JEE-Main-24-06-2022-Shift-1 (Memory Based)

Physics

Question: At what height from the surface of earth the weight of the body is 1/3rd of its weight at the surface?

Options:

(a) 5000 km
(b) 5562.5 km
(c) 4684.8 km
(d) 3600 km

Answer: (c)

Solution:

We know

$$gh = \frac{g}{\left(1+\frac{h}{R}\right)^2} \quad ....(1)$$

So, Q

For weight to be 1/3rd of the weight on the earth surface g_h should be 1/3rd of g. So from eq.

(1) $\frac{g}{3} = \frac{g}{\left(1+\frac{h}{\text{Re}}\right)^2}$

$$\left(1+\frac{h}{\text{Re}}\right)^2 = 3$$

$$\left(1+\frac{h}{\text{Re}}\right) = \sqrt{3}$$

$$\frac{h}{\text{Re}} = 1.732 - 1$$

$$h = 0.732 \times \text{Re} \quad (\text{Re} = 6400 km)$$

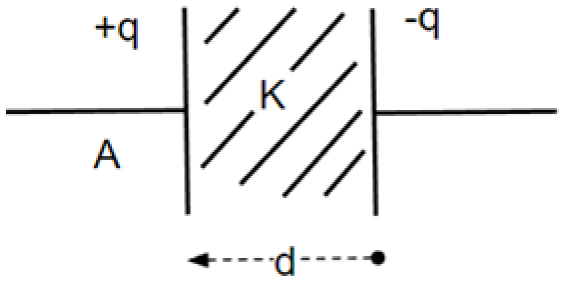
$$h = 0.732 \times 6400 km$$

$$h = 4684.8 km$$

Question: Electric Field Strength = E

Max charge = q

Find K = ?



Options:

(a) $\boxed{K = \dfrac{q}{\varepsilon_0 EA}}$

(b) $\boxed{K = \dfrac{qd}{\varepsilon_0 EA}}$

(c) $\boxed{K = \dfrac{qA}{\varepsilon_0 E}}$

(d) None of these

Answer: (a)

Solution:

$$E = \frac{v}{d} = \frac{q}{c \cdot d} \Rightarrow c = \frac{q}{E \cdot d}$$

$$c = \frac{\varepsilon_0 AK}{d}$$

Therefore, $\dfrac{q}{E \cdot d} = \dfrac{\varepsilon_0 AK}{d}$

Therefore, $\boxed{K = \dfrac{q}{\varepsilon_0 EA}}$

Question: If one end of vertical spring is connected to the ground and other end is connected to horizontal platform at rest. If a ball of mass m is dropped on it from height h above platform compresses spring by h/2. If h = 10 cm find k.

Options:

(a) 120 mg

(b) 200 mg

(c) 180 mg

(d) 130 mg

Answer: (a)

Solution:

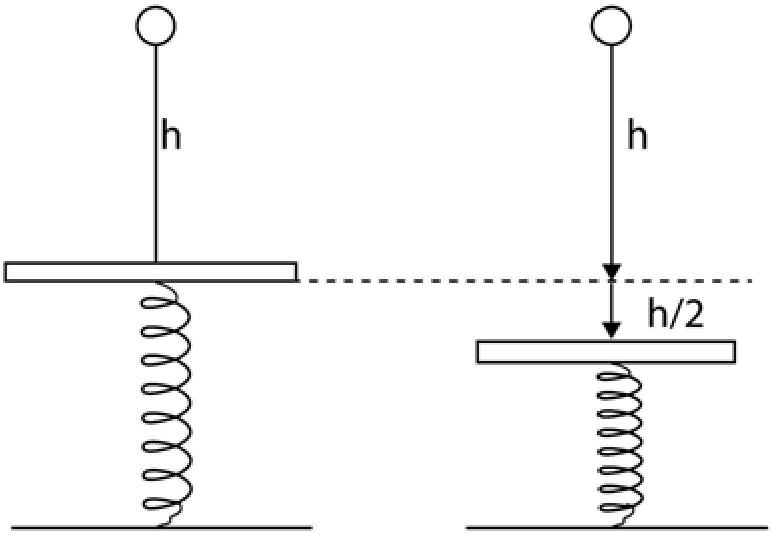
Loss in PE of ball = gain in S.P.E.

$$mg\left(h + \frac{h}{2}\right) = \frac{1}{2}K\left(\frac{h}{2}\right)^2$$

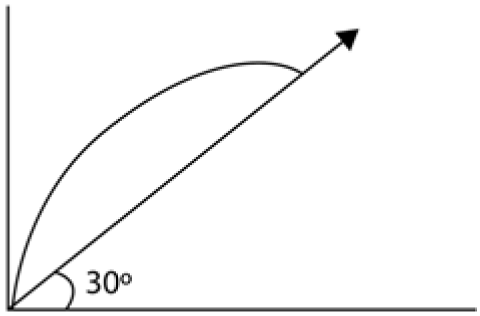
$$mg\left(\frac{3h}{2}\right) = \frac{1}{2}K\frac{h^2}{4}$$

$$\frac{12mg}{h} = K$$

$$K = \frac{12mg}{10\times10^{-2}}$$
$$K = \frac{1200mg}{10}$$
$$K = 120mg$$



Question: The body is projected 10 m/sec. The angle of projection at $30°$. Find range



