.

Solution:

$f(x) = 2x^3 - 15x^2 - 84x - 7$

$\therefore\ f'(x) = \boxed{\qquad\qquad}$

$\therefore\ f'(x) = 6\,(\boxed{\quad})(\boxed{\quad})$

Since $f(x)$ is decreasing function

$\therefore\ f'(x) < 0$

Case1: ( ) > 0 and $(x+2) < 0$

$\therefore x \in$

Case2: ( ) < 0 and $(x+2) > 0$

$\therefore x \in$

$\therefore f(x)$ is decreasing function if and only if $x \in$

4. A manufacturing company produces x items at a total cost of Rs $40 + 2x$. Their price per item is given as $p = 120 - x$ Find the value of x for which (i) revenue is increasing

(ii) profit is increasing (iii) Also find elasticity of demand for price Rs. 80.

Solution: Total cost $C = 40 + 2x$ and Price $p = 120 - x$

(i). Revenue $R = p.x$

Differentiating w.r.t x,

$\therefore \frac{dR}{dx} =$

Since Revenue is increasing $\therefore \frac{dR}{dx} > 0$

$\therefore$ Revenue is increasing for

(ii) Profit $\pi = R - C$

$\therefore \pi =$

Differentiating w.r.t x

$\therefore \frac{d\pi}{dx} =$

Since Profit is increasing $\therefore \frac{d\pi}{dx} > 0$

$\therefore$ Profit is increasing for

(iii) . $p = 120 - x$ $\therefore x = 120 - p$

Differentiating w.r.t p,

$\therefore \frac{dx}{dp} =$

$\therefore$ Elasticity of demand is given by $\eta = -\frac{p}{x} \cdot \frac{dx}{dp}$

$\therefore \eta =$

When $p = 80$, then elasticity of demand $\eta =$

5. INTEGRATION

1. Choose the correct alternative from the following.

1) The value of $\int \frac{dx}{\sqrt{1-x}}$

a) $2\sqrt{1-x}$ + c

b) -2 $\sqrt{1-x}$ + c

c) $\sqrt{x}$ + c

d) x + c

2) $\int \sqrt{1+x}$ dx =

a) $\frac{x}{2}$ $\sqrt{1+x}$ + c

b) $\frac{2}{3}(1+x)^{3/2}$ + c

c) $\frac{2}{\sqrt{1+x}}$ + c

d) $\frac{-3}{2}$ (1 + x) + c

3) $\int x^2\, 3^{x^3}$dx =

a) $(3)^{x^3}$ + c

b) $\frac{(3)^{x^3}}{3\ log3}$ + c

c) log3. $(3)^{x^3}$ + c

d) $x^2(3)^{x^2}$ + c

4) $\int \frac{x+2}{2x^2+6x+5}$ dx = p $\int \frac{4x+6}{2x^2+6x+5}$dx + $\frac{1}{2}\int \frac{1}{2x^2+6x+5}$dx then p = ?

a) $\frac{1}{3}$ b) $\frac{1}{2}$ c) $\frac{1}{4}$ d) 2

5) $\int \frac{dx}{x-x^2}$ =

a) logx – log(1 – x) + c

b) log(1-x^2) +c

c) -logx + log(1-x) + c

d) $\log(x - x^2) + c$

6) $\int \frac{dx}{(x-8)(x+7)} =$

a) $\frac{1}{15}\log(\frac{x+2}{x-1}) + c$

b) $\frac{1}{15}\log(\frac{x+8}{x+7}) + c$

c) $\frac{1}{15}\log(\frac{x-8}{x+7}) + c$

d) (x-8).(x-7) + c

7) $\int (x + \frac{1}{x})^3 dx =$

a) $\frac{1}{4}(x + \frac{1}{x})^4 + c$

b) $\frac{x^4}{4} + \frac{3x^2}{2} + 3\log x - \frac{1}{2x^2} + c$

c) $\frac{x^4}{4} + \frac{3x^2}{2} + 3\log x + \frac{1}{x^2} + c$

d) $(x - x^{-1})^3 + c$

8) $\int (\frac{e^{2x}+e^{-2x}}{e^x}) dx =$

a) $e^x - \frac{1}{3e^{3x}} + c$

b) $e^x + \frac{1}{3e^{3x}} + c$

c) $e^{-x} - \frac{1}{3e^{3x}} + c$

d) $e^{-x}+\frac{1}{3e^{3x}}$ + c

9) $\int(\mathbf{1}-\boldsymbol{x})^{-\mathbf{2}}$**dx=**

a) $(1-x)^{-1}$+ c

b) $(1+x)^{-1}$+ c

c) $(1-x)^{-1}$-1 + c

d) $(1-x)^{-1}$+1 + c

10)$\int\frac{(\boldsymbol{x^3+3x^2+3x+1})}{(\boldsymbol{x+1})^{\mathbf{5}}}$**dx =**

a) $\frac{-1}{x+1}$ + c

b) $(\frac{-1}{x+1})^5$ + c

c) Log(x+1) + c

d) 5Log(x+5)+ c

2. Fill in the blanks. (1 mark each)

1) $\int\frac{1}{x}$ dx=……………+ c

2) $\int\frac{1}{x^2-a^2}$ dx =…………… + c

3) $\int(7x+9)^{13}$dx=………………+ c

4) $\int e^{4x-3}$dx = ……………….+ c

5) $\int 5^{6x+9}$dx = ………………..+ c

6) $\int\frac{5(x^6+1)}{x^2+1}$dx $= x^5-\cdots x^3+5x+c$

7) $\int\frac{x^2+x-6}{(x-2)(x-1)}$dx = x + ………+ c

8) If $f'(x)=\frac{1}{x}$ + x and f(1) = $\frac{5}{2}$ then

f(x) = logx + $\frac{x^2}{2}$ + …………+ c

9) To find the value of $\int \frac{(1+logx)}{x}$dx the proper substitution is

…………………………………

10) $\int \frac{1}{x^3}\,[logx^x]^2\,dx$ = p $(logx)^3$+ c

Then p = …………………………

3. State whether each of the following is TRUE or FALSE. (1 mark each)

1. The proper substitution for $\int x(x^x)^x(2\log x+1)dx$ is $(x^x)^x$ = t

2. If $\int x\,e^{2x}\,dx$ is equal to $e^{2x}f(x)+c$ where C is constant of integration then $f(x)$ is $\frac{(2x-1)}{2}$

3. If $\int x\,f(x)\,dx = \frac{f(x)}{2}$ then $f(x) = e^{x^2}$

4. If $\int \frac{(x-1)\,dx}{(x+1)(x-2)} = A\;log|x+1|+B\;log|x-2|$ then A + B = 1

5. For $\int \frac{x-1}{(x+1)^3}\,e^x dx = e^x\,f(x)+c$, $f(x)$ = $(x+1)^2$.

6. If f′(x) = 3x^2+2x then by definition of Integration , we get f(x) = x^3+x^2+ c

7. If f(x) = k , where k is constant then $\int k\;dx$= 0

8. $\int 3^{2x+3} dx = \frac{3^{2x+3}}{2} + c$

9. $\int e^{4x-7} dx = \frac{e^{4x-7}}{-7} + c$

10. $\int \sqrt{1+x^2}\,.\,x\, dx = \frac{1}{3}(1+x^2)^{\frac{3}{2}} + c$

4. **Solve the following. (3 marks each)**

1) Evaluate $\int (3x^2-5)^2 dx$

2) Evaluate $\int \frac{1}{x(x-1)} dx$

3) Evaluate $\int \frac{1}{xlogx} dx$

4) Evaluate $\int \frac{2e^x+5}{2e^x+1} dx$

5) Evaluate $\int \frac{1}{4x^2-1} dx$

6) Evaluate $\int e^x (\frac{1}{x} - \frac{1}{x^2}) dx$

7) Evaluate $\int \frac{2x+1}{(x+1)(x-2)} dx$

8) Evaluate $\int xlogx\, dx$

9) Evaluate $\int x^2 e^{4x} dx$

10) Evaluate $\int \frac{1}{\sqrt{3x^2+8}} dx$

5. Evaluate the following . (4 marks each)

1) $\int e^x \frac{x}{(x+1)^2} dx$

2) $\int x^3 e^{x^2} dx$

3) $\int \frac{x}{(x-1)^2(x+2)} dx$

4) $\int \frac{logx}{(1+logx)^2} dx$

5) $\int \frac{1}{\sqrt{x^2-8x-20}}dx$

6) $\int \frac{1}{4x^2-20x+17}dx$

7) $\int \frac{1+x}{x+e^{-x}}dx$

8) $\int \frac{x^3}{\sqrt{1+x^4}}dx$

9) $\int \frac{3e^{2t}+5}{4e^{2t}-5}dt$

10) If f′(x) = $4x^3 - 3x^2 + 2x + k$,

f(0) = 1 and f(1) = 4 find f(x)

4. DEFINITE INTEGRATION

I) Choose the correct alternative.

1. $\int_2^3 x^4 dx =$

(a) $\frac{1}{2}$ (b) $\frac{5}{2}$ (c) $\frac{5}{211}$ (d) $\frac{211}{5}$

2. $\int_0^a 3x^2 dx = 8$ then $a =$

(a) 2 (b) 0 (c) $\frac{8}{3}$ (d) a

3. $\int_4^9 \frac{dx}{\sqrt{x}} =$

(a) 9 (b) 4 (c) 2 (d) 0

4. $\int_0^2 e^x dx =$

(a) $e^2 - 1$ (b) $1 - e^2$ (c) $e - 1$ (d) $1 - e$

5. $\int_{-2}^{3} \frac{1}{x+5} dx =$

(a) $log\left(\frac{3}{8}\right)$ (b) $log\left(\frac{8}{3}\right)$ (c) $log\left(\frac{8}{5}\right)$ (d) 0

6. $\int_{2}^{3} \frac{x}{x^{2}-1} dx =$

(a) $log\left(\frac{8}{3}\right)$ (b) $-log\left(\frac{8}{3}\right)$ (c) $\frac{1}{2}log\left(\frac{8}{3}\right)$ (d) $-\frac{1}{2}log\left(\frac{8}{3}\right)$

7. $\int_{a}^{b} f(x)dx =$

(a) $\int_{b}^{a} f(x)dx$ (b) $-\int_{a}^{b} f(x)dx$ (c) $-\int_{b}^{a} f(x)dx$ (d) $\int_{0}^{a} f(x)dx$

8. $\int_{2}^{7} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}} dx =$

(a) $\frac{7}{2}$ (b) $\frac{5}{2}$ (c) 7 (d) 2

9. $\int_{-9}^{9} \frac{x^3}{4-x^2} dx =$

(a) 0 (b) 3 (c) 9 (d) - 9

II) Fill in the blanks.

6. $\int_{1}^{2} x^2 dx =$.............

7. $\int_{0}^{a} 4x^3 dx = 81$ then $a =$………

8. $\int_0^1 e^{2x} dx =$ ....

9. $\int_1^2 \frac{1}{2x+3} dx =$...........

10. $\int_2^4 \frac{x}{x^2+1} dx =$.......

11. $\int_{-7}^{7} \frac{x^3}{x^2+7} dx =$............

III) State whether each of the following is True or False.

1. $\int_0^a 3x^2 dx = 27$ then $a = 2.5$

2. $\int_0^1 \frac{1}{2x+5} dx = \log\left(\frac{7}{5}\right)$

3. $\int_2^3 \frac{x}{x^2+1} dx = \frac{1}{2}\log 2$

4. $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

5. $\int_a^{2a} f(x)dx = \int_0^a f(x)dx + \int_0^a f(a-x)dx$

6. $\int_{-5}^{5} \frac{x}{x^2+7} dx = 10$

Solve the following

3 Marks

1. Evaluate $\int_0^1 \frac{x^2+3x+2}{\sqrt{x}} dx$

2. Evaluate $\int_0^1 \frac{1}{\sqrt{1+x}+\sqrt{x}} dx$

3. If $\int_0^a (2x+1)dx = 2$, find a.

4. If $\int_1^a (3x^2+2x+1)dx = 11$, find the real value of a.

5. Evaluate $\int_1^e \frac{dx}{x(1+logx)^2} dx$

6. Evaluate $\int_1^2 \frac{1}{x^2+6x+5} dx$

7. Evaluate $\int_1^2 \frac{3x}{(9x^2-1)} dx$

8. Evaluate $\int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5}+\sqrt[3]{9-x}} dx$

9. Evaluate $\int_1^2 \frac{\sqrt{x}}{\sqrt{3-x}+\sqrt{x}} dx$

10. Evaluate $\int_0^1 \log\left(\frac{1}{x}-1\right) dx$

4 Marks

1. Evaluate $\int_2^3 \frac{x}{(x+2)(x+3)} dx$

2. Evaluate $\int_1^2 e^{2x}\left(\frac{1}{x}-\frac{1}{2x^2}\right) dx$

3. Evaluate $\int_1^3 logx\ dx$

4. Evaluate $\int_1^3 x^2 . logx\ dx$

5. Evaluate $\int_0^1 e^{x^2} . x^3\ dx$

6. Evaluate $\int_0^a x^2(a-x)^{3/2}\,dx$

7. Evaluate $\int_0^1 x(1-x)^5 dx$

Activity (4 Marks)

1. Completing the following activity, Evaluate $\int_1^2 \frac{x+3}{x(x+2)} dx$

Solution: Let I $= \int_1^2 \frac{x+3}{x(x+2)} dx$

Let $\frac{x+3}{x(x+2)} = \frac{\mathrm{A}}{x} + \frac{\mathrm{B}}{(x+2)}$

$\therefore\ x + 3 = \mathrm{A}(x+2) + \mathrm{B}.x$

$\therefore$ A = $\boxed{}$, B = $\boxed{}$

$$\therefore \mathrm{I} = \int_1^2 \frac{(\quad)}{x} + \frac{(\quad)}{(x+2)} dx$$

$\therefore$ I = [$\boxed{}$ log x + $\boxed{}$ log (x + 2) $]_1^2$

$\therefore$ I = $\boxed{}$

2. By completing the following activity, Evaluate $\int_2^5 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}} dx$

Solution: Let I = $\int_2^5 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}} dx$ …………..(i)

Using the property , $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$, we get

$$I = \int_2^5 \frac{(\ldots\ldots\ldots)}{\sqrt{7-x} \; + \; (\ldots\ldots\ldots)} \, dx \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

Adding equation (i) and (ii) , we get

$$2I = \int_2^5 \frac{\sqrt{x}}{\sqrt{x}+\sqrt{7-x}} dx + (\ldots\ldots.) \, dx$$

$$2I = \int_2^5 \left(\frac{(\ldots\ldots\ldots\ldots) + (\ldots\ldots\ldots..)}{(\ldots\ldots\ldots\ldots) + (\ldots\ldots\ldots..)} \right) dx$$

$$2I = \boxed{\qquad}$$

$\therefore I =$

7. APPLICATIONS OF DEFINITE INTEGRATION

Q.1 A) Select and write the most appropriate answer from the given alternatives for each sub-question.

1) Area of the region bounded by the curve $y = x^3, x = 1 , x = 4$ and the X- axis is ______

a) $\frac{255}{4}$ sq. units. b) $\frac{256}{4}$ sq. units c) $\frac{255}{3}$ sq. units d) $\frac{256}{3}$ sq. units

2) Using the definite integration area of the circle is______

a) 4π sq. units b) 4 sq. units c) 16 sq. units d) 16π sq. units

3) Area of the region bounded by the curve $x = y^2$, the positive Y axis and the lines $y = 1\ and\ y = 3$ is________

a) 26 sq. units b) 3 sq. units c) $\frac{3}{26}$ sq. units d) $\frac{26}{3}$ sq. units

4) Area of the region bounded by the curve $x^2 = 8y$, the positive Y axis and the lines $y = 4\ and\ y = 9$ is ________

a) $\frac{76\sqrt{2}}{3}$ sq. units b) $\frac{76\sqrt{2}}{2}$ sq. units c) $\frac{38\sqrt{2}}{3}$ sq. units d) $76\sqrt{2}$sq. units

5) Area of the region bounded by $y^2 = 16x,\ x = 1\ and\ x = 4$ and the X axis, lying in the first quadrant is _____

a) $\frac{56}{3}$ sq. units b) $\frac{3}{56}$ sq. units c) 56 sq. units d) 63 sq. units

6) Area of the region bounded by $= y^4$, $y = 1\ and\ y = 5$ and the Y axis lying in the first quadrant is _____

a) $\frac{3124}{5}$ sq. units b) $\frac{3142}{5}$ sq. units c) $\frac{3124}{3}$ sq. units d) $\frac{3142}{3}$ sq. units

7) Area of the region bounded by the parabola $y^2 = 25x$ and the lines $x = 5$ is _____

a) $\frac{75\sqrt{5}}{2}$ sq. units b) $\frac{20\sqrt{5}}{3}$ sq. units c) $\frac{100\sqrt{5}}{3}$ sq. units d) $\frac{75\sqrt{5}}{3}$ sq. units

B) State whether the following statements are true or false.

1) $y^2 = 4ax$ is the standard form of parabola when curve lies on X axis.

2) Standard form of parabola is $x^2 = -4by$, when curve lies in the positive Y axis.

3) The area of portion lying below the X axis is negative.

4) The area bounded by the curve $y = f(x)$ lies on the both sides of the X- axis is $\left|\int_a^b f(x)dx\right| + \left|\int_b^c f(x)dx\right|$.

5) The equation of the area of the circle is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

6) The equation of the area of the ellipse is $x^2 + y^2 = a^2$.

7) The area of the shaded region bounded by two curves $y = f(x)$, and $y = g(x)$ and X axis is $\left|\int_a^b f(x)dx + \int_a^b g(x)dx\right|$.

C) Fill in the following blanks.

1) The area bounded by the parabola $x^2 = 9y$ and the lines $y = 4$

and $y = 9$ in the first quadrant is______

2) The area of the circle $x^2 + y^2 = 16$ is _____

3) The area of the region bounded by the curve $y^2 = 4x$, the X axis and the lines $x = 1$ and $x = 4$ is _____

4) The area of the region lying in the first quadrant and bounded by the curve $y = 4x^2$, and the lines $y = 2$ and $y = 4$ is_____

5) The area of the region bounded by the curve $y^2 = x$ and the Y axis in the first quadrant and lines$y = 3$ and $y = 9$ is______

6) The area of the region $x^2 = 4y$, $y = 1$ and $y = 2$ and the Y axis lying in the first quadrant is______

7) The area of the region bounded by $y^2 = 25x, y = 1$ and $y = 2$ and the X axis is____

Q.2 Attempt the following questions. (3 marks each)

1) Find the area of the region bounded by the parabola $y^2 = 25x$ and the line $x = 5$

2) Find the area of the region bounded by the curve $= \sqrt{9 - x^2}$, X- axis and lines $x = 0$ and $x = 3$.

3) Find the area of the region bounded by the curve $y = \sqrt{2x+3}$, the X axis and the lines $x = 0\ and\ x = 2$.

4) Find the area of the region bounded by the curve$4y = 7x + 9$, the X-axis and the lines x=2 and x=8.

5) Find the area of the region bounded by the curve $y = (x^2 + 2)^2$, the X-axis and the lines $x = 1\ and\ x = 3$.

6) Find area of the region bounded by $2x + 4y = 10, y = 2\ and\ y = 4$ and the Y- axis lying in the first quadrant.

7) Find area of the region bounded by the curve $y = -4x$, the Y-axis and the lines $x = -1\ and\ x = 2$.

8) Find area of the region bounded by the parabola

$x^2 = 36y, y = 1\ and\ y = 4$, and the positive Y-axis.

9) Find area of the region bounded by the parabola $x^2 = 4y$, the Y- axis lying in the first quadrant and the lines $y = 3$.

