Total No. of Questions – 24
Total No. of Printed Pages – 3

Regd.
No.

Part – III
MATHEMATICS, Paper-II(B)
(English Version)

Time : 3 Hours]

[Max. Marks : 75

Note : This question paper consists of three sections A, B and C.

SECTION – A

$10 \times 2 = 20$

I. Very short answer type questions :

(i)   Attempt all questions.

(ii) Each question carries two marks.

1. Find the other end of the diameter of the circle $x^2 + y^2 - 8x - 8y + 27 = 0$ if one end of it is $(2, 3)$.

2. Define chord of contact and find the chord of contact of $(1, 1)$ to the circle $x^2 + y^2 = 9$.

3. Find k if the circles $x^2 + y^2 - 5x - 14y - 34 = 0$ and $x^2 + y^2 + 2x + 4y + k = 0$ are orthogonal.

4. Find the equation of the parabola whose vertex is $(3, -2)$ and focus is $(3, 1)$.

5. If $3x - 4y + k = 0$ is a tangent to the hyperbola $x^2 - 4y^2 = 5$, find the value of k.

6. Evaluate : $\int \dfrac{\cos x}{(1+\sin x)^2} \, dx$.

7. Evaluate : $\int x \log x \, dx$ on $(0, \infty)$.

8. Evaluate : $\lim\limits_{n \to \infty} \dfrac{1 + 2^4 + 3^4 + \ldots\ldots + n^4}{n^5}$

9. Find : $\int\limits_{-\pi/2}^{\pi/2} \sin^2 x \cos^4 x \, dx$.

10. Solve : $y(1 + x) \, dx + x (1 + y) \, dy = 0$.

<div align="center">SECTION – B</div> $5 \times 4 = 20$

II. Short answer type questions :

(i) **Attempt any five questions.**

(ii) **Each question carries four marks.**

11. Find the area of the triangle formed by the tangent at $P(x_1, y_1)$ to the circle $x^2 + y^2 = a^2$ with the co-ordinate axes where $x, y, \neq 0$.

12. If the two circles $x^2 + y^2 + 2gx + 2fy = 0$ and $x^2 + y^2 + 2g'x + 2f'y = 0$ touch each other then show that $f'g = fg'$.

13. S and T are the foci of an ellipse and B is one end of the minor axis. If STB is an equilateral triangle, then find the eccentricity of the ellipse.

14. Find the condition for the line

$x \cos \alpha + y \sin \alpha = P$ to be a tangent to the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

15. Find the centre, foci, eccentricity, equation of the directrices of the hyperbola $x^2 - 4y^2 = 4$.

16. Find the area of the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$.

17. Solve : $(x^2 + y^2)dx = 2xy \, dy$.

SECTION - C

$5 \times 7 = 35$

III. **Long answer type questions :**
   (i)   **Attempt any five questions.**
   (ii) **Each question carries seven marks.**

18. Find the equation of the circle which passes through (4, 1), (6, 5) and having the centre on $4x + 3y - 24 = 0$.

19. Find the equation of the circle which touches the circle $x^2 + y^2 - 2x - 4y - 20 = 0$ externally at (5, 5) with radius 5.

20. From an external point P tangents are drawn to the parabola $y^2 = 4ax$ and these tangents make angles θ_1, θ_2 with its axis, such that $\tan \theta_1 + \tan \theta_2$ is a constant b. Then show that P lies on the line $y = bx$.

21. Evaluate : $\int \dfrac{1}{1 + \sin x + \cos x} \, dx$.

22. If $I_n = \int \cos^n x \, dx$, then show that $I_n = \dfrac{1}{n} \cos^{n-1} x \sin x + \dfrac{n-1}{n} I_{n-2}$. (where $n \geq 2$)

23. Show that : $\displaystyle\int_{0}^{\pi/2} \dfrac{x}{\sin x + \cos x} \, dx = \dfrac{\pi}{2\sqrt{2}} \log (\sqrt{2} + 1)$.

24. Solve : $x \log x \dfrac{dy}{dx} + y = 2 \log x$.