Options:
(a) 10/3
(b) 20/3
(c) 10
(d) 40/3
Answer: (b)
Solution:

$$R = U_x \times T + \frac{1}{2}a_x T^2$$

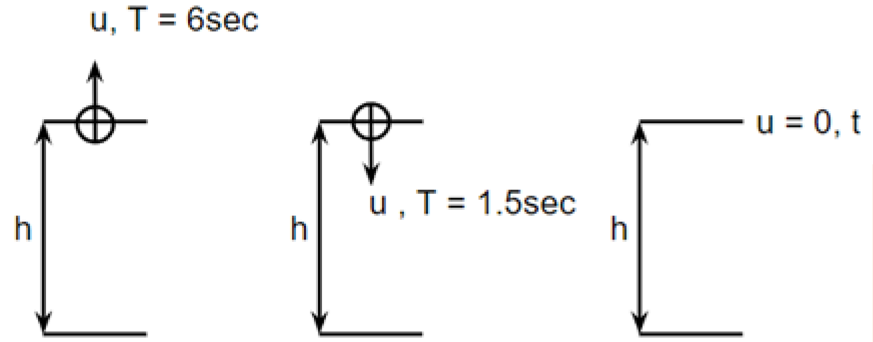
$$R = U\cos 30° \times \frac{2U\sin 30°}{g\cos 30°} - \frac{1}{2}g\sin 30°\left(\frac{2U\sin 30}{g\cos 30°}\right)^2$$

$$R = \frac{2U^2\cos 30°\tan 30°}{g} - \frac{1}{2}g\sin 30°\left(\frac{2U\tan 30°}{g}\right)^2$$

$$R = \frac{2\times100\times\sqrt{3}}{2\times10}\times\frac{1}{\sqrt{3}} - \frac{1}{2}\times10\times\frac{1}{2}\times\frac{4\times100}{100}\times\frac{1}{3}$$

$$R = \frac{100}{10} - \frac{1000}{3 \times 100}$$
$$R = 10 - \frac{10}{3}$$
$$R = \frac{20}{3} m$$

Question: A ball when thrown up from a tower of height h takes 6 seconds to reach ground and when thrown downward with same velocity, it took 1.5 seconds. How much time it will take if ball is dropped from the tower?



Options:
(a) 3 sec
(b) 5 sec
(c) 2 sec
(d) 4 sec
Answer: (a)
Solution:
Ist Condition
$$-h = 64 - \frac{1}{2} \times g \times 36$$
$$h = -6u + 10h$$
$$h = 10g - 6u$$
$$6u = 10g - h$$
$$u = \frac{18g - h}{6} \quad ....(1)$$
IInd Condition
$$-h = -u \times 1.5 - \frac{1}{2} \times g \times 2.25$$
$$h = 1.5u + \frac{1}{2} g \times 2.25$$
$$h = 1.5\left(\frac{18g - h}{6}\right) + \frac{1}{2} g \times 2.25$$
$$h = \frac{18 \times 1.5g}{6} - \frac{1.5h}{6} + \frac{1}{2} g \times 2.25$$
$$\left(h + \frac{1.5h}{6}\right) = \frac{180 \times 1.5}{6} + 5 \times 2.25$$
$$1.25h = 45 + 11.25$$

$h = 45m$

IIIrd Condition

$$h = 0 + \frac{1}{2} \times 10 \times \ell^2$$

$$\frac{45 \times 2}{10} = t^2$$

$$t^2 = 9$$

$$t = 3\sec$$

Question: Find change in kinetic energy if a block is displaced from (1,2) to (2,3) on applying a force of $\vec{F} = 4x^2\hat{i} + 3y^3\hat{j}$.

Options:

(a) 58.08 J

(b)45.08 J

(c) 55.56 J

(d) 32.3 J

Answer:(a)

Solution:

$$WD = \Delta KE = \int F_x dx + \int F_y dy$$

$$= \int_1^2 4x^2 dx + \int_2^3 3y^3 dy$$

$$= \left.\frac{4x^3}{3}\right|_1^2 + \left.\frac{3y^4}{4}\right|_2^3$$

$$\Delta KE = \frac{4}{3}\left(2^3 - 1\right) + \frac{3}{4}\left(3^4 - 2^4\right)$$

$$= \frac{4}{3} \times 7 + \frac{3}{4} \times 65$$

$$= \frac{697}{12}$$

$$= 58.08J$$

Question: The normal reaction 'N' for a vehicle of 800 kg mass, negotiating a turn on a 30° banked road at maximum possible speed without skidding is ___ × 10^3 kg m/s^2. [Given cos 30° = 0.87, μs = 0.2]

Options:

(a) 8.8

(b)5.8

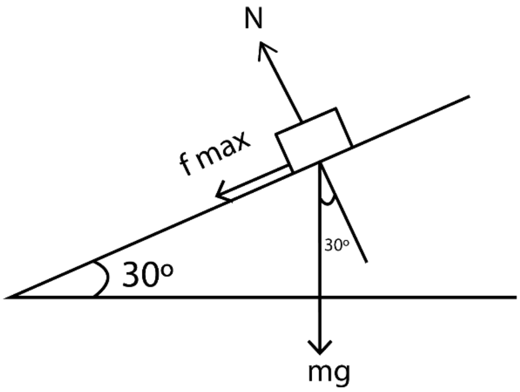
(c) 10.4

(d) 3.2

Answer:(c)

Solution:

At maximum possible speed friction will be limiting



Balancing force invertical

$N\cos 30^{\circ} - mg - \mu N\cos 60^{\circ} = 0$

$N\left(\cos 30^{\circ} - \mu\cos 60^{\circ}\right) = mg$

$\Rightarrow N = \frac{mg}{(0.87 - 0.2\times 0.5)} = \frac{8000}{0.77} = 10.4\times 10^{3}\ \text{N}$

Question: Stopping potential for e- for wavelength 491 nm is 0.410 V incidence wavelength is changed to new value then stopping potential is 1.02 V. New wavelength is

Options:

(a) 234.62 nm

(b)582.65 nm

(c) 104.32 nm

(d) 645.83 nm

Answer:(d)

Solution:

We know that,

$\frac{hc}{\lambda} = \phi + KE$

$\frac{hc}{\lambda_1} = \phi + kE_1$

$\frac{hc}{491nm} = \phi + 0.4100V...(i)$

$\frac{hc}{\lambda_2} = \phi + 1.02eV...(ii)$

(ii) -(i)