10) Find the area of the region bounded by the curve $x = \sqrt{25 - y^2}$, the Y- axis lying in the first quadrant and the lines $y = 0\ and\ y = 5$.

11) Find the area of the region bounded by the curve $y = \sqrt{36 - x^2}$, the X- axis lying in the first quadrant and the lines $x = 0\ and\ x = 6$.

Q.3 Attempt the following questions. (4 marks each)

1) Find area of the ellipse $\frac{x^2}{5^2} + \frac{y^2}{4^2} = 1$.

2) Find the area of the circle $x^2 + y^2 = 6^2$

3) Find the area between the parabolas $y^2 = 5x\ and\ x^2 = 5y$.

4) Find the area of the circle $x^2 + y^2 = 16$

5) Find area of the ellipse $4x^2 + 9y^2 = 36$.

8. DIFFERENTIAL EQUATIONS AND APPLICATIONS

I. Select & write the correct alternative from the given option for each question (1 Marks)

1. Solution of the equation $x\frac{dy}{dx} = y\,logy$ is

   a) $y = ae^{x}$ b) $y = be^{2x}$ c) y $= be^{-2x}$ d) $y = e^{ax}$

2. Bacterial increases at the rate proportional to the number present. If original number M doubles in 3 hours, then number of bacteria will be 4M in

   a) 4 hours b) 6 hours c) 8 hours d) 10 hours

3. The integrating factor of $\frac{dy}{dx} + y = e^{-x}$ is

$a)$ x b) $-x$ c) e^{x} d) $y = e^{-x}$

4. The integrating factor of $\frac{d^2y}{dx^2} - y = e^{x}\ is\ e^{-x}$ then its solution is

a) $ye^{-x} = x + c$ b) $ye^{x} = x + c$

c) $ye^{x} = 2x + c$ d) $ye^{-x} = 2x + c$

5. Differential equation of the function $c + 4yx = 0$ is

a) $xy + \frac{dy}{dx} = 0$ b) $x\frac{dy}{dx} + y = 0$

c) $\frac{dy}{dx} - 4xy = 0$ d) $x\frac{dy}{dx} + 1 = 0$

6. General solution of $y - x\frac{dy}{dx} = 0$

$a)$ $3logx + \frac{7}{y} = c$ $b)$ $2logx + \frac{3}{y} = c$

c) $logx - logy = logc$ d) $3logy + \frac{2}{x} = c$

7. The order and degree of $(\frac{dy}{dx})^3 - \frac{d^3y}{dx^3} + ye^x$ is

   a) 3 , 1 b) 1 , 3 c) 3 , 3 d) 1 , 1

8. The order and degree of $\left(1 + \left(\frac{dy}{dx}\right)^3\right)^{\frac{2}{3}} = 8\frac{d^3y}{dx^3}$ are respectively

   a) 3 , 1 b) 1 , 3 c) 3 , 3 d) 1 , 1

9. The solution of $\frac{dy}{dx}$=1 is

a) $x + y = c$ b) $xy = c$ c) $x^2 + y^2 = c$ d) $y - x = c$

10. The solution of $\frac{dy}{dx} + \frac{x^2}{y^2} = 0$ is

a) $x^3 + y^3 = 7$ b) $x^2 + y^2 = c$ c) $x^3 + y^3 = c$ d) $x + y = c$

II. Fill in the following blanks (1 marks)

1. Order of highest derivative occurring in the differential equation is called
   the …………… of the differential equation.
2. A solution of differential equation which can be obtained from the general
   solution by giving particular values to the arbitrary constant is called ……..Solution.
3. Order and degree of differential equation are always …………integers
4. The degree of a differential equation is the power of higher ordered derivative
   when all the derivatives are made free from negative and / or fractional indices
   if any is called ………….. of the differential equation.

5. The integrating factor of the differential equation $\frac{dy}{dx} - y = x$ is ………..

6. The solution of $\frac{dy}{dx} + y = 3$ is ………………………….

7. Integrating factor of $\frac{dy}{dx} + \frac{y}{x} = x^3 - 3$ is ………………………….

8. Order and degree of differential equation $\left(\frac{d^3y}{dx^3}\right)^{\frac{1}{6}} = 9$ is ………………….

9. The function $y = e^x$ is solution ……….. of differential equation

10. The solution of differential equation $x^2 \frac{d^2y}{dx^2} = 1$ is ………………………….

III. **State whether the following statements are true or false (1 marks)**

1. The integrating factor of the differential equation $\frac{dy}{dx} - y = x \text{ is } e^{-x}$
2. Order and degree of differential equation are always Positive integers
3. The degree of a differential equation is the power of higher ordered derivative when all the derivatives are made free form negative and / or fractional indices if any.
4. Order of highest derivative occurring in the differential equation is called the degree of the differential equation.
5. The degree of a differential equation $e^{-\frac{dy}{dx}} = \frac{dy}{dx} + c$ is not defined

6. A homogeneous differential equation is solved by substituting $y = vx$ and integrating it.
7. Order and degree of differential equation $x\frac{\mathrm{d}^3\mathrm{y}}{\mathrm{d}x^3} + 6\left(\frac{\mathrm{d}^2\mathrm{y}}{\mathrm{d}x^2}\right)^2 + \mathrm{y} = 0$ is (2,2)
8. Number of arbitrary constant in the general solution of a differential equation is equal to order of D.E.
9. A differential equation in which the dependent variable ,*say y*, depends only on one dependent variable , *say x*, is called as ordinary differential equation
10. The function $y = cx$ is the solution of differential equation $\frac{dy}{dx} = \frac{y}{x}$

IV. Attempt the following questions (3 marks)

1. Solve the differential equation $\frac{dy}{dx} + y = e^{-x}$
2. Solve the differential equation $x\frac{dy}{dx} + 2y = x^2 logx$
3. Solve $\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$ $when\ x = \frac{2}{3}, y = \frac{1}{3}$
4. Solve the differential equation $xdx + 2ydx = 0$
5. Solve the differential equation

$$(x^2 - yx^2)dy + (y^2 + xy^2)dx = 0$$

6. Solve the following differential equation

$$\frac{dy}{dx} = x^2y + y$$

7. Find the differential equation by eliminating arbitrary constants from the relation $x^2 + y^2 = 2ax$

8. Find the differential equation by eliminating arbitrary constants from the relation $y = (c_1 + c_2x)e^x$

9. Verify $y=logx+c$ is the solution of differential equation

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$$

10. Solve : $\frac{dy}{dx} + \frac{2}{x}y = x^2$

V. Attempt the following questions (4 marks)

1. For the differential equation, find the particular solution

$$(x - y^2x)dx - (y + x^2y)dy = 0 \;\; when\; x = 2\,, y = 0$$

2. Solve the differential equation $\frac{dy}{dx} + \frac{x-2y}{2x-y} = 0$

3. Find the differential equation from the relation $x^2 + 4y^2 = 4b^2$

4. If the population of a town increases at a rate proportional to the population at that time. If the population increases from 40 thousands to 60 thousands in 40 years, what will be the population in another 20 years?.

(Given $\sqrt{\frac{3}{2}} = 1.2247$)

5. The rate of growth of bacteria is proportional to the number present. If initially, there were 1000 bacteria and the number doubles in 1 hours, find the number of bacteria after 5/2 hours. (Given $\sqrt{2} = 1.414$)

6. Solve the following differential equation

$$yx\frac{dy}{dx} = x^2 + 2y^2$$

7. Solve the following differential equation

$$y\ logy\frac{dy}{dx} = logy - x$$

8. For the differential equation, find the particular solution

$$\frac{dy}{dx} = (4x + y + 1),\ \text{when}\ \ y = 1, x = 0$$

9. Solve the following differential equation

$$y^2\ dx + (xy + x^2)dy = 0$$

10. Solve the following differential equation

$$x^2\frac{dy}{dx} = \ x^2 + xy - y^2$$

VI. Attempt the following questions (Activity) (4 Marks)

1. find the general solution of the equation $\frac{dy}{dx} - y$ =2x

Solution : the equation $\frac{dy}{dx} - y$ =2x

is of the form $\frac{dy}{dx} + Py = Q$

where P = $\boxed{\quad\quad\quad\quad}$ and Q = $\boxed{\quad\quad\quad\quad}$

$$\therefore if\ e^{s-dx} \ = e^{-x}$$

∴ the solution of the linear differential equation is

$$y \quad e^{-x} \quad = \int 2x.e^{-x}dx + c$$

$$\therefore\ y\,e^{-x} \quad = \int 2x.e^{-x}\ dx + c$$

$$y\,e^{-x} \quad = 2\int x\ .\ e^{-x}\ dx$$

$$= 2\left\{x\int e^{-x}dx - \int \boxed{\quad} dx.\frac{d}{dx}\boxed{\quad} dx\right\} + c$$

$$= 2\left\{x\frac{e^{-x}}{-1} - \int \frac{e^{-x}}{-1}.1\right\} dx$$

$$\therefore\ y\, e^{-x} = -2x.e^{-x} + 2\int e^{-x}\, dx + c_1$$

$$\therefore\ e^{-x}\, y = -2x.e^{-x} + 2\boxed{\quad} + c_2$$

$y + \boxed{\quad} + \boxed{\quad} = ce^{x}$ is the required general solution of the

given differential equation.

2. Verify $y = a + \frac{b}{x}$ is solution of $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$

Solution : $y = a + \frac{b}{x}$

$$\frac{dy}{dx} = \boxed{\quad}$$

$$\frac{d^2y}{dx^2} = \boxed{\quad}$$

Consider $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx}$

$$= x\boxed{\quad} + 2\boxed{\quad}$$

$$= \boxed{\quad}$$

Hence $y = a + \frac{b}{x}$ is solution of $x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = 0$

3. The rate of growth of population proportional to the number present. If the population in the last 25 years and present population is 1 lac., when will the city have population 4,00,000?

Solution : Let p be the population at time t.

Then the rate of increase is $\frac{dp}{dt}$ which is proportional to p

$$\therefore \frac{dp}{dt} \ \alpha\ p$$

$$\therefore \frac{dp}{dt} = k\ p, \text{where k is a constant}$$

$$\therefore \frac{dp}{p} = k\ dt$$

on Integrating , we get

$$\therefore \int \frac{dp}{p} = k \int dt$$

$$\therefore logp = kt + c$$

$$Intially, ie\ when\ t = 0, let\ p = 100000$$

$\therefore log100000 = k \times\ 0 + c,$ $\qquad \therefore c =$

$\therefore logp = kt + log100000,$ $\qquad \therefore logp - log100000 = kt$

$$\therefore \log\left(\frac{p}{100000}\right) = kt \dots\dots\dots\dots\dots\dots..(1)$$

$$Since\ the\ number\ double\ in\ 25\ years, i.e. when\ t = 25, p = 200000$$

$\therefore \log\left(\frac{200000}{100000}\right) = 25k,$ $\qquad \therefore\ k =$

$\therefore equation\ (1)\ becomes\ , \log\left(\frac{p}{100000}\right) =$

$when\ p = 400000,$ $\qquad then\ to\ find\ t$

$$\therefore \log\left(\frac{400000}{100000}\right) = \frac{t}{25} log2$$

$$\therefore \log 4 = \frac{t}{25} log2$$

$$\therefore t = 25\frac{log4}{log2} \qquad \therefore \quad t = \boxed{\quad} \; years$$

4. In a certain culture of bacteria, the rate of increase is proportional to the number present. If it is found that the number doubles in 4 hours, find the number of times the bacteria are increased in 12 hours.

Solution : Let x be the number of bacteria in the culture at time t.

Then the rate of increase is $\frac{dx}{dt}$ which is proportional to x

$$\therefore \frac{dx}{dt} \; \alpha \; x$$

$$\therefore \frac{dx}{dt} = k\,x, \text{where k is a constant}$$

$$\therefore \quad \boxed{\qquad\qquad}$$

on Integrating , we get

$$\therefore \int \frac{dx}{x} = k\int dt$$

$$\therefore logx = kt + c$$

$$Intially, ie\ when\ t = 0, let\ x = x_0$$

$$\therefore logx_0 = k \times \ 0 + c, \qquad \therefore c =$$

$$\therefore logx = kt + logx_0 , \qquad \therefore logx - logx_0 = kt$$

$$\therefore \log\left(\frac{x}{x_0}\right) = kt \ldots\ldots\ldots\ldots\ldots\ldots..(1)$$

$Since\ the\ number\ double\ in\ 4\ hours, i.e. when\ t = 4, x = 2x_0$

$$\therefore \log\left(\frac{2x_0}{x_0}\right) = 4k, \qquad \therefore\ k =$$

$$\therefore equation\ (1)\ becomes\ , \log\left(\frac{x}{x_0}\right) = \frac{t}{4}\ log2$$

$when\ t = 12, we\ get$

$$\therefore \log\left(\frac{x}{x_0}\right) = \frac{12}{4} log2 = 3\ log2$$

$$\therefore \log\left(\frac{x}{x_0}\right) = kt$$

$$\therefore \frac{x}{x_0} = 8, \qquad \therefore$$

∴ number of bacteria will be 8 times the original number in 12 hours.

5. Find the population of city at any time t given that rate of increase of population is proportional to the population at that instant & that period of 40 years the population increased from 30000 to 40000.

Solution : Let p be the population at time t.

Then the rate of increase is $\frac{dp}{dt}$ which is proportional to p

$$\therefore \frac{dp}{dt}\ \alpha\ p$$

$$\therefore \frac{dp}{dt} = k\ p, \text{where k is a constant}$$

$$\therefore \frac{dp}{p} = k\ dt$$

on Integrating , we get

$$\therefore \int \frac{dp}{p} = k\int dt$$

$$\therefore logp = kt + c$$

$$Intially, ie\ when\ t = 0, let\ p = 30000$$

$$\therefore log30000 = k \times 0 + c, \qquad \therefore c =$$

$$\therefore logp = kt + log30000, \qquad \therefore logp - log30000 = kt$$

$$\therefore \log\left(\frac{p}{30000}\right) = kt \ldots \ldots \ldots \ldots \ldots \ldots .. (1)$$

$$when\ t = 40, p = 40000$$

$$\therefore \log\left(\frac{40000}{30000}\right) = 40k, \qquad \therefore k =$$

$$\therefore equation\ (1)\ becomes\ , \log\left(\frac{p}{30000}\right) =$$

$$\therefore \log\left(\frac{p}{30000}\right) = \frac{t}{40} \log\left(\frac{4}{3}\right)$$

P=

6. Solve the following differential equation

$$Sec^2 x\ tany\ dx + \sec^2 y\ tanxdy = 0$$

Solution : $Sec^2 x\ tany\ dx + \sec^2 y\ tanxdy = 0$

$$\therefore \frac{Sec^2x}{tanx}dx \ + \qquad\qquad\qquad = 0$$

Integrating, we get

$$\therefore \boxed{\qquad\qquad} + \int \frac{Sec^2y}{tany}\ dy \ = logc$$

Each of these integral is of the type

$$\int \frac{f'(x)}{f(x)dx} = \log|f(x)| + \log c$$

$\therefore$ *the general solution is*

$$\boxed{\qquad\qquad} + \log|tany| = logc$$

$$\therefore \ \log|tanx.\tan y| = \log c$$

$$\boxed{\qquad\qquad\qquad}$$

This is the general solution

7. Solve the following differential equation

$$\frac{dy}{dx} = \cos(x+y)$$

Solution :

$$\frac{dy}{dx} = \cos(x+y) \ldots \ldots \ldots \ldots \ldots \ldots (1)$$

$$Put \qquad \boxed{\qquad\qquad} \therefore 1 + \frac{dy}{dx} = \frac{dv}{dx}$$

$$\therefore \frac{dy}{dx} = \frac{dv}{dx} - 1$$

$\therefore (1) becomes\ \frac{dv}{dx} - 1 = cosv$

$\frac{dv}{dx} = 1 + cosv$

$\therefore \qquad\qquad\qquad dv = \ dx$

Integrating, we get

$\therefore \int \frac{1}{1 + cosv} dv = \int \ dx$

$\therefore \int \frac{1}{2\cos^2(\frac{v}{2})} dv = \int \ dx$

$\therefore \frac{1}{2} \int \boxed{\qquad\qquad} dv = \int \ dx$

$\therefore \frac{1}{2} \frac{\tan\left(\frac{v}{2}\right)}{\frac{1}{2}} = x + c$

$\therefore \qquad \boxed{\qquad\qquad} = x + c$

8. Find the particular solution of the following differential equation

$$\frac{dy}{dx} = e^{2y} \cos x \text{ , when } x = \frac{\pi}{6}, \text{y} = 0$$

Solution :

The given D.E. is $\frac{dy}{dx} = e^{2y} \cos x$

$\therefore \frac{1}{e^{2y}} dy = cosx\ dx$

Integrating, we get

$\int \quad \boxed{\qquad\qquad} dy = cosx\ dx$

$$\therefore \frac{e^{-2y}}{-2} = sinx + c_1$$

$$\therefore e^{-2y} = -2\ sinx - 2c_1$$

$$\therefore \quad \boxed{\quad} = c, where\ c = -2c_1$$

This is general solution

when $x = \frac{\pi}{6}$, $y = 0$, we have

$$e^0 + 2sin\frac{\pi}{6} = c$$

$$\therefore c = \quad \boxed{\quad}$$

$\therefore$ particular solution is $\boxed{\quad}$

9. Bacteria increases at the rate of proportional to the number bacteria present. If the original number N , doubles in 3 hours, find in how many hours the number of bacteria will be 16N.

Solution : Let x be the number of bacteria in the culture at time t.

Then the rate of increase is $\frac{dx}{dt}$ which is proportional to x

$$\therefore \frac{dx}{dt}\ \alpha\ x$$

$$\therefore \frac{dx}{dt} = k\ x, \text{where k is a constant}$$

$$\therefore \frac{dx}{x} = kdt$$

on Integrating , we get

$$\therefore \int \frac{dx}{x} = k\int dt$$

$\therefore logx = kt + c$……………….(1)

$x = a\, e^{kt}$ where $a = e^{c}$

$Intially, ie\ when\ t = 0, let\ x = N$

$\therefore logN = k \times 0 + c, \qquad \therefore a =$ $\boxed{\quad\quad\quad\quad}$

$\therefore a = N\ , x = N\, e^{kt}$ … … … … (2)

$when\ t = 3, x = 2N$

From equation (2), $2N = Ne^{4k}$

$\therefore e^{4k} = 2$

$e^{k} =$ $\boxed{\quad\quad\quad\quad}$

Now we have find out t, when x=16N

From equation (2),

$16N = Ne^{kt}$

$16 = e^{kt}$

$\therefore \frac{t}{4} =$ $\boxed{\quad\quad\quad\quad}$ hours

Hence number of bacteria will be 16N in $\boxed{\quad\quad\quad\quad}$ hours.

10. The population of city doubles in 80 years , in how many year will it be triple when the rate of increase is proportional to the number of inhabitants.

(Given $\frac{log3}{log2} = 1.5894$)

Solution : Let p be the population at time t.

Then the rate of increase is $\frac{dp}{dt}$ which is proportional to p

$$\therefore \frac{dp}{dt} \ \alpha \ p$$

$$\therefore \frac{dp}{dt} = k\ p, \text{where k is a constant}$$

$$\therefore \frac{dp}{p} = k\ dt$$

on Integrating , we get

$$\therefore \int \frac{dp}{p} = k \int dt$$

$$\therefore logp = kt + c$$

$$Intially, ie\ when\ t = 0, let\ p = N$$

$$\therefore logN = k \times 0 + c, \qquad \therefore c = \boxed{\qquad\qquad}$$

When t=80, P=2N

$$\therefore log2N = 80k + logN, \qquad \therefore log2N - logN = 80k$$

$$\therefore \log\left(\frac{2N}{N}\right) = 80k$$

$$\therefore \log(2) = 80k, \qquad \therefore k = \boxed{\qquad\qquad}$$

$$\therefore P = 3N, then\ t = ?$$

$$\therefore \log P = \frac{log2}{80} t + \log N$$

$$Log\ 3N - logN = \boxed{\qquad\qquad}$$

t = $\boxed{}$ = $\boxed{}$ years

Part II

1. Commission, Brokerage and Discount (Commerce 88)

I) Choose the correct alternative.