$\frac{hc}{491nm} - \frac{hc}{\lambda_2} = (1.02 - 0.410)e\text{V}$

$\frac{1240\text{eV}\cdot\text{nm}}{491\text{nm}} - 0.61\text{eV} = \frac{1240\text{eV}\cdot\text{nm}}{\lambda_2}$

$(2.53 - 0.61)\text{eV} = \frac{1240}{\lambda_2}$

$\lambda_2 = \frac{1240}{1.92} = 645.83\,\text{nm}$

Question: If at the centre of circular current carrying coil, magnetic field is B_0 then the magnetic field at distance r/2 on the axis of a coil from centre is (r is the radius)

Options:

(a) $\frac{4}{5\sqrt{5}}B_0$

(b) $\frac{8}{5\sqrt{5}}B_0$

(c) $\frac{4}{5}B_0$

(d) $\frac{8}{\sqrt{5}}B_0$

Answer: (b)

Solution:

We know that magnetic field at center of current carrying coil is $\frac{\mu_0 I}{2r}$

So,

$$B_0 = \frac{\mu_0 I}{2r} ...(i)$$

Magnetic field at a point on the axis of current carrying coil

$$B = \frac{\mu_0 I r^2}{2\left(x^2 + r^2\right)^{3/2}}$$

At $x = \frac{r}{2}$

$$B = \frac{\mu_0 I r^2}{2\left(\frac{r^2}{4} + r^2\right)^{3/2}}$$

$$\Rightarrow B = \frac{\mu_0 I r^2}{2\left(\frac{5}{4}\right)^{3/2} r^3}$$

$$\Rightarrow B = \left(\frac{\mu_0 I}{2r}\right)\frac{8}{5\sqrt{5}}$$

$$\Rightarrow B = \frac{8}{5\sqrt{5}}B_0$$

Question: If B = 10^9 Nm^{-2}& fractional change in volume is 2% find the volumetric stress required

Options:

(a) $1\times10^7\ pa$

(b) $2\times10^7\ pa$

(c) $3\times10^7\ pa$

(d) $5\times10^7\ pa$

Answer: (b)

Solution:

Given $N = 10^9$

Also, $N = \frac{dp}{(dv/v)}$

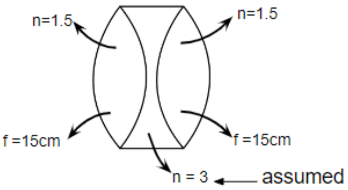
Fractional change = 2%

$\Rightarrow \frac{dv}{v} = 0.02$

$\therefore dp = B\left(\frac{dv}{v}\right) = 0.02 \times 10^9$

$= 2 \times 10^7\ pa$

Question: Find effective focal length



Options:

(a) -7.5 cm

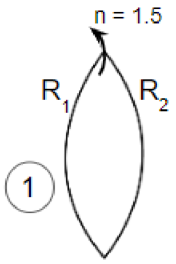
(b) -5.7 cm

(c) -7.4 cm

(d) -6.7 cm

Answer: (a)

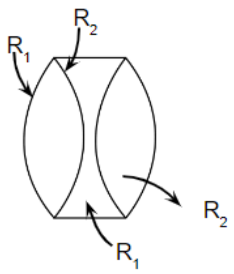
Solution:

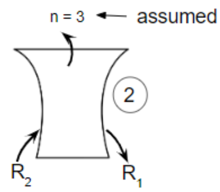


$$\frac{1}{15} = \frac{1}{f_1} = (1.5 - 1)\left(\frac{1}{R_1} - \frac{1}{-R_2}\right)$$

$$\frac{1}{15} = \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{R_1} + \frac{1}{R_2} = \frac{2}{15}$$





$$\frac{1}{f_2} = (3-1)\left(\frac{1}{R_2} - \frac{1}{R_1}\right)$$

$$= -2\left(\frac{1}{R_2} + \frac{1}{R_1}\right)$$

$$= -2 \times \frac{2}{15}$$

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

$$= \frac{1}{15} - \frac{4}{15} + \frac{1}{15}$$

$$\frac{1}{f_{eq}} = -\frac{2}{15}$$

$$f_{eq} = -7.5\,cm$$

Question: Efficiency of carnot engine was 25% at 27° C what will be the temperature to increase its efficiency by 100% more.

Options:

(a) $T_2 = 400K$

(b) $T_2 = 200K$

(c) $T_2 = 100K$

(d) $T_2 = 300K$

Answer: (b)

Solution:

At $27°C = 300K$,

$$\eta_1 = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{T_1}$$

$$\frac{25}{100} = 1 - \frac{300}{T_1}$$

$$-\frac{75}{100} = -\frac{300}{T_1}$$

$$T_1 = 400K$$

To increase η by 100% more

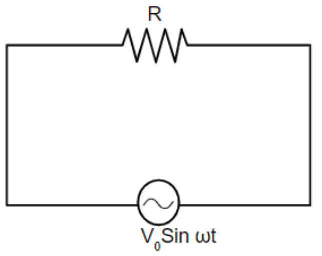
i.e. make the η 50%

$$\frac{50}{100} = 1 - \frac{T_2}{400}$$

$$-\frac{1}{2} = -\frac{T_2}{400}$$

$$T_2 = 200K$$

Question: In a given AC circuit which has maximum voltage V_0 and frequency 50 Hz. Find the time instant where the current in the circuit will be equal to RMS value of circuit



Options:

(a) 1.5 m/s

(b) 2.5 m/s

(c) 4.5 m/s

(d) 3.5 m/s

Answer: (b)

Solution:

In the circuit $i = \frac{V_0 \sin \omega t}{R}$, also $i_{rms} = \frac{i_0}{\sqrt{2}}, \omega = 2\pi f$

Hence at $t = t \Rightarrow \frac{i_0}{\sqrt{2}} = \frac{V_0}{R} \sin \omega t \;= 100\pi$

$$\frac{1}{\sqrt{2}} = \sin \omega t$$

$$\sin \frac{\pi}{4} = \sin \omega t$$

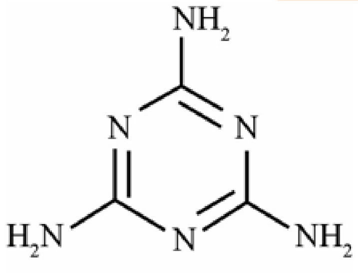
$$\Rightarrow t = \frac{\pi}{4\omega} = \frac{\pi}{4 \times 100\pi} = \frac{1}{400} \;\&= \frac{1000}{400} ms$$

$$\boxed{t = 2.5\, ms}$$

JEE-Main-24-06-2022-Shift-1 (Memory Based)

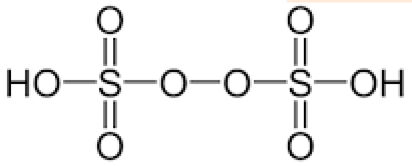
Chemistry

Question: Total Number of lone pair in Melamine
Options:
(a) 6
(b) 3
(c) 2
(d) 4
Answer: (a)
Solution:



Six lone pair

Question: Number of Pi bonds in Marshall's acid
Options:
(a) 6
(b) 3
(c) 2
(d) 4
Answer: (d)
Solution:



Question: Which of these is not a broad spectrum antibiotic?
Options:
(a) Amoxicillin
(b) Ofloxacin
(c) Penicillin G
(d) Chloramphenicol
Answer: (c)
Solution: Gram-positive or Gram-negative bacteria are narrow spectrum antibiotics. If effective against a single organism or disease, they are referred to as limited spectrum antibiotics. Penicillin G has a narrow spectrum. Ampicillin and Amoxycillin are synthetic

modifications of penicillin. These have broad spectrum. Chloramphenicol. isolated in 1947, is a broad spectrum antibiotic.

Question: Statement 1: Emulsion of water and oil is unstable and separates in two layers.
Statement 2: It is stabilized by added excess electrolytes.
Options:
(a) Both S1 and S2 are correct.
(b) S1 is correct but S2 is incorrect.
(c) S1 is incorrect but S2 is correct.
(d) Both S1 and S2 are incorrect.
Answer: (b)
Solution: Emulsions of oil in water are unstable and sometimes they separate into two layers on standing. For stabilization of an emulsion, a third component called emulsifying agent is usually added.

Question: A(g) ⇌ B(g) + C/2(g)
Find relationship between K, alpha, equilibrium pressure P.
Options:

(a) $K_p = \dfrac{\alpha^{\frac{3}{2}} P^{\frac{1}{2}}}{(2+\alpha)^{\frac{1}{2}}}$

(b) $K_p = \dfrac{\alpha^{\frac{1}{2}} P^{\frac{1}{2}}}{(2+\alpha)^{\frac{1}{2}}}$

(c) $K_p = \dfrac{\alpha^{\frac{1}{2}} P^{\frac{1}{2}}}{(2+\alpha)^{\frac{3}{2}}}$

(d) $K_p = \dfrac{\alpha^{\frac{1}{2}} P^{\frac{3}{2}}}{(2+\alpha)^{\frac{3}{2}}}$

Answer: (a)
Solution:

	A(g) ⇌	B(g) +	$\frac{1}{2}$C(g)
initial	1	0	0
final	1–α	α	$\frac{\alpha}{2}$

Total mole = 1 – α + α + $\frac{\alpha}{2}$ = 1 + $\frac{\alpha}{2}$

$$x_A = \left(\frac{1-\alpha}{1+\frac{\alpha}{2}}\right)$$

$$x_B = \frac{\alpha}{1+\frac{\alpha}{2}}$$

$$x_C = \frac{\alpha}{2\left(1+\frac{\alpha}{2}\right)}$$

$$K_p = \frac{P_B - P_C}{P_A}$$

Purity of the value we get the solution

$$K_p = \frac{\left(\frac{\alpha}{1+\frac{\alpha}{2}}\right)P.\left[\frac{\alpha}{2\left(1+\frac{\alpha}{2}\right)}.P\right]^{\frac{1}{2}}}{\left(\frac{1-\alpha}{1+\frac{\alpha}{2}}\right)P}$$

$$K_p = \frac{\alpha\left(\frac{\alpha P}{2\left(1+\frac{\alpha}{2}\right)}\right)^{\frac{1}{2}}}{1-\alpha}$$

$$K_p = \frac{\alpha^{\frac{3}{2}}P^{\frac{1}{2}}}{(2+\alpha)^{\frac{1}{2}}}$$

Question: The molecule which has minimum role in photochemical smog

Options:

(a) HCHO

(b) N_2

(c) NO

(d) O_3

Answer: (b)

Solution: The common components of photochemical smog are ozone, nitric oxide, acrolein, formaldehyde and peroxyacetyl nitrate (PAN). Photochemical smog causes serious health problems. Both ozone and PAN act as powerful eye irritants. Ozone and nitric oxide irritate the nose and throat and their high concentration causes headache, chest pain, dryness of the throat, cough and difficulty in breathing.