1. A salesman receives 3% commission on the sales up to Rs. 50,000 and 4% commission on the sales over Rs. 50,000. His total income on the sale of Rs. 2,00,000 is ___________

a) Rs. 6,000

b) Rs. 7,550

c) Rs. 7,500

d) Rs. 1,500

2. The present worth of Rs.11,660 due 9 months hence is Rs.11,000. The True discount is _______

a) Rs. 660

b) Rs. 750

c) Rs. 400

d) Rs. 5,940

3. If A bill of Rs.6,395 drawn on 15th February 2015 for 10 months was discounted on 28th May 2015 at 8% p.a. interest then legal due date is _____

a) 15^{th} December 2015

b) 15^{th} November 2015

c) 18^{th} December 2015

d) 18^{th} November 2015

4. The date on which the period of the bill expires is called ______

a) Legal Due Date

b) Days of grace

c) The Nominal Due date

d) Date of Drawing

5. The marked price is also called as ______

a) Cost price

b) Selling price

c) Invoice price

d) List price

II) Fill in the blanks.

1. An agent who gives guarantee to his principal that the party will pay the sale price of goods is called ____________

2. The difference between the __________________ and the true discount is called Banker's Gain (B.G). It is equal to the interest on true discount.

3. The buyer is legally allowed ________ days grace period.

4. The date on which the bill is drawn is called as ________

5. When transactions like sale, purchase, auction etc. are done through some middlemen, such middlemen are called ________

III) State whether each of the following is True or False.

1. The trade discount is first calculated on the catalogue (list) price.

2. A factor is an agent who is given the possession of goods and enters a contract for sale in his/her own name.

3. A person can get both, trade discount and cash discount.

4. The sum due is also called as Cash value

5. If only one discount is given then List price = Invoice price.

IV) Solve the following problems. (3 Marks)

1. Find the true discount, Banker's discount and Banker's gain on a bill of Rs.4,240 due 6 months hence at 9% p.a.

2. Ananya gets salary of Rs. 15,000 per month and commission at 8% on the sales over Rs.50,000. If she gets Rs. 17,400 in a certain month, Find the sales made by her in that month.

3. Swastik Distributers allows 15% discount on the list price of washing machine. Further 5% discount is given for cash payment. Find the list price of the washing machine if it was sold for the net amount of Rs. 38,356.25.

4. An agent sold a car and charged 3% commission on sale value. If the owner of the car received Rs.48,500, find the sale value of the car. If the agent charged 2% from the buyer, find his total remuneration.

5. A bill of Rs.65,700 drawn on July 10 for 6 months was discounted for Rs.65,160 at 5% p.a. on what day was the bill discounted?

V) Solve the following problems. (4 Marks)

1. A bill was drawn on 14th April for Rs.7,000 and was discounted on 6th July at 5% p.a. The Banker paid Rs.6,930 for the bill. What is the legal due date.

2. A bill of Rs.51,000 was drawn on 18th February 2010 for 9 months. It was encashed on 28th June 2010 at 5% p.a. Calculate the banker's gain and true discount.

3. A retailer sold a suit for Rs.8,832 after allowing 8% discount on marked price and further 4% cash discount. If he made 38% profit, find the cost price and the marked price of the suit.

4. If difference between true discount and banker's discount on a sum due 4 months hence is Rs 20. Find true discount, banker's discount and amount of bill, the rate of simple interest charged being 5%p.a.

VI) Activity (4 Marks)

1. A bill of Rs.8,000 drawn on 5th January 2019 for 8 months was discounted for Rs.7,680 on a certain date. Find the date on which it was discounted at 10% p.a.

Solution : Banker's Discount

$$\text{B.D.} = \text{F.V.} - \text{C.V.} \quad = 8{,}000 - 7{,}680 = \text{Rs.}320$$

Date of drawing = 5th January 2019

Period = 8 months

Nominal due date = $\boxed{}$

Legal due date = $\boxed{}$

B.D. = Interest on F.V. for n at 10% p.a.

$$\therefore \quad B.D. = \frac{\text{F.V.} \times \frac{n}{365} \times r}{100}$$

$$\therefore \quad 320 = \frac{8{,}000 \times \frac{n}{365} \times 10}{100}$$

$\therefore \quad n =$ $\boxed{}$ days

April	May	June	July	Aug	Sep	Total
15	**31**	**30**	**31**	**31**		

$\therefore$ Date of discounting is $\boxed{}$

2. Three cars were sold through an agent for Rs.2,40,000 , Rs.2,22,000 and Rs.2,25,000 respectively. The rates of commission were 17.5% on the first, 12.5% on the second. If the agent overall received 14% commission on the total sales, find the rate of commission paid on the third car.

Solution :

Total selling Price of three cars = 2,40,000 + 2,22,000 +2,25,000

= $\boxed{}$

Commission on total sale = 14%

$= \frac{14}{100} \times \boxed{}$

$= \boxed{}$

Selling price of First car = Rs. 2,40,000

Rate of commission = 17.5% $= \frac{17.5}{100} \times 2{,}40{,}000 = \boxed{}$

$\therefore$ Commission on first car = Rs. $\boxed{}$

Selling price of Second car = Rs. 2,22,000

Rate of commission = 12.5% $= \frac{12.5}{100} \times 2{,}22{,}000 = \boxed{}$

$\therefore$ Commission on second car = Rs. $\boxed{}$

Selling price of third car = Rs. 2,25,000

Let the rate of commission be x

Commission on third car $= \frac{x}{100} \times 2{,}25{,}000$

$\therefore$ Commission on third car = Total commission −
(commission on first car +

commission on s

$\frac{x}{100} \times 2{,}25{,}000 = \boxed{} - \{ \boxed{} + \boxed{}$
}

$\therefore \quad x = \boxed{}$

2. Insurance and Annuity

Multiple choice questions:

1) Rental payment for an apartment is an example of
   a) Annuity due   b) Perpetuity
   c) Ordinary annuity   d) Installment

2) In an ordinary annuity, payments or receipts occur at
   a)Beginning of each period   b)End of each period
   c)Mid of each period   d) Quarterly basis

3) The amount which can be demanded under the policy is ________.
   a) policy value   b) premium
   c) interest   d) claim

4) A shop valued ` 2,00,000 is insured at 80% of it's value. If the rate of premium is 4%, then the premium is
   a) 6,400   b) 6,000
   c) 6,450   d) 6,500

5) If the claim under the policy is Rs. 4,000 and ratio of property value to policy value is 5:4 then loss occurred is
   a) Rs.4,400   b) Rs.4,000
   c) RS.5,000   d) Rs. 5,500

6) The period for the fire insurance policy is
   a) one year   b) two years
   c) three years   d) four years

7) Premium is paid on ------- value.
   a)property   b) policy
   c)insured   d) both b) and c)

8) In annuity calculations, the interest is usually taken as
   a) simple interest per annum

b) interest compounded every year

c) interest compounded per month

d) simple interest per month

9) If for an immediate annuity r = 10% p.a.,P = Rs. 12,679.46 and A = Rs. 18,564, then the amount of each annuity paid is

a) Rs. 4,000 b) Rs. 4,500

c) Rs. 3,500 d) Rs. 4,200

10) The present value of an immediate annuity of Rs. 10,000 paid each quarter for four quarters at 16% p.a. compounded quarterly is

a) 40,000 b) 36,300 c) 36,286.75 d) 36289.25

True or False

1) Premium is the amount paid to the insurance company every month.

2)An installment of money paid for insurance is called Premium

3)The value of insured property is called policy value

4)A sinking fund is a fund established by financial organization

5) The relation between accumulated value ‘A’and present value ‘P’ is $A= P(1+i)^n$

6)The future value of an annuity is the accumulated values of all instalments

7)Annuity contingent begins and ends on certain fixed dates

8)An annuity where payments continue forever is called perpetuity.

Fill in the blanks:

1)In an ordinary annuity, payments or receipts occur at----

2)The present value of an immediate annuity for4 years at 10% p.a. compounded annually isRs. 23,400. It’s accumulated value after 4 years would be ` ________.

3) If for an immediate annuity r = 10% p.a.,P = ` 12,679.46 and A = ` 18,564, then the amount of each annuity paid is------

4) An annuity in which each payment is made at the end of period is called-------

5) If payments of an annuity fall due at the beginning of every period, the series is called annuity ___________.

6) The intervening time between payment of two successive installments is called as ___________.

7) --------- insurance is not covered by general insurance(mcq1)

3 marks Questions:

1) A shop and a godown worth Rs.1,00,000 and Rs.2,00,000 respectively were insured through an agent who was paid 12% of the total premium. If the shop was insured for 80% and the godown for 60% of their respective values, find the agent's commission, given that the rate of premium was 0.80% less 20% .
2) The rate of premium is 2% and other expenses are 0.075%. A cargo worth Rs.3,50,100 is to be insured so that all its value and the cost of insurance will be recovered in the event of total loss.
3) A merchant takes fire insurance policy to cover 80 % of the value of his stock. Stock worth Rs.80,000 was completely destroyed in a fire. while the rest of stock was reduced to 20% of its value. If the proportional compensation under the policy was Rs.67,200, find the value of the stock.
4) The rate of premium is 2% and other expenses are 0.075%. A cargo worth Rs.3,50,100 is to be insured so that all its value and the cost of insurance will be recovered in the event of total loss.
5) A 35-year old person takes a policy for Rs.1,00,000 for a period of 20 years. The rate of premium is Rs.76 and the average rate of bonus is Rs.7 per thousand p.a. If he dies after paying 10 annual premiums, what amount will his nominee receive?

6) A person invested Rs.5,000 every year in finance company that offered him interest compounded at 10% p.a., what is the amount accumulated after 4 years? [Given $(1.1)^4 = 1.4641$]

7) Find the amount of an ordinary annuity if a payment of Rs. 500 is made at the end of every quarter for 5 years at the rate of 12% per annum compounded quarterly.

8) An annuity immediate is to be paid for some years at 12% p.a. The present value of the annuity is Rs.10,000 and the accumulated value is Rs.20,000. Find the amount of each annuity payment.

9) A person sets up a sinking fund in order to have Rs.1,00,000 after 10 years. What amount should be deposited bi-annually in the account that pays him 5%p.a.compounded semi-annually?[Given $(1.025)^{20} = 1.675$]

10) A company decides to set aside a certain sum at the end of each year to create a sinking fund, which should amount to Rs. 4 lakhs in 4 years at 10% p.a. Find the amount to be set aside each year?

Activity based Question

1) Property value = Rs.12,50,000

Rate of premium ,r = Rs. 3%

If i) property is fully insured ,the policy value is same as property value

therefore policy value = --------

Premium = $\frac{r}{100}\ X\ \ policy\ value$

$$= \frac{---}{100}\ X\ \ 12{,}50{,}000$$

= ---------

ii) property is 80% insured

Policy value = 80 % of its property value

$$= \frac{-----}{100} \; X \;\; 12,50,000$$

= Rs.10,00,000

$$\text{Premium} = \frac{--}{100} \;\; X \;\; 10,00,000$$

= Rs.

2) Policy value = Rs. 80,000

Period of policy = 20 year

Amount of money paid in 10 years = ------

Annualised average rate per bonus

= R. 20 per thousand per year

$$\text{For one year ,bonus} = \frac{----}{1000} \; X \;\; 80,000$$

= Rs.1,600

Bonus for 10 years = 10 x

= Rs.16,000

Total amount after 10 years

+ 160 = Rs.

3) For annuity due,

C = Rs.20,000 ,n = 3, I = 0.1

Therefore , P = $\frac{\quad}{0.1}$ x [1- $(1+0.1)^{----}$]

= 2,00,000 [1- 0.7513]

= Rs.------------

4) The future amount A = RS. 10,00,000

Period , n = 20 , r = 5 %

$$A = \frac{C}{I}[(1+i))^{n} - 1]$$

$I=\frac{5}{200}$ $- -$ as interest is calculated semi-annually

$$A= 10,00,000 = \frac{C}{I}[(1+i))^{n} - 1]$$

$$10,00,000= \frac{C}{0.025}[(1+0.025))^{--} - 1]$$

$$=\frac{C}{0.025}[1.675 - 1]$$

$$10,00,000 = \frac{C\ X\ 0.675}{0.025}$$

C = Rs.-------

3. LINEAR REGRESSION

I Select and write the most appropriate answer from given alternatives of the following sub questions:

1.If for a bivariate data, byx= -1.2 and bxy=-0.3 then r =-----

a. -0.06
b. 0.06
c. 0.6
d. -0.6

2. If the regression equation x on y is 3x+2y=26 then bxy equals to

a. 3/2

b 2/3

c -3/2

d -2/3

3. If byx <0 and bxy <0 then r is---

a <0

b >0

c =0

d >1

4. |byx+bxy| ≥…..

a. |r|
b 2|r|
c r
d -r

5. Find the value of the covariance between x and y, if the regression coefficient of y on x is
3.75 and σ_x =2, σ_y =8

a. 7
b. 30
c 15
d 1.875

6. The slope of the line of regression of y on x is called the----

a regression coefficient of x on y
b correlation coefficient between y and x

c covariance between y and x

d regression coefficient of y on x

7. Regression analysis is the theory of

a Estimation

b Prediction

c Both a and b

d Calculation

8. We can estimate the value of one variable with the help of other known variable only if they are

a Correlated

b Positively correlated

c Negatively correlated

d Uncorrelated

9. There are ___________ types of regression equations.

a 4

b 2

c 3

d 1

10. In the regression equation of X on Y

a X is independent and Y is dependent.

b Y is independent and X is dependent.

c Both X and Y are independent.

d Both X and Y are dependent.

11. bxy and byx are ___________

a Independent of change of origin and scale

b Independent of change of origin but not of scale

c Independent of change of scale but not of origin d.Affected by change of origin and scale

12.If the lines of regression of y on x is y=x/4 and x on y is x=y/9 +1, then the value of r is

a.1/6

b.0

c. -1/4

d.-1/6

13. If r= 0.5, σ_x =3, σ_y^2 =16, then b_{yx}=…

a. 0.375

b. 0.667

c. 2.667

d. 0.093

14. The regression line is obtained by

a Minimizing the sum of squares of deviations of the predicted values from the observed values.

b Minimizing the sum of deviations of the predicted values from the observed values.

c Maximizing the sum of squares deviations of the predicted values from the observed values.

d. Maximizing the sum of deviations of the predicted values from the observed values..

15. u = $\frac{x-20}{5}$ and v =$\frac{y-30}{4}$,then bxy=

a. $\frac{4}{5}$ bvu

b. $\frac{4}{5}$ buv

c. $\frac{5}{4}$ buv

d $\frac{5}{4}$ bvu

16. y = 5 - 2.8x and x = 3 – 0.5y be the regression lines ,then the value of byx is

a. -0.5
b. -2.8
c. 0.5
d. 2

17. If r= 0.5, σ_x =3, σ_y^2 =16, then b_{xy}= ---

a..0.375
b. 0.667
c. 2.667
d. 0.093

II State whether the following statements are true or false:

1. The equations of two regression lines are 10x-4y=80 and 10y-9x=40 . then bxy = 0.9
2. y = 5 + 2.8x and x = 3 + 0.5y be the regression lines of y on x and x on y respectively ,then byx = - 0.5
3. Both the regression coefficients cannot exceed 1
4. If byx=1.5 and bxy=1/3 then r = 1/2. the given data is consistent.

5. Correlation analysis is the theory of games

6. If bxy < 0 and byx < 0 then 'r' is >0

7. The following data is not consistent:
   byx+ bxy =1.3 and r= 0.75

8. .If u=x-a and v= y-b then bxy= buv.

9. If equation of regression lines are 3x+2y-26=0 and 6x+y-31=0 then mean of x is 7.

10. bxy is the slope of regression line of y on x.

11..Corr(x.x) = 0

12..Corr(x,x) =1

13. Cov(x,x)=Variance of x.

14.Regression analysis is used for measuring the degree of the relationship between the variables.

15.Regression coefficient of x on y is the slope of regression line of x on y

16.The variable used for predicting the response is called the independent variable.

III. Fill in the following blanks:

1. If n = 5,Σxy=76 ,Σx^2= Σy^2 =90, Σx=20=Σy then covariance=…..

2. |bxy+ byx| ≥__________

3. Among the given regression lines 6x+y-31=0 and 3x+2y-26=0,the regression line of x on y is….

4. If u = $\frac{x-a}{c}$ and v =$\frac{y-b}{d}$, then bxy=______

5. If the regression equations are 8x – 10y + 66 = 0 and 40x – 18y = 214, the mean value of y is…..

6. If the sign of the correlation coefficient is negative, then the sign of the slope of the
respective regression line is----

7. The value of product moment correlation coefficient between x and x is---

8. Arithmetic mean of positive values of regression coefficients is greater than or equal to ….

9. If u = $\frac{x-20}{5}$ and v =$\frac{y-30}{4}$, then byx=______

10. The geometric mean of negative regression coefficients is ----

11. Dependent variables are also known as----

12. b_{yx} is the ….. of regression line of y on x

IV.Answer the following:

1.The equations of two lines of regression are $3x+2y-26=0$ and $6x+y-31=0$. find variance of
x if variance of y is 36.

2.Given the following information about the production and demand of a commodity.
Obtain the two regression lines :

	ADVERTISEMNT(x) (₹.in lakhs)	DEMAND(y) (₹ in lakhs)
MEAN	10	90
VARIANCE	9	144

Coefficient of correlation between x and y is 0.8.

What should be the advertising budget if the company wants to attain the sales target of Rs.150 lakhs?

3.The equations of the two lines of regression are $2x + 3y - 6 = 0$ and $5x + 7y - 12 = 0$
a. Identify the regression lines.
b. Find the value of the correlation coefficient (Given $\sqrt{0.933} = 0.9667$.)

4.The age in years of 7 young couples is given below. Calculate husband's age when
wife's age is 38 years.

Husband(x)	21	25	26	24	22	30	20
Wife(y)	19	20	24	20	22	24	18

5.Given the following information about the production and demand of a commodity.

Obtain the two regression lines :

	PRODUCTION(X)	DEMAND(Y)
MEAN	85	90
VARIANCE	25	36

Coefficient of correlation between X and Y is 0.6. Also estimate the demand when the production is 100 units.

6.The equations of the two lines of regression are $6x + y - 31 = 0$ and $3x + 2y - 26 = 0$.

a. Identify the regression lines
b. Find the value of the correlation coefficient.
c .Calculate the mean values of x and y.

7.Two samples from bivariate populations have 15 observations each. The sample means of X and Y are 25 and 18 respectively. The corresponding sum of squares of deviations from means are 136 and 148 respectively. The sum of product of deviations from respective means is 122.Obtain the regression equation of x on y.

8. For 50 students of a class, the regression equation of marks in Statistics(X) on the marks in accountancy(Y) is 3y-5x+180 =0.The mean marks in accountancy is 44 and the variance of marks in Statistics is $\left(\frac{9}{16}\right)^{th}$ of variance of marks in Accountancy .Find the mean marks in Statistics and the correlation coefficient between the marks of the two subjects.

9. If $n = 5, \Sigma x = \Sigma y = 20, \Sigma x^2 = \Sigma y^2 = 90$, $\Sigma xy = 76$

Find

a. Covariance (x,y)

b. Find the regression equation of x on y.

10. If $n = 6$, $\Sigma x = 36$, $\Sigma y = 60, \Sigma xy = -67$, $\Sigma x^2 = 50$, $\Sigma y^2 = 106$,

Estimate y when x is 13.

11.Compute the appropriate regression equation for the following data:

x(Dependent Variable)	10	12	13	17	18
y(Independent Variable)	5	6	7	9	13

12.For a certain bivariate data of a group of 10 students, the following information gives the

internal marks obtained in English (X)and Hindi (Y) :

	X	Y
MEAN	13	17
STANDARD DEVIATION	3	2

If $\Sigma(x-\bar{x})(y-\bar{y}) = 36$, Estimate x when y =16 and y when x=10.

V Activity questions:

(1)

X	Y	x-$\bar{x}$	y-$\bar{y}$	(x-$\bar{x}$) (y-$\bar{y}$)	(x-$\bar{x}$)^2	(y-$\bar{y}$)^2
1	5	-2	-4	8	4	16
2	7	-1	-2	$\boxed{\ }$	1	4
3	9	0	0	0	0	0
4	11	1	2	2	4	4
5	13	2	4	8	1	16
Total=15	Total=45	Total=0	Total= 0	Total=$\boxed{\ }$	Total=10	Total=40

Mean of x = $\bar{x}$ $\boxed{\ }$

Mean of y = $\bar{y}$ = $\boxed{\ }$

$b_{xy} = \frac{\square}{\square}$

$b_{yx} = \frac{\square}{\square}$

Regression equation of x on y is $x-\bar{x} = b_{xy}(y-\bar{y})$

∴Regression equation of x $\boxed{}$s

Regression equation of y on x is $y-\bar{y} = b_{yx}(x-\bar{x})$

∴Regression equation of y $\boxed{}$s

(2). Mean of x= 53

Mean of y = 28

Regression coefficient of y on x = - 1.2

Regression coefficient of x on y= - 0.3

a. r = $\boxed{}$

b. When x=50,

$y- \boxed{} = \boxed{} (50- \boxed{})$

$\therefore y = \boxed{}$

c. When y= 25,

$x - \boxed{} \quad = \quad \boxed{} \quad (25 - \boxed{})$

$\therefore x = \boxed{}$

(3). Mean of x= 25

Mean of y =20

$\sigma_x = 4$

$\sigma_y = 3$

r = 0.5

$b_{yx} = \boxed{}$

$b_{xy} = \boxed{}$

when x= 10,

$y - \boxed{} \quad = \quad \boxed{} \quad (10 - \boxed{})$

$\therefore y = \boxed{}$

(4).The regression equation of y on x is 2x-5y +60 =0.

Mean of x = 18

$2\,\square\; - 5\,\bar{y} + 60 = 0$

$\therefore \bar{y} = \square$

$\sigma_x : \sigma_y = 3:2$

$b_{yx} = \square$

$\therefore b_{yx} = r\frac{\square}{\square}$

$\therefore r = \square$

(5). The regression equation of x on y is 40x-18y = 214… (i)

The regression equation of y on x is 8x -10y +66 =0….. (ii)

Solving equations i and ii,

$\bar{x} = \square$

$\bar{y} = \square$

$\therefore b_{yx} = \frac{\square}{\square}$

$\therefore b_{xy} = \frac{\square}{\square}$

$\therefore r = \square$

Given variance of x = 9

$\therefore b_{yx} = \dfrac{\square}{\square}$

$\therefore \sigma_y = \square$

(6)

X	y	xy	x^2	y^2
6	9	54	36	81
2	11	22	4	121
10	5	50	100	25
4	8	32	16	64
8	7	$\square$	64	49
Total=30	Total=40	Total $\square$	Total= 220	Total= $\square$

$b_{xy} = \dfrac{\square}{\square}$

$b_{yx} = \dfrac{\square}{\square}$

$\therefore$Regression equation of x $\square$ is

$\therefore$Regression equation of y on x is $\square$

4. TIME SERIES

I) Choose the correct alternative.

1. Which of the following can't be a component of a time series?

(a) Seasonality (b) Cyclical (c) Trend (d) Mean

2. Which component of time series refers to erratic time series movements that follow no recognizable or regular pattern?

(a) Trend (b) Seasonal (c) Cyclical (d) Irregular

3. The following trend line equation was developed for annual sales from 1984 to 1990 with 1984 as base or zero year.

$Y = 500 + 60X$ (in 1000 ₹). The estimated sales for 1984 (in 1000 ₹) is:

(a) 500 (b) 560 (c) 1,040 (d) 1,100

4. An overall upward or downward pattern in an annual time series would be contained in which component of the times series

(a) Trend (b) Cyclical (c) Irregular (d) Seasonal

5. Moving averages are useful in identifying

(a) Seasonal component (b) Irregular component

(c) Trend component (d) cyclical component

II) Fill in the blanks

1. __________ components of time series is indicated by a smooth line.

2. __________ component of time series is indicated by periodic variation year after year.

3. The complicated but efficient method of measuring trend of time series is _________

4. The simplest method of measuring trend of time series is ___________

5. The method of measuring trend of time series using only averages is _____

III) State whether each of the following is True or False.

1. The secular trend component of time series represents irregular variations.

2. Seasonal variation can be observed over several years.

3. Cyclical variation can occur several times in a year.

4. Moving average method of finding trend is very complicated and involves several calculations.

5. Least squares method of finding trend is very simple and does not involve any calculations.

IV) Solve the following problems.

1. Following table shows the amount of sugar production (in lac tons) for the years 1971 to 1982.

Year	1971	1972	1973	1974	1975	1976
Production	1	0	1	2	3	2
Year	1977	1978	1979	1980	1981	1982
Production	4	6	5	1	4	10

Fit a trend line by the method of least squares.

2. Obtain trend values for data in Problem 1 using 4-yearly centered moving averages.

3. The following table gives the production of steel (in millions of tons) for years 1976 to 1986.

Year	1976	1977	1978	1979	1980	1981
Production	0	4	4	2	6	8
Year	1982	1983	1984	1985	1986	
Production	5	9	4	10	10	

Fit a trend line by the method of least squares. Also, obtain the trend value for the year 1990.

4. Obtain the trend values for the above data using 3-yearly moving averages.

5. Use the method of least squares to fit a trend line to the data in Problem 6 below. Also, obtain the trend value for the year 1975.

6. The following table shows the production of gasoline in U.S.A. for the years 1962 to 1976.

Obtain trend values for the above data using 5-yearly moving averages.

Year	1962	1963	1964	1965	1966	1967	1968	1969
Production(million barrels)	0	0	1	1	2	3	4	5
Year	1970	1971	1972	1973	1974	1975	1976	
Production(million barrels)	6	8	9	9	8	9	10	

V) Activity based questions

1. Following table shows the all India infant mortality rates (per '000) for years 1980 to 2000.

Year	1980	1985	1990	1995
IMR	10	7	5	4
Year	2000	2005	2010	
IMR	3	1	0	

Fit a trend line by the method of least squares.

Solution: Let us fit equation of trend line for above data.

Let the equation of trend line be y = a + b.x ... (i)

Here n = 7 (odd), one middle yea $\boxed{}$ and h =1

Year	**IMR (y)**	**x**	$\mathbf{X^2}$	**x.y**
1980	10	-3	9	-30
1985	7	-2	4	-28
1990	5	-1	1	-5
1995	4	0	0	0
2000	3	1	1	3
2005	1	2	4	4
2010	0	3	9	0
Total	**30**	**0**	**28**	**-56**

The normal equations are

$\Sigma y = na + b\,\Sigma x$

As, $\Sigma x = 0$, a = $\boxed{}$

Also, $\Sigma xy = a\,\Sigma x + b\,\Sigma x^2$

As, $\Sigma x = 0$, b = $\boxed{}$

$\therefore$The equation of trend line is y = $\boxed{}$

2. Obtain trend values for data in Problem 1 using 3-yearly moving averages.

Solution:

Year	**IMR**	**3 yearly moving total**	**3-yearly moving average (trend value)**
1980	10	-	-
1985	7		7.33
1990	5	16	
1995	4	12	4
2000	3	8	
2005	1		1.33
2010	0	-	-

3. Fit equation of trend line for the data given below.

Year	**Production (y)**	**x**	**X^2**	**x.y**
2006	19	-9	81	-171
2007	20	-7	49	-104
2008	14	-5	25	-70
2009	16	-3	9	-48
2010	17	-1	1	-17
2011	16	1	1	16

2012	18	3	9	54
2013	17	5	25	85
2014	21	7	49	147
2015	19	9	81	171
Total	**177**	**0**	**330**	**33**

Let the equation of trend line be y = a + bx..... (i)

Here n = ☐ (even), two middle ☐ are ☐ and 2011, and h =

The normal equations are $\Sigma y = na + b\Sigma x$

As $\Sigma x = 0$, a = ☐

Also, $\Sigma xy = a\Sigma x + b\Sigma x^2$

As $\Sigma x = 0$, b = ☐

Substitute values of a and b in equation (i) the equation of trend line is ☐

To find trend value for the year 2016, put x = ☐ in the above equation.

y = ☐

4. Complete the table using 4 yearly moving average method.

Year	**Production**	**4 yearly moving total**	**4 yearly centered total**	**4 yearly centered moving average (trend values)**
2006	19		-	-
		☐		
2007	20		-	☐

		72		
2008	17		142	17.75
		70		
2009	16		$\square$	17
		$\square$		
2010	17		133	$\square$
		67		
2011	16		$\square$	$\square$
		$\square$		
2012	18		140	17.5
		72		
2013	17		147	18.375
		75		
2014	21		-	-
		-		
2015	19		-	-

5. INDEX NUMBERS

I) Choose the correct alternative.

1. Price Index Number by using Weighted Aggregate Method is given by

(a) $\frac{\sum p_1 q}{\sum p_0 q}$ x 100

(b) $\sum p_1\, w$ x 100

(c) $\frac{\sum p_1 w}{\sum p_0 w}$ x 100

(d) $\frac{\sum p_0 w}{\sum p_1 w}$ x 100

2. The formula $P_{01} = \frac{\sum p_1 q_0}{\sum p_0 q_0}$ x 100 is for
   (a) Laspeyre's Price Index Number
   (b) Paasche's Price Index Number
   (c) Fisher's Price Index Number
   (d) Walsh's Price Index Number

3. Dorbish–Bowley's Price Index Number is
   (a) P_{01} (L) + P_{01} (P)
   (b) P_{01} (L) - P_{01} (P)
   (c) $\frac{P_{01\,(L)} + P_{01\,(P)}}{2}$ x 100
   (d) $\frac{P_{01\,(L)} + P_{01\,(P)}}{2}$

4. $\frac{\sum p_1 q_1 w}{\sum p_0 q_0 w}$ x 100 gives
   (a) Value Index Number by Simple Aggregate method
   (b) Value Index Number by Weighted Aggregate method
   (c) Cost of Living Index Number
   (d) Laspeyre's Index Number

5. Walsh's Price Index Number is given by
   (a) $\frac{\sum q_0 \sqrt{p_0 p_1}}{\sum q_1\ \sqrt{p_0 p_1}}$ x 100

(b) $\frac{\sum p_0 \sqrt{q_0 q_1}}{\sum p_1 \sqrt{q_0 q_1}}$ x 100

(c) $\frac{\sum q_1 \sqrt{p_0 p_1}}{\sum q_0 \sqrt{p_0 p_1}}$ x 100

(d) $\frac{\sum p_1 \sqrt{q_0 q_1}}{\sum p_0 \sqrt{q_0 q_1}}$ x 100

6. Quantity Index Number by Simple Aggregate Method is given by

   (a) $\sum \frac{q_1}{q_0}$ x 100

   (b) $\sum \frac{q_0}{q_1}$ x 100

   (c) $\frac{\sum q_1}{\sum q_0}$ x 100

   (d) $\frac{\sum q_0}{\sum q_1}$ x 100

7. Fisher's Price Index Number is

   (a) $\sqrt{P_{01}(L) \ \ x \ P_{01}(P)}$

   (b) $P_{01}(L) \ \ x \ P_{01}(P)$

   (c) $\sqrt{P_{01}(L) \ \ x \ P_{01}(P)}$ x 100

   (d) $\sqrt{P_{01}(L) + P_{01}(P)}$

8. The Cost of Living Index Number using Weighted Relative Method is given by

   (a) $\frac{\sum IW}{\sum W}$ x 100

   (b) $\frac{\sum IW}{\sum W}$

   (c) $\frac{\sum W}{\sum IW}$ x 100

(d) $\frac{\sum W}{\sum IW}$

9. The Cost of Living Index Number by Aggregate Expenditure Method is same as
   (a) Fisher's Price Index Number
   (b) Laspeyre's Price Index Number
   (c) Paasche's Price Index Number
   (d) Dorbish-Bowley's Price Index Number

II) Fill in the blanks.

1. Price Index Number by Simple Aggregate Method is given by ....
2. Value Index Number by Simple Aggregate Method is given by ....
3. Fisher's Price Index Number is given by ....
4. Marshall-Edgeworth's Price Index Number is given by …………..
5. The Cost of Living Index Number by Aggregate Expenditure Method is given by …………
6. The average of Laspeyre's and Paasche's Price Index Numbers is called……………..Price Index Number.
7. Quantity Index Number by Weighted Aggregate Method is given by ....
8. Price Index Number by Weighted Aggregate Method is given by ....

III) State whether each of the following is True or False.

1. Walsh's Price Index Number is given by
$\frac{\sum p_1 \sqrt{q_0 q_1}}{\sum p_0 \sqrt{q_0 q_1}}$ x 100
2. The three types of Index numbers are i) Price Index Number, ii) Quantity Index Number, iii) Value Index Number.
3. For Cost of Living Index Number CLI = $\frac{\sum IW}{\sum W}$, where
I = $\frac{p_0}{p_1}$ x 100 and w = $p_0 q_0$
4. Purchasing power of money = $\frac{1}{Cost\ of\ Living\ Index\ Number}$
5. $\sum \frac{q_1}{q_0}$ x 100 is the Quantiy Index Number by Simple Aggregate Method.
6. $\frac{\sum p_1 q_1}{\sum p_0 q_1}$ x 100 is Paasche's Price Index Number.
7. $\frac{\sum p_0 \sqrt{q_0 + q_1}}{\sum p_1 \sqrt{q_0 + q_1}}$ x 100 is Marshall-Edgeworth Price Index Number.
8. **[** $\sqrt{\frac{\sum p_1 q_1}{\sum p_0 q_1}} + \frac{\sum \sqrt{q_0 q_1}}{\sum (p_0 + p_1)}$] x 100 is Fisher's Price Index Number.
9. $\sum \frac{p_1 q_1}{p_0 q_0}$ x 100 is the Value Index Number by Simple Aggregate Method.

IV) Solve the following problems.

1. Find Price Index Number using Simple Aggregate method by taking 2005 as base year.

commodity	P	Q	R	S	T
Price in 2005 (in Rs.)	10	25	14	20	30

Price in 2015 (in Rs.)	32	40	20	45	70

2. Find Quantity Index Number using Simple Aggregate method.

Commodity	A	B	C	D	E
Base year Quantity	170	150	100	195	205
Current year Quantity	90	70	75	150	95

3. Calculate Value Index Number for the following using Simple Aggregate Method.

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	30	13	40	15
B	40	15	70	20
C	10	12	60	22
D	50	10	90	18
E	20	14	100	16

4. Calculate Quantity Index Number using Simple Aggregate method.

Commodity	I	II	III	IV	V
Base year Quantity	140	120	100	200	225
Current year Quantity	100	80	70	150	185

5. Find Price Index Number using Simple Aggregate method by taking 2000 as base year.

Commodity	Price (in Rs.) for year 2000	Price (in Rs.) for year 2007
Watch	900	1475
Shoes	1760	2300
Sunglasses	600	1040
Mobile	4500	8500

6. Find x if the Price Index Number by Simple Aggregate Method is 125.

Commodity	P	Q	R	S	T
Base Year Price (in Rs.)	10	8	12	24	18
Current Year Price (in Rs.)	14	10	x	28	22

7. Find values x and y if the Price Index Number by Simple Aggregate Method by taking 2001 as base year is 120, given

$\sum p_1 = 300$.

Commodity	A	B	C	D
Price (in Rs) in 2001	90	x	90	30
Price (in Rs) in 2004	95	60	y	35

8. Find x from following data if the Value Index Number is 200

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	10	10	20	10
B	8	20	22	15
C	2	X	8	10
D	9	10	16	10
E	5	6	3	10

9. Calculate a) Laspeyre’s, b) Paasche’s, and c) Dorbish-Bowley’s Price Index Numbers for following data.

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	10	9	50	8
B	20	5	60	4
C	30	7	70	3
D	40	8	80	2

10. Calculate Marshall-Edgeworth Price Index Number for following.

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity

A	8	20	11	15
B	7	10	12	10
C	3	30	5	25
D	2	50	4	35

11. Calculate Walsh's price Index Number for the following data.

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
I	10	12	40	3
II	20	2	25	8
III	30	3	50	27
IV	60	9	90	36

12. If $P_{01}(L) = 40$ and $P_{01}(P) = 90$, find P_{01}(D-B) and P_{01}(F).

13. If Laspeyre's and Paasche's Price Index Numbers are 50 & 72 respectively, find Dorbish-Bowley's and Fisher's Price Index Numbers.

14. Given P_{01}(M-E) =120, $\sum p_1 q_1 = 300$, $\sum p_0 q_0 = 120$, $\sum p_0 q_1 = 320$, Find $P_{01}(L)$.

15. Find the missing price if Laspeyre's and Paasche's Price Index Numbers are equal for following data.

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	1	10	2	5
B	1	5	–	12

16. If $\sum p_0q_0 = 150$, $\sum p_0q_1 = 250$, $\sum p_1q_1 = 375$ and $P_{01}(L) = 140$. find $P_{01}(M\text{-}E)$.

17. Calculate the Cost of Living Index Number for the following data.

Group	Base Year		Current Year
	Price	Quantity	Price
Food	40	5	20
Clothing	30	10	35
Fuel and Lighting	20	17	10
House Rent	60	22	10
Miscellaneous	70	25	8

18. Calculate the Cost of Living Index by Family Budget method in following example where W are wages of base year and I are current year price relatives.

Group	Food	Clothing	Fuel and Lighting	House Rent	Miscella-neous
I	150	140	100	120	200
W	4	3	3	4	6

19. Find the missing wage if the Cost of Living Index for the following data is 150.

Group	Food	Clothing	Fuel and Lighting	House Rent	Miscella - neous
I	200	150	140	100	120
W	6	4	x	3	4

20. The Cost of Living Index Numbers for years 2003 and 2008 are 150 and 200 respectively. A person earned Rs. 18,000 per month in year 2003. What should be his earning per month in year 2008, so as to maintain same standard of living as 2003?

V) Activity

1. Given the following table, find Walsh's Price Index Number by completing the activity.

Commodity	P_0	Q_0	P_1	Q_1	Q_0Q_1	$\sqrt{Q_0\ Q_1}$	P_0	P_1

							$\sqrt{Q_0\, Q_1}$	$\sqrt{Q_0\, Q_1}$
I	20	9	30	4	36	…..	…..	180
II	10	5	50	5	…..	5	50	…..
III	40	8	10	2	16	….	160	
IV	30	4	20	1	…..	2	….	40
Total	-	-	-	-	-		390	

Walsh's price Index Number is

$$P_{01}(W) = \frac{\ldots\ldots}{\sum p_0 \sqrt{q_0 q_1}} \times 100$$

$$= \frac{510}{\ldots\ldots} \times 100$$

$$=$$

2. Given the following table, find the Cost of living Index Number using Aggregate Expenditure Method by completing the activity.

Group	P_0	Q_0	P_1	P_0Q_0	P_1Q_0
A	23	4	25	…..	100
B	15	5	20	75	…..
C	5	9	8	….	72
D	12	5	18	60	…..