Question: The difference between the oxidation number of Cr in chromate and dichromate ion is

Options:

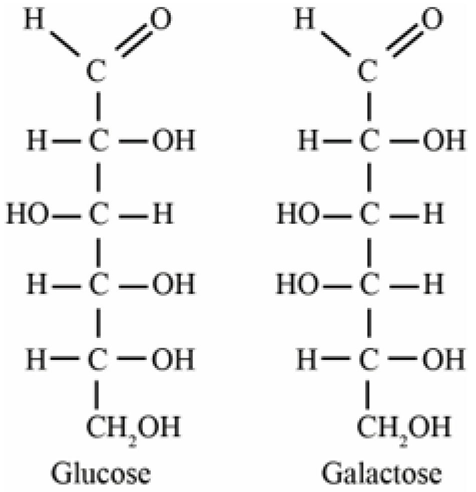
(a) 0

(b) 1

(c) 2

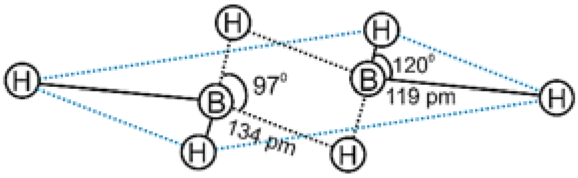
(d) 3
Answer: (a)
Solution: Oxidation number of Cr in chromate ion (CrO_4^{2-}) is +6
Oxidation number of Cr in dichromate ion ($Cr_2O_7^{2-}$) is also +6

Question: Galactose is which epimer of Glucose
Options:
(a) C-1
(b) C-2
(c) C-3
(d) C-4
Answer: (d)
Solution:



Question: Which of the following is stable nitrogen halide?
Options:
(a) NF_3
(b) NCl_3
(c) NBr_3
(d) NI_3
Answer: (a)
Solution: Nitrogen is an element of second period and it cannot hold 3 larger halogen atoms efficiently, hence NF_3 is the only stable halide.

Question: Which of the following in correct statement?
Options:
(a) B_2H_6 is Lewis Acid
(b) All the B-H bonds in B_2H_6 are equal
(c) B_2H_6 has planar structure
(d) Maximum no. of hydrogen in one plane is six
Answer: (a)
Solution: The structure of diborane is shown in Fig. The four terminal hydrogen atoms and the two boron atoms lie in one plane. Above and below this plane, there are two bridging hydrogen atoms. The four terminal B-H bonds are regular two centre-two electron bonds while the two bridge (B-H-B) bonds are different and can be described in terms of three



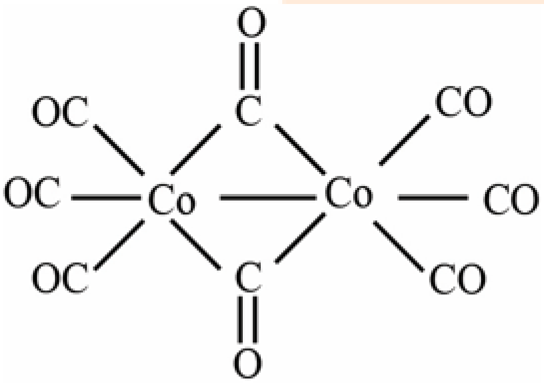
Structure of diborane, B_2H_6

Question: Match the ore with its formula.

Column-I	Column-II
(A) Calamine	(P) PbS
(B) Galena	(Q) $ZnCO_3$
(C) Sphalerite	(R) $FeCO_3$
(D) Siderite	(S) ZnS

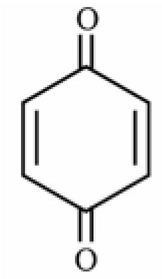
Options:
(a) (A) → (P); (B) → (Q); (C) → (R); (D) → (S)
(b) (A) → (Q); (B) → (P); (C) → (R); (D) → (S)
(c) (A) → (Q); (B) → (P); (C) → (R); (D) → (S)
(d) (A) → (P); (B) → (Q); (C) → (S); (D) → (R)
Answer: (c)
Solution:
Calamine → $ZnCO_3$
Galena → PbS
Sphalerite → ZnS
Siderite → $FeCO_3$

Question: In the structure of $[Co_2(CO)_8]$, x is the number of Co-Co bonds and y is the no of Co-CO terminal bonds. x + y?
Options:
(a) 6
(b) 8
(c) 4
(d) 7
Answer: (d)
Solution:

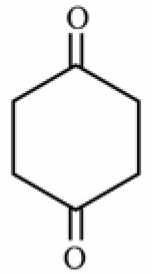


x = 1 and y = 6

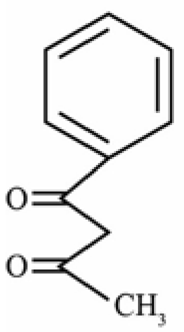
Question: Which is conjugate dione?
Options:
(a)



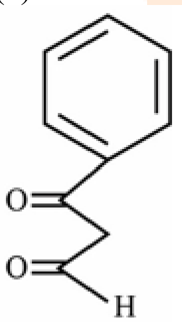
(b)



(c)

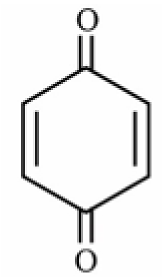


(d)



Answer: (a)

Solution:



It is a diketone with conjugation intact between both the functional groups.

Question: 2, 7 - dimethyl - 2, 6 octadiene $\xrightarrow[\Delta]{H^+}$. Find the number of sp^2 hybridized carbon in the product 'A'?

Options:

(a) 2

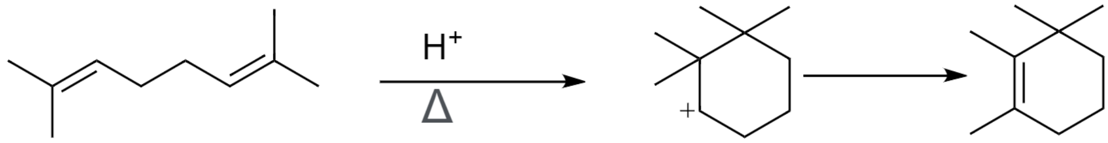
(b) 4

(c) 6

(d) 5

Answer: (a)

Solution:



2, 7 - dimethyl - 2, 6 octadiene

Question: Which of the following is a polyester?