E	8	6	13	….	78
Total	-	-	-	320	$\boxed{}$

Therefore Cost of Living Index using Aggregate Expenditure method is

$$CLI = \frac{\sum p_1 q_0}{\sum p_0 q_0} \boxed{}$$

$$= \boxed{\frac{}{}} \quad x\ 100$$

$$= \boxed{}$$

6. LINEAR PROGRAMMING PROBLEMS

1. Select the most appropriate option for each of the following.

i) If LPP has optimal solution at two point then,

a) LPP will give unique solution

b) LPP will give two solutions.

c) LPP will give infinite solutions.

d) LPP will not give any convex set.

ii) The feasible region is

a) common region determined by all the constraints

b) common region determined by the non-negativity constraints

c) either of the options a or b

d) both the options a & b

iii) The maximum value of Z = 3x + 5y subjected to the constraints x +y $\leq$ 2,

4x + 3y $\leq$ 12, x$\geq$ 0, $y \geq 0$ is

a)10. b) 9. c)15. d) 20

iv) The minimum value of Z = 4x + 5y subjected to the constraints x + y $\geq$ 6,

5x + y $\geq$ 10, x, y $\geq$ 0 is

a) 28. b) 29. c) 30. d) 31

v) The point at which the minimum value of Z = 8x + 12y subject to the constraints 2x +y $\geq$ 8, x + 2y$\geq$ 10 x$\geq$ 0, $y \geq 0$ is obtained at the point

a) (8,0) b) (9, 1) c) (2, 4) d) (10, 0)

vi) The point at which the maximum value of Z = 4x + 6y subject to the constraints 3x + 2y $\leq$ 12, x + y $\geq$ 4, x $\geq$ 0, $y \geq 0$ is obtained at the point

a) (0, 6) b) (6,0) c) (0, 4) d) (4, 0)

vii) Z = 9x + 13y subjected to constraints 2x +3y $\leq$18, 2x + y $\leq$ 10, 0 $\leq$ x, y was found to be maximum at the point

a) (3,4) b) (0, 6) c) (5, 0) d) (9, 0)

viii) The Corner points of feasible region for the inequations , x + y $\leq$5,x +2y$\leq$ 6, x$\geq$ 0, $y \geq 0$ is

a)(0,3),(5,0),(0,5),(6,0) b) (0,3),(5,0),(4,1),(0,0) c) (0,0),(1,4),(5,0),(0,3)

d) (3,0),(0,5)(0,0),(4,1)

ix) The corner points of the feasible region are (0,3), (3,0), (8,0), (12/5,38/5) and (0,10), then the point of maximum z = 6x + 4y= 48 is at

a) (0.10) b) (8,0) c) (12/5,38/5) d) (3,0)

x) The corner points of the feasible region are (4, 2), (5,0), (4, 1) and (6,0) then the point of minimum z = 3.5x + 2y= 16 is at

b) (4, 2)) b) (5,0) c) (6,0) d) (4, 1)

xi) The constraint that in a college there are more scholarship holders in FYJC class(X) than in SYJC class (Y) is given by

a) $X > Y$ b) $X < Y$ c) $X = Y$ d) $X \neq Y$

xii) How does a constraint,” A washing machine can hold up to 8 kilograms of cloths(X)” can be given?

a) $X \geq 8$ b) $X \leq 8$ c) $X \neq 8$ d) $X = 8$

B State whether each of the following statement is TRUE or FALSE

i) The maximum value of Z = 5x + 3y subjected to constraints $3x+y \leq 12$, $2x + 3y \leq 18$, $0 \leq x , y$ is 20.

ii) Objective function of LPP is a relation between the decision variables.

iii) If LPP has two optimal solutions then the LPP has infinitely many solution

iv) LPP is related to efficient use of limited resources

v) A convex set includes the points but not the segment joining the points.

vi) If the corner points of the feasible region are (0, 7/3),(2, 1),(3, 0) & (0,0) then the maximum value of $Z = 4x + 5y$ is 12 .

vii) If the corner points of the feasible region are (0, 10),(2, 2) & (4, 0) then the minimum value of $Z = 3x + 2y$ is at (4, 0)

viii) The half plane represented by $3x + 4y \geq 12$ includes the point (4,3)

ix) Corner point method is most suitable method for solving the LPP graphically

x) Of all the points of feasible region, the optimal value is obtained at the boundary of the feasible region

xi) The point (6,4) does not belong to the feasible region bounded by $8x + 5y \leq 60, 4x + 5y \leq 40, 0 \leq x, y$

xii) The Graphical solution set of the inequation $0 \leq y, x \geq 0$ lies in second quadrant.

C Fill in each of the following blanks

i) The variables involved in LPP are called _____________

ii) Constraints are always in the form of ____________ or___________

iii) A set of values of variables satisfying all the constraints of LPP is known as _____

iv) By spending almost ₹ 250 ,Rakhi bought some kg grapes(x) & some dozens of bananas(y), then as a constraint this information can be expressed by_______

v) Tyco Cycles Ltd manufactures bicycles(x) & tricycles(y). The profit earned from the sales of each bicycle & a tricycle is ₹400 & ₹200 respectively, then the total profit earned by the manufacturer will be given as ____________

vi) A doctor prescribed 2 types of vitamin tablets, T_1 & T_2 for Mr. Dhawan. The tablet T_1 contains 400 units of vitamin & T_2 contains 250 units of vitamin. If his requirement of vitamin is at least 4000 units then the inequation for his requirement will be _______

vii) The feasible region represented by the inequations $x \geq 0, y \leq 0$ lies in ______ quadrant.

viii) Heramb requires at most 400 calories from his breakfast. Every morning he likes to take oats & milk. If each bowl of oats & a glass of milk provides him 80 calories & 50 calories respectively, then as a constraint this information can be expressed as ____________

ix) Ganesh owns a godown used to store electronic gadgets like refrigerator(x) & microwave(y),If the godown can accommodate at most 75 gadgets, then this can be expressed as a constraint by _________________

x) Ms. Mohana want to invest at least ₹55000 in Mutual funds & fixed deposits, Mathematically this information can be written as ____________

xi) If the feasible region is bounded by the inequations $2x + 3y \leq 12$, $2x + y \leq 8$, $0 \leq x, 0 \leq y$ then point (5,4) is a

________ of the feasible region

xii) The constraint that in a particular XII class, number of boys are les s than number of girls is given by_______________

D Solve the following problems

i) A company manufactures 2 types of goods P & Q that requires copper & brass. Each unit of type P requires 2 grams of brass & 1 gram of copper while one unit of type Q requires 1 gram of brass & 2 grams of copper. The company has only 90 grams of brass & 80 grams of copper. Each unit of type P & Q brings profit of ₹400 & ₹500 respectively. Find the number of units of each type the company should produce to maximize its profit

ii) A dealer deals in two products X & Y. He has ₹1,00,000/- to invest & space to store 80 pieces. Product X costs ₹ 2500/- & product Y costs ₹ 1000/- per unit. Construct the LPP and find the number of units of each product to be purchased.

iii) A company manufactures two types of ladies dresses ,C & D. The raw material & labour available per day is given in the table.

Resources	Dress C(x)	Dress D(y)	Max. availability
Raw material	5	4	60
Labour	5	3	50

P is the profit, if P = 50x + 100y, solve this LPP to find x & y to get the maximum profit.

iv) Smita is a diet conscious house wife, wishes to ensure certain minimum intake of vitamin A, B & C for the family. The minimum daily needs of vitamin A, B & C for the family are 30, 20, & 16 units respectively. For the supply of the minimum vitamin requirements Smita relies on 2 types of foods F1 & F2. F_1 provides 7, 5 & 2 units of A, B, C vitamins per 10 grams & F_2 provides 2, 4 & 8 units of A, B & C vitamins per 10 grams. F_1 costs ₹ 3 & F_2 costs ₹ 2 per 10 grams. How many grams of each F1 & F_2 should buy every day to keep her food bill minimum.

v) A chemist has a compound to be made using 3 basic elements X, Y, Z so that it has at least 10 litres of X, 12 litres of Y & 20 litres of Z. He makes this compound by mixing two compounds (I) & (II). Each unit compound (I) had 4 litres of X, 3 litres of Y. Each unit compound (II) had 1 litres of X, 2 litres of Y & 4 litres of Z. The unit costs of compound (I) & (II) are ₹400 & ₹600 respectively. Find the number of units of each compound to be produced so as to minimize the cost.

vi) A wholesale dealer deals in two kinds of mixtures A & B of nuts. Each kg of mixture A contains 60 grams of almonds, 30 grams of cashew & 30 grams of hazel nuts. Each kg of mixture B contains 30 grams of almonds, 60 grams of cashew & 180 grams of hazel nuts. A dealer is contemplating to use mixtures A & B to make a bag which will contain at least 240 grams of almonds, 300 grams of cashew and 540 grams of hazel nuts Mixture A costs ₹8 & B costs ₹12 per kg. How many kgs of each mixture should he use to minimize the cost of the kgs.

vii) Maximize Z =2x + 3y subject to constraints

$x + 4y \leq 8$, $3x + 2y \leq 14$, $x \geq 0$, $y \geq 0$

viii) Maximize $Z = 5x + 10y$ subject to constraints

$x + 2y \leq 10$, $3x + y \leq 12$, $x \geq 0$, $y \geq 0$

ix) Maximize $Z = 400x + 500y$ subject to constraints

$x + 2y \leq 80$,$2x + y \leq 90$, $x \geq 0$, $y \geq 0$

x) Minimize $Z = 24x + 40y$ subject to constraints

$6x + 8y \geq 96$, $7x + 12y \geq 168$, $x \geq 0$, $y \geq 0$

xi) Minimize $Z = x + 4y$ subject to constraints

$x + 3y \geq 3$, $2x + y \geq 2$, $x \geq 0$, $y \geq 0$

xii) Minimize $Z = 2x + 3y$ subject to constraints

$x + y \geq 6$, $2x + y \geq 7$, $x + 4y \geq 8$, $x \geq 0$, $y \geq 0$

E Activities

i) Amartya wants to invest ₹45,000 in Indira Vikas Patra (IVP) & in Public Provident fund (PPF). He wants to invest at least ₹ 10,000 in PPF & at least ₹5000 in IVP. If the rate of interest on PPF is 8% per annum & that on IVP is 7% per annum. Formulate the above problem as LPP to determine maximum yearly income.

Solu: Let x be the amount (in ₹) invested in IVP & y be the amount (in ₹) invested in PPF

$x \geq 0$, $y \geq 0$

As per the given condition, $x + y$ ___ 45000

He wants to invest at least ₹ 10,000 in PPF

$\therefore$ y __ 10000

Amartya wants to invest at least ₹5000 in IVP

∴ x __ 5000

Total interest(Z) = ____________

The formulated LPP is

Maximize Z = ___________   subject to

____

ii) Solve the following LPP graphically :

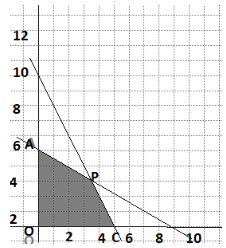
Maximize Z = 9x + 13y subject to constraints

$2x + 3y \leq 18$, $2x + y \leq 10$, $x \geq 0$, $y \geq 0$

Solu: Convert the constraints into equations & find the intercept made by each one of it.

Inequation	Equation	X intercept	Y intercept	Region
$2x + 3y \leq 18$	$2x + 3y = 18$	(9, 0)	(0,--)	Towards origin
$2x + y \leq 10$	$2x + y = 10$	(--, 0)	(0,10)	Towards origin
$x \geq 0$, $y \geq 0$	X =0, y = 0	X axis	Y axis	--------------

The feasible region is OAPC   Where O(0,0), A(0,6), P(--,--),C(5,0)



The optimal solution is in the following table

Point	Coordinates	Z = 9x + 13y	Values	Remark
O	(0,0)	9(0)+13(0)	0	
A	(0,6)	9(0)+13(6)	-----	
P	(--,--)	9(--)+13(--)	-----	-------------
C	(5,0)	9(5)+13(0)	-----	

∴ Z is maximum at __ (__ , __) with the value __

iii) Solve the LPP graphically:

Minimize Z = 4x + 5y

Subject to the constraints $5x + y \geq 10$, $x + y \geq 6$, $x + 4y \geq 12$, $x , y \geq 0$

Solu: Convert the constraints into equations & find the intercept made by each one of it.

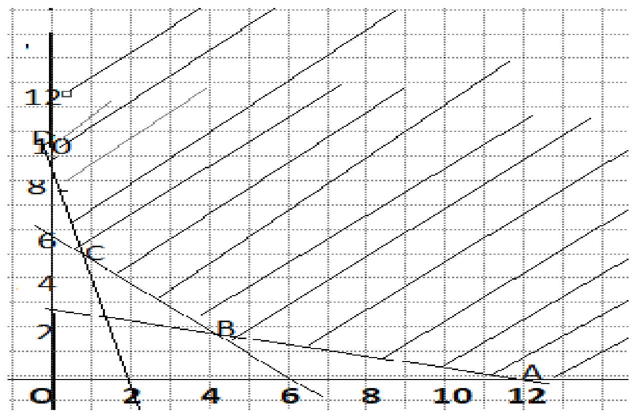
Inequations	Equations	X intercept	Y intercept	Region
$5x + y \geq 10$	$5x + y = 10$	(--,0)	(0, 10)	Away from origin
$x + y \geq 6$	$x + y = 6$	(6, 0)	(0,--)	Away from origin

x + 4y ≥ 12	x + 4y = 12	(12, 0)	(0, 3)	Away from origin
x , y ≥ 0	X =0, y = 0	X = 0	Y = 0	Ist quadrant

∵ Origin has not satisfied the inequation

∴Solution of the inequations

is away from origin.



The feasible region is unbounded

area which is satisfied by all

constraints

In the figure, ABCD represents

The set of feasible solution where

A (12,0), B (--, --), C (--, --) &

D (0, 10)

The coordinates of B are

Obtained by solving equations

x + 4y = 12 & x + y = 6

The coordinates of C are

Obtained by solving equations

$5x + y = 10$ & $x + y = 6$

Hence the optimum solution lies at the extreme points

The optimal solution is in the following table

Point	Coordinates	Z = 4x + 5y	Values	Remark
A	(12,0)	4(12) + 5(0)	48	
B	(--,--)	4(--) + 5(--)	-----	------------
C	(--,--)	4(--) + 5(--)	-----	
D	(0, 10)	4(0) + 5(10)	50	

∴ Z is maximum at __ (__ , __) with the value __ .

7. ASSIGNMENT PROBLEMS AND SEQUENCING

I) Select and write the most appropriate answer from the given alternatives for each sub question. [1 mark each]

1. The assignment problem is generally defined as a problem of …
   (a) maximization (b) minimization (c) allocation (d) restriction

2. Assignment Problem is special case of ….
   (a) Linear Programming Problem (b) Unbalanced Problem
   (c) Restricted Problem (d) Sequencing Problem

3. The assignment problem is solved by ….
   (a) Simplex method (b) Hungarian method
   (c) vector method (d) Graphical method

4. The cost matrix of an unbalanced assignment problem is not a …
   (a) Unit matrix (b) triangular matrix
   (c) rectangular matrix (d) square matrix

5. When an assignment problem has more than one solution, then it is…
   (a) double optimal solution (b) infinite optimal solution
   (c) multiple optimal solution (d) dual optimal solution

6. The assignment problem is said to be balanced if…
   (a) no. of rows = no. of columns (b) no. of rows ≠ no. of columns
   (c) no. of rows < no. of columns (d) no. of rows> no. of columns

7.

Machine	Job			
	A	B	C	D
I	5	6	8	4
II	4	7	9	10

The optimal sequence for above data is
   (a) CDBA (b) DBCA (c) BCDA (d) ABCD

8. In sequencing, an optimal path that minimizes ….
   (a) Elapsed time (b) Idle time
   (c) Both (a) and (b) (d) Ready time

9. If there are 3 machines A, B and C, conditions for reducing a 3 machine problem to a 2 machine problem with respect to minimum processing time is…
   (a) Min $A_i \geq$ Max B_i **OR** Min $C_i \geq$ Max B_i, i=1,2,3…n.
   (b) Min $A_i \leq$ Max B_i **OR** Min C $\leq$ Max B_i, i=1,2,3,...n.
   (c) Max $A_i \geq$ Min B_i **OR** Max B $\geq$ Min A_i, i=1,2,3,…n.
   (d) Max $A_i \leq$ Min B_i **OR** Max B $\leq$ Min Ai, i=1,2,3,…n.

10. The objective of sequencing problem is
   (a) to find the order in which jobs are to be made
   (b) to find the time required for the completing all the job on hand
   (c) to find the sequence in which jobs on hand are to be processed to minimize the total time required for processing the jobs.
(d) to maximization the cost.

11. If there are n jobs and m machines, then there will be …. sequence of doing jobs.
   (a) mn   (b) m(n!)   (c) n^m (d) $(n!)^m$

12. In solving 2 machine and n jobs sequencing problem, the following assumption is wrong
   (a) No passing is allowed
   (b) Processing times are known
   (c) Handling time is negligible
   (d) The time of passing depends on the order of machining.

II) Fill in the blanks. [1 mark each]

1. If the given matrix is …… matrix, the assignment problem is called balanced problem.

2. An unbalanced assignment problems can be balanced by adding dummy rows or columns with …… cost.

3. A …. assignment problem does not allow some worker(s) to be assign to some job(s).

4. In an assignment problem if number of rows is greater than number of columns, then dummy ….. is added.

5. The Hungarian method is an …. algorithm that solves an assignment problem.

6. In assignment problem each worker or machine is …… capable of handling any job.

7. In sequencing problems one has to …. the total processing time or cost.

8. In sequencing problem the time which required to complete all the jobs i.e. entire task is called ….

9. ….. time is the time when the machine is available but is waiting for a job to be processed.

10. In sequencing problem the time required to transfer a job from one machine to another is…..

11.

Books	A	B	C	D
Printing	5	8	10	7
Data Entry	7	4	3	6

The optimum sequence for the above data is …….

12. In sequencing problem one has to determine the …. in which jobs has to processed through the machines.

III) State whether each of the following is true or false.[1 mark each]

1. The objective of an assignment problem is to assign number of jobs to equal number of persons at maximum cost.

2. To convert the assignment problem into maximization problem, the smallest element in the matrix is to deducted from all other elements.

3. Optimal assignments are made in the Hungarian method to cells in the reduced matrix that contain a zero.

4. In assignment problem, if number of column is greater than number of rows, then a dummy row is added.

5. The Hungarian method is used to assign n jobs on 2 machines to get the optimal sequence.

6. In assignment problem each worker or machine is assigned only one job.

7. One of the assumptions is made while sequencing n jobs on 2 machines is: two jobs must be loaded at a time on any machine.

8. The Total Elapsed is the time required to complete all the jobs i.e. entire task.

9. The idle time for a machine is the when machine is available but is not waiting for any job to be processed.

10. In sequencing problem the processing times are dependent of order of processing the jobs on machine.

11. In sequencing problem each job once started on any machine must be processed still its completion.

12. In sequencing problem each machine is of different type.

IV) Attempt the following questions.[3 or 4 marks each]

1. Four new machines M_1, M_2, M_3 and M_4 are to be installed in machine shop. There are five vacant places A, B, C, D and E available. Because of limited space, machine M_2 cannot be placed at C and M_3 cannot be placed at A. The cost of matrix given below:

Machines	**Places**				
	A	**B**	**C**	**D**	**E**
M_1	4	6	10	5	6
M_2	7	4	-	5	4
M_3	-	6	9	6	2
M_4	9	3	7	2	3

Find the optimal assignment schedule.

2. Solve the following assignment problem to maximize sales:

Salesmen	**Territoris**				
A	**I**	**II**	**III**	**IV**	**V**
B	11	16	18	15	15
C	7	19	11	13	17
D	9	6	14	14	7
E	13	12	17	11	13

3. Consider the problem of assigning of five operators to five machines. The assignment costs are given in the following table:

Operator	**Machine**				
	1	**2**	**3**	**4**	**5**
A	6	6	-	3	7
B	8	5	3	4	5
C	10	4	6	-	4
D	8	3	7	8	3
E	7	6	8	10	2

4. Find the assignments of salesman to various district which will yield maximum profit.

Salesman	District			
	1	2	3	4
A	16	10	12	11
B	12	13	15	15
C	15	15	11	14
D	13	14	14	15

5. A dairy plant has five milk tankers, I, II, III, IV and V. Three milk tankers are to be used on five delivery routes A, B, C, D & E. The distances (in kms) between the diary plant and the delivery routes are given in the following distance matrix.

	I	II	III	IV	V
A	150	120	175	180	200
B	125	110	120	150	165
C	130	100	145	160	170
D	40	40	70	70	100
E	45	25	60	70	95

How should the milk tankers be assigned to the chilling center so as to minimize the distance travelled?

6. Find the sequence that minimizes the total elapsed time to complete the following jobs. Each job is processed in the order AB:

Machines	Jobs (Processing times in minutes)						
	I	II	III	IV	V	VI	VII
Machine A	12	6	5	11	5	7	6
Machine B	7	8	9	4	7	8	3

Determine the sequence for the jobs so as to minimize the processing time. Find the total elapsed time and the idle times for both the machines.

7. Find the optimal sequence that minimizes total time required to complete the following jobs in the order ABC. The processing times are given in hours:

Jobs	I	II	III	IV	V
Lathe	4	1	5	2	5
Surface Grinder	3	2	4	2	6

Jobs	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**
Machine A	6	7	5	11	6	7	12
Machine B	4	3	2	5	1	5	3
Machine C	3	8	7	4	9	8	7

8. Five jobs must pass through a lathe and a surface grinder, in that order. The processing time in hours are shown below. Determine the optimal sequence of the jobs. Also, find the idle time of each machine:

9. A toy manufacturing company produces five types of toys. Each toy has to go through three machines A, B, C in the order ABC. The time required in hours for each process is given in the following table:

Type	**1**	**2**	**3**	**4**	**5**
Machine A	16	20	12	14	22
Machine B	10	12	4	6	8
Machine C	8	18	16	12	10

Solve the problem for minimizing the total elapsed time.

10. Find the sequence that minimizes total elapsed time to complete the following jobs in the order XY. Find the total elapsed time and idle times for each machine.

Jobs	**A**	**B**	**C**	**D**	**E**
Machine X	10	2	18	6	20
Machine Y	4	12	14	16	8

V) Activities [4 marks each]

1. For the following assignment problem minimize total man hours:

Subordinates	**Required hours for task**			
	I	**II**	**III**	**IV**
A	7	25	26	10
B	12	27	3	25
C	37	18	17	14
D	18	25	23	9

Subtract the ☐ element of each ☐

From
every element of that ☐

Subordinates	**Required hours for task**			
	I	**II**	**III**	**IV**
A	0	18	19	3
B	9	24	0	22
C	23	4	3	0
D	9	16	14	0

Subtract the smallest element in each column from ☐

of that column.

Subordinates	**Required hours for task**			
	I	**II**	**III**	**IV**
A	□	□	19	□
B	□	□	0	□
C	□	□	3	□
D	□	□	14	□

The lines covering all zeros is ______ to the order of matrix ______

The assignment is made as follows:

subordinates	**Required hours for task**			
	I	**II**	**III**	**IV**
A	0	14	9	3
B	9	20	0	22
C	23	0	3	0
D	9	12	14	0

Optimum solution is shown as follows:

A→□, □→III, C→□, □→IV

Minimum hours required is ______ hours

2. Solve the following problem of sequencing for minimizing the total elapsed time and idle time for both the machines.

Job	**P**	**Q**	**R**	**S**	**T**	**U**
M_1	1	4	6	3	5	2
M_2	3	6	8	8	1	5

The optimal sequence of the jobs as follows:

Total elapsed time is obtained as follows:

Job sequence	**Machine A**		**Machine B**		**Idle time for Machine B**
	Time In	**Time Out**	**Time In**	**Time Out**	
P	□	1	1	□	□
U	□	3	4	□	□
S	□	6	9	□	□
Q	□	10	17	□	□
R	□	16	23	□	□
T	□	21	31	□	□

Total elapsed time T = □ minutes
Idle time for Machine A = T- □ = □ minutes
Idle time for Machine B = □

8. PROBABILITY DISTRIBUTIONS

I) Choose the correct alternative.

1) The variance of a Binomial distribution is given by

a) np b) pq c) npq d) $\sqrt{npq}$

2) F(x) is c.d.f. of discrete r.v. X whose distribution is

x_i	-2	-1	0	1	2
p_i	0.2	0.3	0.15	0.25	0.1

then F(-3) = ...

a) 0 b) 1 c) 0.2 d) 0.15

3) X : is number obtained on upper most face when a fair die is

thrown then E(x) = ……….

a) 3.0   b) 3.5   c) 4.0   d) 4.5

4) If p.m.f. of r. v. X is given below.

x	0	1	2
$P(x)$	q^2	$2pq$	p^2

then Var(x) = ...

a) p^2 (b) q^2 (c) pq (d) $2pq$

5) The expected value of the sum of two numbers obtained when two fair dice are rolled is ....

a) 5   b) 6   c) 7   d) 8

6) If X ~ B (20, $\frac{1}{10}$) , then E (x) =.............

a) 2   b) 5   c) 4   d) 3

7) A sequence of dichotomous experiments is called a sequence of Bernoulli trials if it satisfies ----------

a) The trials are independent.
b) The probability of success remains the same in all trials.
c) a but not b
d) both a and b

8) For the Poisson distribution

a) Mean= E(X) = m
b) Var(X) = m
c) Mean = E(X) = m and Var(X) = m

d) Mean = E(X) $\neq$ m and Var (X) = m.

9) A discrete random variable X is said to have the Poisson distribution with parameter m if its p.m.f. is given by P(x) = $\frac{e^{-m}m^{x}}{x\,!}$ the condition for m is --

a) m > 0 b) m ≥ 0 c) m $\neq 1$ d) m = 0.

II) Fill in the blanks.

1. The values of discrete r.v. are generally obtained by ....
2. The values of continuous r.v.are generally obtained by ....
3. If X is discrete random variable takes the values $x_1, x_2, x_3, \ldots x_n$ then $\sum_{i=1}^{n} P(x_i) = \cdots$ ^{n}
4. E(x) is considered to be...........of the probability distribution of x.
5. In Binomial distribution probability of success ------------------ from trial to trial.
6. In Binomial distribution if n is very large and probability of success of p is very small such that $np = m$ (constant) then.............. distribution is applied
7. When n is very large and p is very small in the binomial distribution, then X follows the Poisson distribution with parameter m = ……

III) State whether each of the following is True or False.

1) X is the number obtained on upper most face when a die is thrown then E(x) = 3.5.

2) If $f(x) = k\,x\,(1 - x)$, for $0 < x < 1$

$= 0$, otherwise , is the p.d.f. of a r. v. X then k = 12 .

3) If X ~ B(n , p) and $n = 6$ and P(x =4) = P(x = 2) then $p = \frac{1}{2}$

4) If a r. v. X assumes the values 1,2,3,…………,9 with equal probabilities then E(x) = 5.

5) Let X ~ B(n,p) then the mean or expected value of r. v. X is denoted by E(X). It is also denoted by $E(X)$ and is given by μ= E(X) = npq .

6) A discrete random variable X is said to follow the Poisson distribution with parameter m $\geq$ 0 if its p. m. f. is given by P(X = x) = $\frac{e^{-m}m^{x}}{x}$, x = 0, 1, 2, ...

7) For the Binomial distribution , Mean E(X) = m and Variance = Var(X) = m.

8) If n is very large and p is very small then X follows Poisson distribution with n = m p.

9) The cumulative distribution function (c.d.f.) of a continuous random variable X is denoted by F and defined by

$$F(x) = 0 \quad \text{for all } x \leq a,$$
$$= \int_a^x f(x)\,dx \quad \text{for all } x \geq a$$

Solve the following.

3 MARKS

1. Find the probability distribution of a) number of heads in two tosses of a coin, b) number of tails in three tosses of a coin c)

number of heads in four tosses of a coin.

2.A sample of 4 bulbs is drawn at random with replacement from a lot of 30 bulbs which includes 6 defective bulbs . Find the probability distribution of the number of defective bulbs.

3. Find the expected value and variance of X using the following p. m. f.

x	-2	-1	0	1	2
$P(x)$	0.2	0.3	0.1	0.15	0.25

4. Find the mean of number of heads in three tosses of a fair coin.

5. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

6. A pair of dice is thrown 3 times. If getting a doublet is considered a success, find the probability of two successes.

7. Given that $X \sim B(n,p)$ if $n = 10$ and $p = 0.4$, find $E(X)$ and $\text{Var}(X)$.

8. If X has Poisson distribution with m=1, then find $P(X \leq 1)$ given $e^{-1} = 0.3678$.

9. If X has Poisson distribution with parameter m and $P(X = 2) = P(X = 3)$, then find $P(X \geq 2)$.

Use $e^{-3} = 0.0497$.

4 MARKS

1 .A random variable X has the following probability distribution:

x	1	2	3	4	5	6	7
$P(x)$	k	$2k$	$2k$	$3k$	$k2$	$2k^2$	$7k^2 + k$

Determine (i) k, (ii) $P(X < 3)$,(iii) $P(0 < X < 3)$, (iv) $P(X > 4)$.

2. The p. d. f. of a continuous r. v. X is

$$f(x) = \begin{cases} \frac{3x^2}{8} & , 0 < x < 2 . \\ 0 , otherwise & . \end{cases}$$

Determine the c.d.f. of X and hence find

(i)$P(X<1)$,(ii)$P(X<-2)$,(iii)$P(X>0)$,(iv) $P(1 < X < 2)$.

3. If a r. v. X has p. d.f. $f(x) = \begin{cases} \frac{c}{x} & , 1 < x < 3 \ , \ c > 0 \\ 0 , otherwise . \end{cases}$

Find c, $E(X)$, and Var (X). Also find $F(x)$.

4. A die is thrown 4 times. If 'getting an odd number' is a success , find the probability of

   i) 2 successes (ii) atleast 3 successes (iii) at most 2 successes.

5. The probability that a bulb produced by a factory will fuse after 200 days of use is 0.2. Let X denote the number of bulbs (out of 5) that fuse after 200 days of use . Find the probability of (i) $X = 0$, (ii) $X \leq 1$, (iii) $X > 1$, (iv) $X \geq 1$.

6.The number of complaints which a bank manager receives per day follows a Poisson distribution with parameter m = 4. Find the probability that the manager receives

a) only two complaints on a given day, b) at most two complaints on a given day.

Use e^{-4} =0.0183.

7. Defects on plywood sheet occur at random with the average of one defect per 50 Sq. ft. Find the probability that such a sheet has (i) no defect, (ii) at least one defect.

Use e^{-1} =0.3678.

8.It is known that, in a certain area of a large city, the average number of rats per bungalow is five. Assuming that the number of rats follows Poisson distribution, find the probability that a randomly selected bungalow has

i) exactly 5 rats (ii) more than 5 rats (iii) between 5 and 7 rats, inclusive.

Given e^{-5} =0067.

ACTIVITIES .

1. The probability distribution of a discrete r. v. X is as follows.

x	1	2	3	4	5	6
$P(X = x)$	k	$2k$	$3k$	$4k$	$5k$	$6k$

Complete the following activity .

Solution :

Since $\sum p_i = 1$

i) k = $\boxed{}$

$\boxed{}$ $\boxed{}$

ii)$P(X \leq 4) =$ $\quad + \boxed{\quad} + \quad + \boxed{\quad} = \boxed{\quad}$

iii)$P(X \geq 3) = \boxed{\quad} - \boxed{\quad} - \boxed{\quad} = \boxed{\quad}$

2. Using the following activity , find the expected value and variance of the r. v. X if its probability distribution is as follows.

x	1	2	3
$P(X = x)$	1/5	2/5	2/5

<u>Solution :</u>

$\mu = E(X) = \sum_{i=1}^{3} x_i p_i$

$E(X) = \boxed{\quad} + \boxed{\quad} + \boxed{\quad} = \boxed{\quad}$

$\text{Var}(X) = E(X^2) - \{E(X)\}^2 = \sum X_i^2 P_i - [\sum X_i P_i]^2 = \boxed{\quad} - \boxed{\quad}$

$= \boxed{\quad}$

3. Let X ~ B(n , p) , If n = 10 and E(X) = 5 , Using the following activity find p and Var (X).

<u>Solution</u> : $E(X) = \boxed{\quad} = 5$ $\boxed{\quad}$ $p = \boxed{\quad}$, $q = \boxed{\quad}$

$\text{Var}(X) = \boxed{\quad} = \boxed{\quad}$

4. The probability that a bomb will hit the target is 0.8. Using the following activity ,find the probability that ,out of 5 bombs , exactly 2 will miss the target.

Solution : Let p = probability that bomb miss the target $\therefore$ q = $\boxed{}$, p = $\boxed{}$, n = 5 .

X ~ B (5 ,) , P(x) = $n_{c_x}\, p^x\, q^{n-x}$.

P (X = 2) = $5_{C_2}\boxed{}$ = $\boxed{}$

5. If X follows Poisson distribution such that P (X = 1) = 0.4 and P (X = 2) = 0.2, Using the following activity find the value of m .

Solution : X : Follows Poisson distribution . $\therefore$P(x) = $\frac{e^{-m}m^x}{x\,!}$, $P(X = 1) = 0.4$ and $P(X = 2) = 0.2$.

$\therefore$ $P(X = 1) = \boxed{}\ P(X = 2)$.

$\frac{e^{-m}m^1}{1\,!} = \boxed{}\ \frac{e^{-m}m^2}{2\,!}$, $e^{-m} = \boxed{}\ e^{-m}\ \frac{m}{2}$, m ≠ 0

$\boxed{}$

ANSWER KEY :

PART I

MATHEMATICAL LOGIC

Q1) i) c ii) c iii) c iv) b v) b vi) c vii) d viii) b ix) b x) b

Q2) i) F ii) F iii) F iv) T v) F vi) T vii) T viii) T ix) T x) F

Q3) i) $p \wedge q$ ii) All men are not animals iii) False iv) True v) $q \rightarrow \sim p$

Q 4) i) An angle is a right angle and it is not of measure 90^0 , or an angle is of measure 90^0 and it is not a right angle.

ii) a) $p \leftrightarrow \sim q$ b) $p \rightarrow q$ c) $\sim p \wedge q$

iii) a) $\forall$ n ε N , $n^2 \geq 1$ is a true statement

b) **∃ x ε** N such that $3x - 4 < 9$ is a true statement

c) **∃ y ε** N such that $y + 4 > 6$ is a true statement

iv) Contingency

v) proof

vi) a) 13 is a prime number or India is a democratic country

b)) $(p \vee \sim q) \wedge (\sim p \vee q) \equiv (p \wedge q) \vee \sim (p \vee q)$

vii) Converse: If they do not drive the car, then it snows

Inverse: If it does not snow, then they drive the car

Contrapositive: If they drive the car, then it does not snow

Q 5 i) Contradiction

ii) Proof

iii) Proof

iv) [(p ∧~q) v (q ∧~p)] ∧ (~q ∧ r)

v)

P	q	r	~r	P ∧q	(p ∧q) v ~r
T	T	T	F	T	T
T	T	F	T	T	T
T	F	T	F	F	F
T	F	F	T	F	T
F	T	T	F	F	F
F	T	F	T	F	T
F	F	T	F	F	F
F	F	F	T	F	T

vi) Solution

p ↔q and p →q are true if p and q has $\boxed{T}$ t $\boxed{T}$ al $\boxed{F}$ $\boxed{F}$ or

i) P v q

a) If both p and q are true then

P v q = V $\boxed{T}$ = $\boxed{T}$ $\boxed{T}$

b) If both p and q are false, then

P v q = V $\boxed{F}$ = $\boxed{F}$ $\boxed{F}$

ii) p ∧ q

a) If both p and q are true then

$P \wedge q = \wedge \quad \wedge \boxed{T} \quad \boxed{T} \quad \boxed{T}$

b) If both p and q are false then $\boxed{F}$

$P \wedge q = \quad \wedge \boxed{F} \quad \boxed{F}$

vii)

i) $(p \wedge q) \wedge r = \boxed{T} (\quad \boxed{T} \wedge \quad \boxed{F}) \wedge \quad)$

$= \quad \wedge \boxed{T} \quad \boxed{F}$

$= \quad \boxed{F}$

ii) $\sim (p \wedge r) = \boxed{T} (\quad \boxed{F} \wedge \quad)$

$= \sim \quad = \boxed{F} \quad \boxed{T}$

iii) $P \rightarrow q = \boxed{T} \quad \boxed{T} \quad \boxed{T} \quad =$

viii)

p	q	r	q→r	r→p	(q→r)v (r→p)
T	T	T	T	$\boxed{T}$	T
T	T	F	F	$\boxed{T}$	$\boxed{T}$
T	F	T	T	$\boxed{T}$	T
T	F	F	T	$\boxed{T}$	$\boxed{T}$
F	T	T	$\boxed{T}$	F	T

F	T	F	F	T	T
F	F	T	T	F	T
F	F	F	T	T	T

The given statement pattern is a Tautology

2. Matrices

(I)	Answers	(II)	Answers	(III)	Answers
1)	b) p=0 q=6	1)	False	1)	$A=\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}$
2)	a) k=1	2)	True	2)	Zero
3)	d) x= - 4 and y=$\frac{3}{5}$.	3)	False	3)	$A-3B=\begin{bmatrix}-2 & 7\\ -7 & 24\end{bmatrix}$
4)	c) [1]	4)	True	4)	$AB=\begin{bmatrix}3 & 4\\ -3 & -2\end{bmatrix}$ $\lvert AB\rvert=6$
5)	b) I	5)	True	5)	-2
6)	c) Symmetric matrix	6)	True	6)	2
7)	a) $B^T A^T$	7)	False	7)	$A=\begin{bmatrix}3 & -1\\ 4 & 1\end{bmatrix}$
8)	c)	8)	False	8)	R_2-2R_1
9)	a)	9)	False	9)	$\lvert adj(A)\rvert=\lvert A\rvert^2=2^2=4$
10)	a)	10)	True	10)	$\lvert A\rvert=7$

	Solve the following…. [3 Marks]
1)	a=2 and b=-1
2)	$A=\begin{bmatrix} 3 & \frac{-14}{3} & -8 \\ -2 & 1 & 3 \end{bmatrix}$ $\mathbf{B}=\begin{bmatrix} 0 & \frac{-10}{3} & \frac{-16}{3} \\ 0 & 0 & 5 \end{bmatrix}$
3)	
4)	k=1
5)	
6)	$A^{-1}=\frac{1}{40}\begin{bmatrix} 19 & 5 & -27 \\ -2 & 10 & -14 \\ -3 & -5 & 19 \end{bmatrix}$
7)	
8)	
	Solve the following….[4 Marks]
1)	$A^{-1}=\begin{bmatrix} \frac{2}{3} & \frac{-1}{3} & \frac{1}{3} \\ \frac{-1}{2} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{3} & \frac{-1}{3} \end{bmatrix}$
2)	$x=\frac{26}{7}, \quad y=\frac{30}{7}$
3)	$x=\frac{1}{6}, \quad y=\frac{-1}{3}, \quad z=\frac{5}{6}$
4)	X=1, y=2, z=3
5)	Cost of T.V is ₹ 3000 and selling price is ₹ 4000 where as Cost price of VCR is ₹ 13000 and its selling price is ₹ 13500.
	ACTIVITIES
1)	Given $A=\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ then $\lvert A\rvert = 2(3)-0(15)+(-1\ )(5)=6-0-5=1\neq 0$