Options:

(a) Dacron

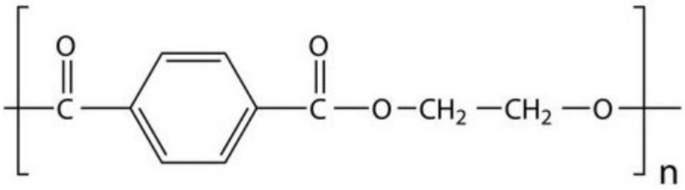
(b) Polyethene

(c) Teflon

(d) DNA

Answer: (a)

Solution:



Question: Which of the following have the maximum melting point?

Options:

(a) Acetic acid

(b) Formic acid

(c) Propanoic acid

(d) Butanoic acid

Answer: (a)

Solution: Carboxylic acids with even number carbon atoms have higher melting point than those with odd number carbons atom. Among acetic acid and butanoic acid the latter molecules face more repulsion hence acetic acid has higher melting point.

Question: In the production of which of the following compound, H_2 is used?

Options:

(a) CO_2

(b) NH_3

(c) P_4

(d) SO_2

Answer: (b)

Solution: $N_2 + 3H_2 \rightleftharpoons 2NH_3$

Question: X is hcp, Y is 2/3 of tetrahedral voids…. Find percentage of X in the lattice
Answer: 42.85
Solution: X is hcp
So atom per unit cell = 6

Y at $\frac{2}{3}$ of T-void = $\frac{2}{8} \times 12 = 8$

% of X = $\frac{6}{14} \times 100 = 42.85\%$

JEE-Main-24-06-2022-Shift-1 (Memory Based)

MATHEMATICS

Question: If the sum of the squares of the reciprocal of the roots α and β of the equation $3x^2+\lambda x+1=0$ is 15, then $6\left(\alpha^3+\beta^3\right)^2$ is equal to

Options:
(a) 18
(b) 24
(c) 36
(d) 96

Answer: (b)

Solution:

Given, $3x^2+\lambda x-1=0$

$\alpha+\beta=-\frac{\lambda}{3}$

$\alpha\beta=\frac{-1}{3}$

Also, $\frac{1}{\alpha^2}+\frac{1}{\beta^2}=1$

$\therefore \frac{\left(\alpha+\beta\right)^2-2\alpha\beta}{\left(\alpha\beta\right)^2}=15$

$\frac{\frac{\lambda^2}{9}+\frac{2}{3}}{\frac{1}{9}}=15$

$\lambda^2+6=15$

$\lambda^2=9$

$\lambda=\pm 3$

Now, $\left(\alpha^3+\beta^3\right)^2=6\left[\left(\alpha+\beta\right)^3-3\alpha\beta\left(\alpha+\beta\right)\right]^2$

$=6\left[\left(-1\right)^3-3\left(\frac{-1}{3}\right)\left(-1\right)\right]^2$

$=6\left[-1-1\right]^2$

$=24$

Question: Find the remainder when 3^{2022} is divided by 5:

Options:

(a) 1

(b) 2

(c) 4

(d) 0

Answer: (c)

Solution:

Given, 3^{2022}

$\left(3^2\right)^{1011}$

$(9)^{1011}$

$(10-1)^{1011}$

${}^{1011}C_0 10^{1011} - {}^{1011}C_1 10^{1010} + ... + {}^{1011}C_{1010} 10^1 - {}^{1011}C_{1011}$

$\therefore\ 10(\text{Integer})-1$

or $10(\text{Integer})-1-4+4$

$\Rightarrow 10(\text{Integer})-5+4$

$\Rightarrow 5(2\,\text{Integer}-1)+4$

$\therefore$ Remainder when it is divided by 5 will be '4'.

Question: The Boolean expression $(p \Rightarrow q) \wedge (q \Rightarrow \sim p)$ is equivalent to:

Answer: ()

Solution:

p	q	$p \rightarrow q$	$q \rightarrow \sim p$	$(p \rightarrow q) \wedge (q \rightarrow \sim p)$
T	T	T	F	F
T	F	F	T	F
F	T	T	T	T
F	F	T	T	T

Question: $\left(\tan^{-1} x\right)^3 + \left(\cot^{-1} x\right)^3 = k\pi^3$, find the range of k:

Options:

(a) $\left(\frac{1}{32}, \frac{9}{8}\right)$

(b) $\left[\frac{1}{32}, \frac{7}{8}\right)$

(c)

(d)

Answer: (b)

Solution:

$$\left(\tan^{-1}x\right)^3+\left(\cot^{-1}x\right)^3=k\pi^3$$

Now, $\left(\tan^{-1}x+\cot^{-1}x\right)^3-3\tan^{-1}x\cot^{-1}x\left(\tan^{-1}x+\cot^{-1}x\right)$

$$\frac{\pi^3}{8}-3\tan^{-1}x\left(\frac{\pi}{2}-\tan^{-1}x\right)\left(\frac{\pi}{2}\right)$$

Let $\tan^{-1}x=t$

$$\Rightarrow\frac{\pi^3}{8}-\frac{3\pi^2}{4}t+\frac{3\pi}{2}t^2$$

$$\Rightarrow\frac{3\pi}{2}\left[t^2-\frac{\pi}{2}t\right]+\frac{\pi^3}{8}$$

$$\Rightarrow\frac{3\pi}{2}\left[t^2-2\frac{\pi}{4}t+\frac{\pi^2}{16}-\frac{\pi^2}{16}\right]+\frac{\pi^3}{8}$$

$$\Rightarrow\frac{3\pi}{2}\left(\tan^{-1}x-\frac{\pi}{4}\right)^2-\frac{3\pi^3}{32}+\frac{\pi^3}{8}$$