Cofactor of all elements of matrix A are

$$A_{11}=(-1)^2\begin{vmatrix}(1) & (0)\\ (1) & (3)\end{vmatrix}=(3),\quad A_{12}=(-1)^3\begin{vmatrix}5 & (0)\\ (0) & 3\end{vmatrix}=-15,$$

$$A_{13}=(-1)^4\begin{vmatrix}5 & (1)\\ (0) & 1\end{vmatrix}=5,\quad A_{21}=-1,\quad A_{22}=6,\quad A_{23}=-2,$$

$$A_{31}=(-1)^4\begin{vmatrix}(0) & (-1)\\ (1) & (0)\end{vmatrix}=(1),\quad A_{32}=(-1)^5\begin{vmatrix}2 & (-1)\\ (5) & 0\end{vmatrix}=(-5),$$

$$A_{33}=(-1)^6\begin{vmatrix}2 & (0)\\ (5) & 1\end{vmatrix}=2.$$

Cofactor of matrix A= $\begin{bmatrix}3 & (-15) & 5\\ -1 & 6 & -2\\ 1 & -5 & 2\end{bmatrix}$, adj(A)= $\begin{bmatrix}3 & -1 & 1\\ -15 & 6 & -5\\ 5 & -2 & 2\end{bmatrix}$

A. adj(A)= $\begin{bmatrix}2 & 0 & -1\\ 5 & 1 & 0\\ 0 & 1 & 3\end{bmatrix}.\begin{bmatrix}(3) & -1 & 1\\ -15 & (6) & -5\\ (5) & -2 & (2)\end{bmatrix}=\begin{bmatrix}1 & 0 & (0)\\ (0) & (1) & (0)\\ 0 & (0) & (1)\end{bmatrix}=|A|I$

2)

$$\begin{bmatrix}2 & 0 & -1\\ 5 & 1 & 0\\ 0 & 1 & 3\end{bmatrix}B^{-1}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$C_1+C_3$$

$$\begin{bmatrix}1 & 0 & -1\\ 5 & 1 & 0\\ 3 & 1 & 3\end{bmatrix}B^{-1}=\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 1 & 0 & 1\end{bmatrix}$$

$$C_3+C_1$$

	$$\begin{bmatrix}1&0&0\\5&1&5\\3&1&6\end{bmatrix}B^{-1}=\begin{bmatrix}1&0&1\\0&1&0\\1&0&2\end{bmatrix}$$ $C_1-5C_2,\ C_3-5C_2$ $$\begin{bmatrix}1&0&0\\0&1&0\\-2&1&1\end{bmatrix}B^{-1}=\begin{bmatrix}1&0&1\\-5&1&-5\\1&0&2\end{bmatrix}$$ $C_1+2C_3,\ C_2-C_3$ $$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}B^{-1}=\begin{bmatrix}3&-1&1\\-15&6&-5\\5&-2&2\end{bmatrix}$$ $$B^{-1}=\begin{bmatrix}3&-1&1\\-15&6&-5\\5&-2&2\end{bmatrix}$$ $$\begin{bmatrix}2&0&-1\\5&1&0\\0&1&3\end{bmatrix}.\bullet\begin{bmatrix}3&-1&1\\-15&6&-5\\5&-2&2\end{bmatrix}=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$
3)	The cost of 4 kg potato, 3kg wheat and 2kg rice is ₹ 60 . The cost of 1 kg potato, 2 kg wheat and 3kg rice is ₹45. The cost of 6 kg potato, 3 kg rice and 2 kg wheat is ₹ 70. Find the per kg cost of each item by matrix method. **Solution:** Let the cost of potato, wheat and rice per kg are x, y and z respectively. Therefore by given condition,

$$4x+3y+2z=60$$
$$x+2y+3z=45$$
$$6x+2y+3z=70$$

Matrix form of above equation is,

$$\begin{bmatrix} 4 & 3 & 2 \\ 1 & (2) & 3 \\ (6) & 2 & (3) \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} (60) \\ 45 \\ (70) \end{bmatrix}$$

$$R_1 \leftrightarrow R_2$$

$$\begin{bmatrix} 1 & 2 & 3 \\ (4) & (3) & (2) \\ 6 & 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} (45) \\ 60 \\ (70) \end{bmatrix}$$

$$R_2-4R_1,\quad R_3-6R_1$$

$$\begin{bmatrix} 1 & 2 & 3 \\ (0) & -5 & (-10) \\ 0 & (-10) & -15 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 45 \\ (-120) \\ -200 \end{bmatrix}$$

$$\frac{-1}{5}R_2,\quad \frac{-1}{5}R_3$$

$$\begin{bmatrix} (1) & 2 & 3 \\ 0 & (1) & 2 \\ 0 & 2 & (3) \end{bmatrix}\begin{bmatrix} x \\ (y) \\ z \end{bmatrix}=\begin{bmatrix} 45 \\ 24 \\ 40 \end{bmatrix}$$

$$R_3-2R_2$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} (45) \\ (24) \\ (40) \end{bmatrix}$$

By Re multiplying we get,

$x+2y+(3)z=(45)$.......(1) $y+2z=24$.......(2) $-z=(-8)$......(3)

From (3), we get, z=(8)

From (2), we get, y=(8)

From (3), we get, x=(5)

3. Differentiation

I. Choose the correct alternatives :

1	2	3	4	5	6	7	8	9	10	11
d	a	b	b	a	d	a	b	a	a	b

II. Fill in the blanks :

1. $9(5x^3-4x^2-8x)^8(15x^2-8x-8)$
2. $a^{(1+logx)}\ loga\ .\frac{1}{x}$
3. $\frac{dx}{dy} = \frac{1}{(-x^2e^{-x}+2xe^{-x}+2)}$
4. $\frac{e^x}{1-x}$
5. $10x^9$
6. $2e^{(2x+5)}$
7. $-\sqrt{\frac{y}{x}}$
8. $x.5^x log5$
9. $x.e^x$
10. 2

III. State whether each of the following is True or False :

1	2	3	4	5	6	7	8	9	10	11
False	True	True	False	True	False	False	False	False	True	True

IV. Solve the following : (3 Marks)

1. $\frac{2\log[\log(logx)]}{x.logx.\log(logx)}$
2. $10(6x^3-3x^2-9x)^9\ (18x^2-6x-9)$
3. $-\frac{(2x-13)^2}{79}$
4. $\frac{dx}{dy} = \frac{1}{(-x^2e^{-x}+2xe^{-x}+2)}$
5. $\frac{y}{x}\left(\frac{x.logy-y}{y.logx-x}\right)$
6. $-\frac{y}{x}$
7. $\frac{2}{\sqrt{1+u^2}}$
8. t^t
9. $4.e^{(2x+1)}$

V. Solve the following : (4 Marks)

1.$(logx)^x\left[\frac{1}{logx}+\log(logx)\right]-x^{logx}\left[\frac{2logx}{x}\right]$ 6.

$\frac{1}{3}\sqrt[3]{\frac{(3x-1)}{(2x+3)(5-x)^2}}\left[\frac{3}{3x-1}-\frac{2}{2x+3}+\frac{2}{5-x}\right]$

2. $\frac{4}{5}(3x^2+8x+5)^{\frac{-1}{5}}(6x+8)$ 7. $\frac{dy}{dx}=\ \frac{y}{x}$

3. x^x (1+logx) + $(7x-1)^x\left[\log(7x-1)+\frac{7x}{7x-1}\right]$ 8. $\frac{dy}{dx}=\ \frac{y}{x}$

4. $\frac{dx}{dy}=\frac{\left(2x^2+5\right)^2}{(-6x^2-28x+15)}$ 9. $\frac{dy}{dx}=\ \frac{-9x}{4y}$

5. $x^{x^x}.x^x\left[\frac{1}{x}+\ \log(x).(1+\log x)\right]$ 10. $\frac{d^2y}{dx^2}=\frac{80}{(3x+y)^3}$

4. Applications Of Derivative

I) Choose the correct alternative.

1. (c) 16 2. (c) $\frac{3}{2}$3. (a) increasing for all x ∈ R, x ≠ 1

4. (d) 42 5. (c) D <20 6. (c) unitary elastic 7. (b) in elastic

II) Fill in the blanks.

1. Gradient 2. R 3. -5 4. 36 5. $x > 100$

III) State whether each of the following is True or False.

1. True 2. True 3. True 4. True 5. False 6. False

Solve the following

3 Marks

1. $5x - y - 2 = 0, x + 5y - 16 = 0$ 2. $(-\infty, 2) \cup (3, \infty)$ 3. $(-3,8)$

5. 10,10 6. (i) $\eta = 6.5$, elastic (ii) $\eta = \frac{7}{20}$, inelastic

7. 3.6 8. (i) $x < 10$ (ii) $x > 10$

4 Marks

1. Max value= -3 at $x = 1$, Min value = -128 at $x = 6$ 2. 27 cm, 27cm

3.MPC= 0675, MPS= 0.325, APC= 0.375, APS= 0.625 4. (i) $x < 120$ (ii) $x < 118$

6. $2x - y + 5 = 0, 2x - y - 5 = 0$ 7. $x - 2y - 2 = 0$

8. $2x + y + 9 = 0, \quad 2x - y - 1 = 0$

5. Integrations

1. 1) b 2) b 3) b 4) c 5) a 6) c 7) b 8) a 9) b 10) a.

2. 1) logx + c,

   2) $\frac{1}{2a}$log ($\frac{x-a}{x+a}$) + c

   3) $\frac{(7x+9)^{14}}{91}$ + c

   4) $\frac{e^{4x-3}}{4}$ + c

   5) $\frac{5^{6x+9}}{6\ log5}$ + c

   6) - $\frac{5}{3}$

   7) 4 log(x-1)

   8) 5

   9) 1+ logx = t

   10) $\frac{1}{3}$

3. 1. True 2. False 3. True 4. True 5. False

6. True 7.False 8. False 9. False 10. True

4. 1) x^3- 4$\sqrt{x}$ + c

   2) $\frac{9x^5}{5}$ - 10x^3+ 25 x + c

   3) log(logx) +c

   4) 5x – 8 log (2$e^x + 1) + c$

5) $\frac{1}{4}$ log ($\frac{2x-1}{2x+1}$) + c

6) $\frac{e^x}{x}$ + c

7) $\frac{1}{3}$ log(x+1) + $\frac{5}{3}$ log (x – 2) + c

8) $\frac{x^2}{2}$ logx - $\frac{x^2}{4}$ + c

9) $\frac{e^{4x}}{4}$ ($x^2 - \frac{x}{2} + \frac{1}{8}$) + c

10) $\frac{1}{\sqrt{3}}$ log ($\sqrt{3}$ x + $\sqrt{3x^2+8}$) + c

5. **1) $e^x \frac{1}{(x+1)^2}$ + c**

2) $\frac{1}{2}$ { (x^2- 1) e^{x^2}} + c

3) $\frac{2}{9}$ log ($\frac{x-1}{x+2}$) - $\frac{1}{3\,(x-1\,)}$ + c

4) $\frac{x}{(\,1+logx\,)}$ + c

5) log { (x- 4) + $\sqrt{x^2-8x-20}$ } + c

6) $\frac{1}{8\sqrt{2}}$ log ($\frac{2x-5-2\sqrt{2}}{2x-5+2\sqrt{2}}$) + c

7) log (xe^x+ 1) + c

8) $\frac{1}{2}\sqrt{(1+x^4)}$ +c

9) – t + $\frac{7}{8}$log ($4e^{2t} - 5$) + c

10) f(x) = x^4- x^3+ x^2+2x + 1

6. DEFINITE INTEGRAL

I) Choose the correct alternative.

1. (d) $\frac{211}{5}$ 2. (a) 2 3. (c) 2 4. (a) $e^2 - 1$

5. (b) $log\left(\frac{8}{3}\right)$ 6. (c) $\frac{1}{2}log\left(\frac{8}{3}\right)$ 7. (c) $-\int_b^a f(x)dx$ 8. (b) $\frac{5}{2}$

9.(a)0

II) Fill in the blanks.

1. $\frac{7}{3}$. 2. $a = 3$ 3. $\frac{1}{2}(e^2 - 1)$ 4. $\frac{1}{2}log\left(\frac{7}{5}\right)$

5. $\frac{1}{2}log\left(\frac{17}{5}\right)$ 6. 0

III) State whether each of the following is True or False.

1.False 2. False 3. True 4. True

5.False 6.True

Solve the following

3 Marks

1. $\frac{32}{5}$ 2. $\frac{4}{3}\left(\sqrt{2} - 1\right)$ 3. -2,1 4. $a = 2$

5. $\frac{1}{2}$ 6. $\frac{1}{4}log\left(\frac{9}{7}\right)$ 7. $\frac{1}{6}log\left(\frac{35}{8}\right)$ 8. 1

9. $\frac{1}{2}$ 10. 0

4 Marks

1. $log\left(\frac{3456}{3125}\right)$ 2. $\frac{e^4}{4}-\frac{e^2}{2}$ 3. $log27-2$ 4. $9log3-\frac{26}{3}$

5. $\frac{1}{2}$ 6. $\frac{16}{315}a^{\frac{9}{2}}$ 7. $\frac{1}{42}$

7. APPLICATIONS OF DEFINITE INTEGRAL

Q.1) A) Select and write the most appropriate answer from the given alternatives for each sub-**question.(1 mark each)**

1) a) $\frac{255}{4}$
2) d) $16\pi sq.units$
3) d) $\frac{26}{3}$ sq. units
4) a) $\frac{76\sqrt{2}}{3}$
5) a) $\frac{56}{3}$sq. units
6) a) $\frac{3124}{5}$ sq. units
7) c) $\frac{100\sqrt{5}}{3}$ sq. units

B) State whether the following statements are True or False.(1 mark each)

1) True
2) False
3) True
4) True
5) False

6) False

C) Fill in the following blanks.(1 mark each)

1) 38

2) $25\pi\ qu.units$

3) $\frac{28}{3}$ sq. units

4) $\frac{2}{3}(4-\sqrt{2})$ sq. units

5) $\frac{702}{3}$ sq. units

6) $\frac{4}{3}\left(2\sqrt{2}-1\right)$ sq. units

7) $\frac{10}{3}(5\sqrt{5}-1)$ sq. units

Q. 2) Attempt the following equations. (3 marks each)

1) $\frac{100\sqrt{5}}{3}$ sq. units

2) $\frac{9\pi}{4}\ sq.units$

3) $\frac{1}{3}(7\sqrt{7}-3\sqrt{3})$ sq. units

4) 66 sq. units

5) $\frac{1250}{18}$ sq. units

6) -10 sq. units (wrong question)

7) 6 sq. units

8) $\frac{14}{3}$ sq. units

9) $8\sqrt{3}\ sq.units$

10) $\frac{25\pi}{4}$ sq. units

11) $\frac{36\pi}{4}$ sq. units

Q. 3) Attempt the following equations. (4 marks each)

1) $\frac{8}{5}$ sq. units

2)$36\pi\ sq.units$

3) 75 sq. units

4) $16\pi\ sq.units$

5) $12\pi\ sq.units$

8. DIFFERENTIAL EQUATIONS AND APPLICATIONS

I. Select & write the correct alternative from the given option for each question (1 Marks)

Ques. No	Answer	Ques. No	Answer
1	d) $y = e^{ax}$	6	c) $logx - logy = logc$
2	b) 6 hours	7	a) 3 , 1
3	c) e^{x}	8	c) 3 , 3
4	a) $ye^{-x} = x + c$	9	d) $\mathrm{y - x = c}$
5	b) $x\frac{dy}{dx} + y = 0$	10	c) $x^3 + y^3 = c$

II. Fill in the following blanks (1 marks)

Ques. No	Answer	Ques. No	Answer
1	Order of Differential Equation	6	y-3 = ce^{-x}
2	Particular solution	7	x
3	Positive	8	3,1
4	Degree of Differential Equation.	9	$dy/dx = y$
5	e^{-x}	10	$y = 1 - logx$

III. State whether the following statements are true or false (1 marks)

Ques. No	Answer	Ques. No	Answer
1	True	6	True
2	True	7	False
3	True	8	True
4	False	9	True
5	True	10	True

IV. Attempt the following questions (3 marks)

Ques. No	Answer
1	$ye^x = x + c$
2	$x^2y = \frac{x^4}{16}(logx - 1) + c$
3	$log\lvert x + y\rvert = \ y - x + \frac{1}{3}$
4	$x^2 + 2y^2 = c$
5	$logx - logy = \frac{1}{x} + \frac{1}{y} + c$
6	$logy = \frac{x^3}{3} + x + c$
7	$2xy\frac{dy}{dx} = \ y^2 - x^2$
8	$\frac{d^{2y}}{dx^2} - \frac{2dy}{dx} + y = 0$
9	Verified
10	$5x^2y = x^5 + c$

V. Attempt the following questions (4 marks)

Ques. No	Answer

1	$(1-x^2)(1-y^2)=5$
2	$log\left(\frac{x+y}{x-y}\right)-\frac{1}{2}\log(x^2-y^2)+2logx=logc$
3	$x+4y\frac{dy}{dx}=0$
4	73482
5	45248
6	$x^2+y^2=c^2x^4$
7	$2xlogy=\ (logy)^2+c$
8	$log\left\|\frac{4x+y+5}{6}\right\|=x+c$
9	$logx+\frac{1}{4}\log\left\|\frac{2y^2+xy}{x^2}\right\|+\frac{3}{4}\log\left\|\frac{2y}{x+2y}\right\|=c$
10	$\frac{x+y}{x-y}=\ cx^2$

VI. Attempt the following questions (Activity) (4 Marks)

Ques. No	Answer			
1	$-1,2x$	e^{-x},x	$\frac{e^{-x}}{-1}$	$2x,2$
2	$-\frac{b}{x^2}$	$\frac{2b}{x^3}$	$\frac{2b}{x^3},-\frac{b}{x^2}$	0
3	log 100000	$\frac{1}{25}\ log2$	$\frac{t}{25}\ log2$	50
4	$\frac{dx}{x}=kt$	$log\ x_0$	$\frac{1}{4}\ log2$	$8x_0$
5	log 30000	$\frac{1}{40}\ \log(\frac{4}{3})$	$\frac{t}{40}\ \log(\frac{4}{3})$	$\frac{t}{40}$ log($\frac{4}{3}$)+log30000
6	$\frac{sec^2y}{tany}dy$	$\int\frac{sec^2x}{tanx}dx$	$log\|tanx\|$	$tanx\ tany=c$

7	$x+y=v$	$\frac{1}{1+cosv}$	$\sec^2(\frac{v}{2})$	$tan\left(\frac{x+y}{2}\right)$
8	e^{-2y}	$e^{-2y}+2sinx$	2	e^{-2y}+2sin$x$ =2
9	N	$2^{\frac{1}{4}}$	4	16
10	log N	$\frac{1}{80}\ log2$	$\frac{t}{80}\ log2$	127.15

Part II

1.Commission, Brokerage and Discount

I)

1. **b** Rs. 7,550 **2.** **a)**Rs. 660 **3.** **c)** 18^{th} December 2015

4. **c)**The Nominal Due date **5.d)**List price

II)

1.Del Credere Agent **2.**Banker's discount. **3.**3day **4.** date of bill **5.**An agent

III)

1. True **2**. True, **3.**True 4.**False** 5.**False**

IV)

1. Rs 182.58 , Rs 190.80 and Rs 8.22

2. Rs 80,000

3. Rs 47,500

4. Rs 50,000 , Rs 2,500

5. 14^{th} November

V)

1. 14/09
2. Rs 20, Rs 1,000
3. Rs 6,400 and Rs 10,000
4. Rs 1,200 , Rs 1,220 and Rs 73,200

VI)

1. Banker's Discount

$$\text{B.D.} = \text{F.V.} - \text{C.V.} \quad = 8{,}000 - 7{,}680 = \text{Rs.}320$$

Date of drawing = 5th January 2019

Period = 8 months

Nominal due date = 05/09/2019

Legal due date = 08/09/2019

B.D. = Interest on F.V. for n at 10% p.a.

$$\therefore \quad B.\text{D.} = \frac{\text{F.V.} \times \frac{n}{365} \times \text{r}}{100}$$

$$\therefore \quad 320 = \frac{8{,}000 \times \frac{n}{365} \times 10}{100}$$

$\therefore \quad n =$ 146 days

April	May	June	July	Aug	Sep	Total
15	**31**	**30**	**31**	**31**	**8**	**146**

$\therefore$ Date of discounting is 15th April 2019

2.
Total selling Price of three cars = 2,40,000 + 2,22,000 +2,25,000

= Rs 6,87,000

Commission on total sale = 14%

$$= \frac{14}{100} \times 6{,}87{,}000$$

= Rs 96180

Selling price of First car = Rs. 2,40,000

Rate of commission = 17.5% = $\frac{17.5}{100} \times 2{,}40{,}000 =$ Rs 42,000

$\therefore$ Commission on first car = Rs. R42,000

Selling price of Second car = Rs. 2,22,000

Rate of commission = 12.5% = $\frac{12.5}{100} \times 2{,}22{,}000 = 27{,}750$

$\therefore$ Commission on second car = Rs. 27,750

Selling price of third car = Rs. 2,25,000

Let the rate of commission be x

Commission on third car $= \frac{x}{100} \times 2,25,000$

$\therefore$ Commission on third car $=$ Total commission $-$ (commission on first car $+$ commission on secon

$\frac{x}{100} \times 2,25,000 =$ 96,180 $- \{$ 42,000 $+$ 27,750 $\}$

$\therefore$ x $=$ Rs 47,500

2. INSURANCE AND ANNUITY

MCQ

1)b (2)b (3)d (4)a (5)c

(6)a (7)b (8) b (9) a (10)b

True or false:

(1) T (2) T (3) F (4) T (5) T
(6) T (7) F (8) T

Fill in the blanks:

1)End of each period (2)34,259.94 (3)4000

(4)Immediate annuity (5)Due (6)payment period

(7)life insurance

3 marks question:

(1)153.6 (2)Rs.3,57,518.5090 (3)Rs.85,000 (5)Rs.1,07,000
(6)23,205 (7) Rs.2267.60 (8)Rs.2400

(9) Rs.3,703.70 (10)Rs.86188.32

3. LINEAR REGRESSION

I.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
d	d	a	b	c	d	c	a	b	b	b	A	b	a	c	d	a

II.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
F	F	T	T	F	F	T	T	F	F	F	T	T	F	F	T

III.

1)-0.8 2) $2|r|$ 3)$6x + y - 31 = 0$. 4) $\frac{c}{d}$ b_{uv} 5)17 6)negative 7)1

8) $|r|$ 9) $\frac{4}{5}$ b_{vu} 10)-r 11)responses 12)slope

1V

1. 4

2. Regression line of x on y is x =0.2y -8 and y on x is y =3.2x+58
x= 22 lakhs.

3. a. 2x + 3y− 6 = 0 is y on x and 5x + 7y− 12 = 0 is x ony
   b. r = -0.9667

4. 43.5 years

5. Regression line of x on y is x =0.5y +40 and y on x is y =0.72 x+28.8

y = 100.8 units.

6. Regression line of x on y is 6x+y-31=0 and y on x is 3x+2y-26=0

r= - 0.5

7. x = 0.82y+10.24

8. 62.4,0.8

9. a. -0.8 b.x+0.4y=5.6

10. 0.62

11. x = y+6

12. x = 12.6, y = 14.3

V

1. 2, 20, 3, 9, $\frac{1}{2}$, 2x-y+3 = 0, 2x-y+3=0

2. a. -0.6, b. 28, -1.2, 53, 31.6 c. 53, -0.3, 28, 53.9

3. 0.375, 0.667, 20, 0.375, 25, 14.375

4. 18, 19.2, $\frac{2}{5}$, 0.6

5. 13, 17, $\frac{4}{5}$, $\frac{9}{20}$, 0.6, 4.

6. 56, 214, 340, $\frac{-13}{10}$, $\frac{-13}{20}$, x=-1.3y +16.4,
y = -0.65x + 11.9

4. TIME SERIES

I)

1. (d) 2. (a) 3. (a) 4. (a) 5. (c)

II)

1. Trend 2. Seasonal 3. Least square 4. Graphical 5. Moving average

III)

1. F 2. T 3. F 4. F 5. F

IV)

1. y = 3.08+0.25x

2.

1971	1972	1973	1974	1975	1976	1977	1978	1979	1980	1981	1982
-	-	1.25	1.75	2.375	3.25	4	4.125	4	4.5	-	-

3. y = 6+0.7x , y = 12.3

4.

1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986

-	3.33	3.33	4	6	7	8	6	7.67	8	-

5. y = 5+0.8x , y = 9.8

6.

1962	1963	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	1974	1975	1976
-	-	0.8	1.4	2.2	3	4	5.2	6.4	7.4	8	8.6	9	-	-

V)

1. Middle year is 1995

a= 4.2857, b = -2

The equation of trend line is y = 4.2857 -2x.

2. 3 yearly moving total = 22, 4.

3-yearly moving average (trend value) = 5.33, 2.67.

3. n = 10, two middle years are 2010 and 2011 and h = 2

a = 17.7 and b = 0.1

The equation of trend line is y = 17.7 + 0.1x

Put x = 11 then y = 18.8

4. 4 yearly moving total = - , 66, 68

4 yearly centered total = 136,135

4 yearly centered moving average (trend values) = - , 16.625, 16.875

5. Index Numbers

I) Correct alternative

1. c)
2. a)

3. d)
4. b)
5. d)
6. c)
7. a)
8. b)
9. b)

II) Fill in blanks

1. $\frac{\sum p_1}{\sum p_0}$ x 100
2. $\frac{\sum p_1 q_1}{\sum p_0 q_0}$ x 100
3. $\sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} x \frac{\sum p_1 q_1}{\sum p_0 q_1}}$ x 100
4. $\frac{\sum p_1 \sqrt{q_{0+} q_1}}{\sum p_0 \sqrt{q_0 + q_1}}$ x 100
5. $\frac{\sum p_1 q_1}{\sum p_0 q_0}$ x 100
6. Dorbish-Bowley's Price Index Number
7. $\frac{\sum q_1 w}{\sum q_0 w}$ x 100
8. $\frac{\sum p_1 w}{\sum p_0 w}$ x 100

III) 1. T

2. T
3. F
4. T
5. F

6. T
7. F
8. F
9. F

IV) Solve

1. 209.09
2. 58.54
3. 346.03
4. 74.52
5. 171.59
6. x = 16
7. x = 40, y = 110
8. x = 10
9. 261.11, 306.06, 283.59
10. 179.19
11. 161.74
12. D-B = 65, F = 60
13. 61,60
14. 190.0
15. 2
16. 146.25
17. 26.59
18. 150
19. x = 3
20. 24000

V) Activity

1. 130.77
2. 137.5

6. LINEAR PROGRAMMING PROBLEMS

A. Select the most appropriate option from the following

i) (c), ii) (d), iii) (a), iv) (b), v) (c), vi) (a), vii) (a), viii) (b), ix) (b), x) (d), xi) (a), xii) (b)

B. State whether each of the following statement is true or false

TRUE: (i), (ii), (v), (vi), (vii), (x), (xii),

FALSE: (iii), (iv), (viii), (ix), (xi).

C. Fill in each of the following blank

(i) Decision variables, (ii) equations or inequations, (iii) solution of LPP,

(iv) $x + y \leq 250$,(v) Total Profit = $400x + 200y$, (vi) $400x + 250y \geq 4000$,

(vii) IV quadrant, (viii) $80x + 50y \leq 400$,(ix) $x + y \leq 75$, (x) $x + y \geq 55000$,

(xi) not a part ,(xii) $y \leq x$,

D. Solve graphically

(i) Max Z = 25000 at x =100/3 , y = 70/3
(ii) Max Z =12000 at x = 40 , y = 0
(iii) Max Z =1500 at x = 0 , y = 15
(iv) Min Z =14 at x = 4, y = 1
(v) Min Z = 3500 at x = 5/4 , y = 5
(vi) Min Z =64 at x = 2 , y = 4
(vii) Max Z = 37/3 at x =14/3 , y = 0
(viii) Max Z = 50 at x =14/5, y = 18/5 & x =0, y = 5. Hence it has infinite solutions
(ix) Max Z = 25000 at x =100/3 , y = 70/3
(x) Min Z =576 at x = 24, y = 0
(xi) Min Z =3 at x = 3, y = 0
(xii) Min Z =38/3 at x = 16/3, y = 2/3

E. Activities

i) Amartya wants to invest ₹45,000 in Indira Vikas Patra (IVP) & in Public Provident fund (PPF). He wants to invest at least ₹ 10,000 in PPF & at least ₹5000 in IVP. If the rate of interest on PPF is 8% per annum & that on IVP is 7% per annum. Formulate the above problem as LPP to determine maximum yearly income.
Solu: Let x be the amount (in ₹) invested in IVP & y be the amount (in ₹) invested in PPF

$x \geq 0$, $y \geq 0$

As per the given condition, $x + y \leq 45000$

He wants to invest at least ₹ 10,000 in PPF

$\therefore y \geq 10000$

Amartya wants to invest at least ₹5000 in IVP

$\therefore x \geq 5000$

Total interest(Z) = _0.07x + 0.08y

The formulated LPP is

Maximize $Z = 0.07x + 0.08y$ subject to

$x + y \leq 45000$

$y \geq 10000$

$x \geq 5000$

$x \geq 0$, $y \geq 0$

ii) Solve the following LPP graphically :

Maximize $Z = 9x + 13y$ subject to constraints

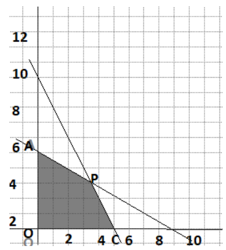
$2x + 3y \leq 18$, $2x + y \leq 10$, $x \geq 0$, $y \geq 0$

Solu: Convert the constraints into equations & find the intercept made by each one of it.

Inequation	Equation	X intercept	Y intercept	Region
$2x + 3y \leq 18$	$2x + 3y = 18$	(9, 0)	(0,6)	Towards origin
$2x + y \leq$	$2x + y = 10$	(5, 0)	(0,10)	Towards

10				origin
x ≥ 0, y ≥ 0	X =0, y = 0	X axis	Y axis	I st quadrant

The feasible region is OAPC Where O(0,0), A(0,6), P(3,4),C(5,0)



The optimal solution is in the following table

Point	Coordinates	Z = 9x + 13y	Values	Remark
O	(0,0)	9(0)+13(0)	0	
A	(0,6)	9(0)+13(6)	78	
P	(3,4)	9(3)+13(4)	79	maximum
C	(5,0)	9(5)+13(0)	45	

∴ Z is maximum at P (3,4) with the value 79

iii)Solve the LPP graphically:

Minimize Z = 4x + 5y

Subject to the constraints 5x + y ≥ 10, x + y ≥ 6, x + 4y ≥ 12, x , y ≥ 0

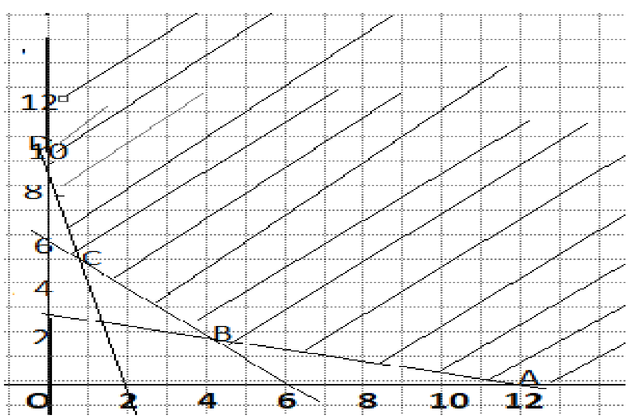
Solu: Convert the constraints into equations & find the intercept made by each one of it.

Inequations	Equations	X intercept	Y intercept	Region
$5x + y \geq 10$	$5x + y = 10$	(2, 0)	(0, 10)	Away from origin
$x + y \geq 6$	$x + y = 6$	(6, 0)	(0, 6)	Away from origin
$x + 4y \geq 12$	$x + 4y = 12$	(12, 0)	(0, 3)	Away from origin
$x , y \geq 0$	$X = 0, y = 0$	$X = 0$	$Y = 0$	Ist quadrant

∵ Origin has not satisfied the inequation

∴ Solution of the inequations

is away from origin.



The feasible region is unbounded

area which is satisfied by all

constraints

In the figure, ABCD represents

The set of feasible solution where

A (12,0), B (4, 2), C (1, 5) &

D (0, 10)

The coordinates of B are

Obtained by solving equations

$x + 4y = 12$ & $x + y = 6$

The coordinates of C are

Obtained by solving equations

$5x + y = 10$ & $x + y = 6$

Hence the optimum solution lies at the extreme points

The optimal solution is in the following table

Point	Coordinates	$Z = 4x + 5y$	Values	Remark
A	(12,0)	4(12) + 5(0)	48	
B	(4, 2)	4(4) + 5(2)	26	minimum
C	(1, 5)	4(1) + 5(5)	29	
D	(0, 10)	4(0) + 5(10)	50	

$\therefore$ Z is minimum at $B\ (4, 2)$ with the value 26.

7. ASSIGNMENT PROBLEM AND SEQUENCING

I)Choose the correct alternative.

1. b) minimization
2. a) Linear Programing Problem
3. b) Hungarian method
4. d) square matrix
5. c) multiple optimal solution
6. a) No. of rows = No. of columns
7. b) DBCA
8. c) Both (a) and (b)
9. a) $\text{Min}A_i \geq \text{Max}B_i$ OR $\text{Min}C_i \geq \text{Max}B_i$ $i=1,2,3\ldots n$.
10. c) to find the sequence in which jobs on hand are to be processed to minimize the total time required for processing the jobs.
11. d) $(n!)^m$
12. d) The time of passing depends on the order of machining.

II) Fill in the Blanks.

1. square 2. zero 3. restricted 4.column 5. Optimization

6. Independently 7. Minimize 8.Total Elapsed Time 9. Idle

10. negligible 11. A-D-B-C 12. Order

III) True or False

1. False 2. False 3. True 4. True 5.False 6. True
7. False 8. True 9.False 10. False 11. True 12. True

IV) Attempt the following

The following optimum solution obtained:

1.

Machines	M_1	M_2	M_3	M_4	M_5

Places	A	B	E	D	C
Cost (Rs.)	4	4	2	2	0

Total cost = 4+4+2+2+0= 12 Rs

2.

Salesman	A	B	C	D	E
Territories	V	II	IV	III	I
Sales	15	19	14	17	0

Maximum sale = 15+19+14+17+0=65 units

3.

Operators	A	B	C	D	E
Machines	4	3	2	5	1
Cost	3	3	4	3	6

OR

Operators	A	B	C	D	E
Machines	4	3	5	2	1
Cost	3	3	4	3	6

Total cost = 3+3+4+3+6 = 19 Rs.

4.

Salseman	A	B	C	D
District	1	3	2	4
Profit	16	15	15	15

Total Profit = 16+15+15+15 = 61 Rs.

5.

Routes	A	B	C	D	E
Milk Tankers	II	III	V	I	IV
Distance (km)	120	120	170	40	70

Min distance travelled = 120+120+170+40+70= 520km.

6. The optimal sequence of jobs

III	V	II	VI	I	IV	VII

Total Elapsed Time T = 55 minutes
Idle Time for machine A =3 minutes
Idle Time for machine B = 9 minutes

7. The optimal sequence of jobs

III	V	II	VI	VII	IV	I

OR

V	III	II	VI	VII	IV	I

Total Elapsed Time T = 61 hours
Idle Time for machine A = 7 hours
Idle Time for machine B = 38 hours
Idle Time for machine C = 15 hours

8. The optimal sequence of jobs

II	IV	V	III	I

Total Elapsed Time T = 21 hours
Idle Time for lathe =4 hours
Idle Time for grinding = 3 hours

9. The optimal sequence of jobs

3	2	5	4	1

Total Elapsed Time T = 102 hours
Idle Time for machine A = 18 hours
Idle Time for machine B = 62 hours
Idle Time for machine C = 38 hours

10. The optimal sequence of jobs

B	D	C	E	A

Total Elapsed Time T = 60 hours
Idle Time for machine X = 4 hours
Idle Time for machine Y = 6 hours

V) Activities [4 marks each]

1.

Subordinates	A	B	C	D
Tasks	I	III	II	IV
Required hrs	7	3	18	9

Minimum hours required = 7+3+18+9 = 37 hours.

2. The optimal sequence of jobs

P	U	S	Q	R	T

Total Elapsed Time T = 32 hours
Idle Time for machine M_1 = 11 hours
Idle Time for machine M_2 = 1 hours

8. Probability Distribution

MCQ	ANSWER KEY
1	c
2	a
3	b
4	d
5	c
6	a
7	d
8	c
9	a
FILL IN THE	
1	Counting
2	Measurement
3	1
4	Centre of gravity
5	Remains constant/ Independent
6	Poisson
7	np
TRUE OR FALSE	
1	TRUE
2	FALSE
3	TRUE
4	TRUE
5	FALSE

	6	FALSE
	7	FALSE
	8	FALSE
	9	TRUE

ANSWERS OF 3 MARKS QUESTIONS

1) a) X : 0 1 2 b) 0 1 2 3 c) 0 1 2 3 4

P(X) : ¼ 2/4 ¼ 1/8 3/8 3/8 1/8 1/16 4/16 6/16 4/16 1/16

2) X : 0 1 2 3 4

P(X) : 506/1305 4048 /9135 92/609 32/1827 1/1827

3) E(X) = -0.05 V (X) = 2.2475

4) X 0 1 2 3 E(X)= 1.5

P(X) 1/8 3/8 3/8 3/8

5) X : 0 1 2 E(X)= 1/3

P(X) : 25/36 10/36 1/36

6) P(X=2) = 5/72 7) V = 2.4 8) $P (X \leq 1) = 0.7356$ 9) $P (X \geq 2) = 0.8012$.

ANSWERS OF 4 MARKS QUESTIONS

1) i) k = 1/10 ii) 3/10 III) 3/10 IV) 1/5

2) c.d.f. = $x^3 / 8$ i) 1/8 ii) 0 iii) 0 iv) 7/8

3) C = 1/log 3 , f(x)=1/xlog3 , V= 4/log3 – 4 /$(\log 3)^2$ F (x) = logx/log3

4) i) 3/8 ii) 5/16 iii) 11/16

5) i) $(0.8)^5$ ii) 1.8 x $(0.8)^4$ iii) 1 - 1.8 x $(0.8)^4$ iv) 1 – $(0.8)^5$.

6) a) 0.1464 b) 0.2379

7) i) 0.3678 ii) 0.6322

8) I) 0.17 II) 0.39 III) 0.42

ACTIVITIES

1) K = 1/21 , $P(X \leq 4) = 10/21$, $P(X \geq 3) = \frac{6}{7}$
2) E(X) = 11/5 V(X) = 14/25
3) p =1/2 , q = ½ V = 2.5
4) P=0.2 , P(2) = 128/625
5) m = 1.