Now, minimum value when $\tan^{-1}x=\frac{\pi}{4}$

$$\Rightarrow\frac{3\pi}{2}(0)+\frac{\pi^3}{32}$$

$$=\frac{\pi^3}{32}$$

Maximum when $\tan^{-1}x=-\frac{\pi}{2}$

$$\frac{3\pi}{2}\left(\frac{9\pi^2}{16}\right)+\frac{\pi^3}{32}$$

$$\Rightarrow\frac{28\pi^3}{32}$$

$$\Rightarrow 7\frac{\pi^3}{8}$$

So, $k\in\left[\frac{1}{32},\frac{7}{8}\right]$

Question: If $f(\theta)=\sin\theta+\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\sin\theta+t\cos\theta)\cdot f(t)\,d\theta$, then $\left|\int_0^{\frac{\pi}{2}}f(\theta)\,d\theta\right|$ is

Options:

(a) $1+\pi t\,f(t)$

(b) $1-\pi t f(t)$

(c)

(d)

Answer: (a)

Solution:

Given, $f(\theta)=\sin\theta+\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(\sin\theta+t\cos\theta)\cdot f(t)\,d\theta$

$=\sin\theta+\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin\theta\cdot f(t)\,d\theta+\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}t\cos\theta f(t)\,d\theta$

$=\sin\theta+f(t)\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin\theta\,d\theta+t\,f(t)\int\limits_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos\theta\,d\theta$

$\sin\theta+0+t\,f(t)[\sin\theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$

$f(\theta)=\sin\theta+2t\,f(t)$

Now, $\left|\int\limits_0^{\frac{\pi}{2}}f(\theta)\,d\theta\right|=\left|\int\limits_0^{\frac{\pi}{2}}(\sin\theta+2t\,f(t))\,d\theta\right|$

$=\left|\int\limits_0^{\frac{\pi}{2}}\sin\theta d\theta+2t\,f(t)\int\limits_0^{\frac{\pi}{2}}1\cdot d\theta\right|$

$=\left|[-\cos\theta]_0^{\frac{\pi}{2}}+2t\,f(t)\frac{\pi}{2}\right|$

$=|1+t\,f(t)(x)|$

$=1+\pi t\,f(t)$

Question: $\langle a_i\rangle$ sequence is an A.P. with common difference 1 and $\sum\limits_{i=1}^{n}a_i=192$, $\sum\limits_{i=1}^{\frac{n}{2}}a_{2i}=120$, then find the value of n, where n is an even integer.

Options:

(a) 48

(b) 96

(c) 18

(d) 36

Answer: (b)

Solution:

Given, $\sum_{i=1}^{n} a_i = 192 \Rightarrow a_1 + a_2 + ... + a_n = 192$

$$\Rightarrow \frac{n}{2}(a_1 + a_n) = 192$$

$$a_1 + a_n = \frac{384}{n} \qquad ...(1)$$

Also, $\sum_{i=1}^{\frac{n}{2}} a_{x_i} = 120$

$$\Rightarrow a_2 + a_4 + a_6 + ... + a_n = 120$$

$$\Rightarrow \frac{\frac{n}{2}}{2}[a_2 + a_n] = 120$$

$$\Rightarrow \frac{n}{4}[a_1 + 1 + a_n] = 120$$

$$a_1 + a_n + 1 = \frac{480}{n} \qquad ...(2)$$

Now, $\frac{384}{n} + 1 = \frac{480}{n}$

$$\Rightarrow \frac{96}{n} = 1$$

$$n = 96$$

Question: Find domain: $\dfrac{\cos^{-1}\left(\dfrac{x^2 - 5x + 6}{x^2 - 9}\right)}{\ln\left(x^2 - 3x + 2\right)}$

Answer: $\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \{3\}$

Solution:

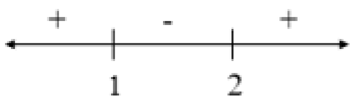
Given, $\dfrac{\cos^{-1}\left(\dfrac{x^2 - 5x + 6}{x^2 - 9}\right)}{\ln\left(x^2 - 3x + 2\right)}$

For domain

$$x^2 - 3x + 2 > 0$$

$$x^2 - 2x - x + 2 > 0$$

$$(x-2)(x-1) > 0$$



$x \in (-\infty, 1) \cup (2, \infty)$

&

$$-1 \le \frac{x^2 - 5x + 6}{x^2 - 9} \le 1$$

$$-1 \le \frac{(x-2)(x-3)}{(x-3)(x+3)} \le 1$$

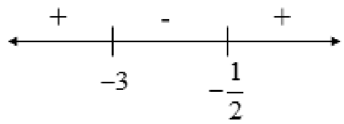
$x \ne 3$, then

$$-1 \le \frac{x-2}{x+3} \le 1$$

$$\frac{x-2}{x+3} \ge -1 \text{ and } \frac{x-2}{x+3} \le 1$$

$$\frac{x-2+x+3}{x+3} \ge 0 \text{ and } \frac{x-2-x-3}{x+3} \le 0$$

$$\frac{2x+1}{x+3} \ge 0 \text{ and } \frac{-5}{x+3} \le 0$$



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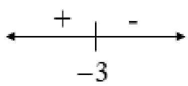

So, $x \in \left[-\frac{1}{2}, \infty\right)$

Combining both we get

$$\left[-\frac{1}{2}, 1\right) \cup (2, \infty) - \{3\}$$

Question: If $A = \begin{bmatrix} 1 & 0 & a \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$, where $a \in N$ from 1 to 50 and $\sum_{a=1}^{50} |adj\ A| = 100k$, then the value of k is:

Options:

(a) $\frac{1723}{2}$

(b) $\frac{1717}{2}$

(c) $\frac{1719}{4}$

(d) $\frac{1821}{4}$

Answer: (d)

Solution:

Given, $\sum_{a=1}^{50}|adj\ A| = 100k$

Now, $|adj\ A| = |A|^{n-1} = |A|^2$

Here, $|A| = a+1$

$\therefore |A|^2 = (a+1)^2$

$\sum |A|^2 = \sum_{a=1}^{50}(a+1)^2 = 2^2+3^2+4^2+...+51^2$

$= 1^2+2^2+...+51^2-1^2$

$= \frac{(51)(52)(103)}{6} - 1$

$= \frac{1821}{4}$

Question: A tangent $ax - \mu y = 2$ to hyperbola $\frac{a^4x^2}{\lambda^2} - \frac{b^2y^2}{1} = 4$, then the value of $\left(\frac{\lambda}{a}\right)^2 - \left(\frac{\mu}{b}\right)^2$ is:

Options:

(a) 0

(b) 1

(c) 2

(d) 3

Answer: (b)

Solution:

Given, $\frac{a^4x^2}{\lambda^2} - \frac{b^2y^2}{1} = 4$

$\Rightarrow \frac{x^2}{4\left(\frac{\lambda^2}{a^4}\right)} - \frac{y^2}{4\left(\frac{1}{b^2}\right)} = 1$

$\Rightarrow \frac{x^2}{\left(\frac{2\lambda}{a^2}\right)^2} - \frac{y^2}{\left(\frac{2}{b}\right)^2} = 1$

Now, tangent to above hyperbola is

$y = mx \pm \sqrt{a^2m^2 - b^2}$

Here, $ax - \mu y = 2$

$\Rightarrow \mu y = ax - 2$

$y=\frac{a}{\mu}x-\frac{2}{\mu}$

$\Rightarrow -\frac{2}{\mu}=\sqrt{\frac{4\lambda^2}{a^4}\cdot\frac{a^2}{\mu^2}-\frac{4}{b^2}}$

$\Rightarrow \frac{4}{\mu^2}=\frac{4\lambda^2}{a^2\mu^2}-\frac{4}{b^2}$

$\Rightarrow \left(\frac{\lambda}{a}\right)^2-\left(\frac{\mu}{b}\right)^2=1$

Question: A tangent at (x_1, y_1) to the curve $y=x^3+2x^2+4$ and passes through origin, then (x_1, y_1) is:

Options:

(a) $(0,4)$

(b) $(-1,5)$

(c) $(1,7)$

(d) $(2,20)$

Answer: (c)

Solution:

Given, $y=x^3+2x^2+4$

$\therefore \frac{dy}{dx}=3x^2+4x$

At $(x_1, y_1)\frac{dy}{dx}=3x_1^2+4x_1$

Now, slope of tangent is also

$m=\frac{y_1-0}{x_1-0}=\frac{y_1}{x_1}$

$\Rightarrow \frac{y_1}{x_1}=3x_1^2+4x_1$

$y_1=3x_1^3+4x_1^2$

$\Rightarrow x_1^3+2x_1^2+4=3x_1^3+4x_1^2$

$2x_1^2+2x_1^2-4=0$

$x_1^3+x_1^2-2=0$

Here, $x_1=1$, satisfying equation

$\therefore x_1=1,\ y_1=7$

$(1,7)$

Question: A circle of equation $x^2+y^2+ax+by+c=0$ passes through (0, 6) and touches $y=x^2$ at (2, 4). Find $a+c$.

Answer: 16.00

Solution:

Given, $x^2+y^2+ax+by+c=0$ passes through (0, 6)

Then, $0+36+0+6b+c=0$

$6b+c=-36 \quad ...(1)$

Touches $y=x^2$ at $(2,4)$

Thus, passes through $(2,4)$ and also tangent at $(2,4)$ is having same slope.

$\therefore\ 4+16+2a+4b+c=0$

$2a+4b+c=-20 \quad ....(2)$

Now, for slope

$\frac{dy}{dx}=2x$

$\left.\frac{dy}{dx}\right|_{(2,4)}=4$

For circle

$2x+2y\frac{dy}{dx}+a+b\frac{dy}{dx}=0$

$\frac{dy}{dx}=\frac{-a-2a}{2y+b}$

$\left.\frac{dy}{dx}\right|_{(2,4)}=\frac{-a-4}{8+b}$

$\therefore\ 4=\frac{-a-4}{8+b}$

$4b+32=-a-4$

$a+4b=-36 \quad ....(3)$

Putting in (2)

$a+a+4b+c=-20$

$a+(-36)+c=-20$

$a+c=16$

Question: $S=\left\{\theta:\theta\in[-\pi,\pi]-\left\{\pm\frac{\pi}{2}\right\} \text{ \& } \sin\theta\tan\theta+\tan\theta=\sin 2\theta\right\}$. Let $T=\sum\cos 2\theta$ where $\theta\in S$, then $T+n(S)=?$

Answer: 9.00

Solution:

Here, $\sin\theta\tan\theta+\tan\theta-\sin\theta=0$

$\Rightarrow \sin\theta[\tan\theta+\sec\theta-2\cos\theta]=0$

$\tan\theta(2\sin^2\theta+\sin\theta-1)=0$

$\tan\theta=0,\ \sin\theta=-1,\ \sin\theta=\frac{-1}{2}$

$\theta=0,\pi,-\pi,\ \theta=-\frac{\pi}{2},\ \theta=\frac{\pi}{6},\frac{5\pi}{6}$

But $\theta\neq-\frac{\pi}{2}$

$\therefore n(S)=5$

Now $T=\sum\cos 2\theta,\ \theta\in S$

$\Rightarrow T=\cos 2(-\pi)+\cos(2\pi)+\cos(0)+\cos 2\left(\frac{\pi}{6}\right)+\cos 2\left(\frac{5\pi}{6}\right)$

$=1+1+1+\frac{1}{2}+\frac{1}{2}$

$=4$

$\therefore T+n(S)=4+5=9$

Question: Image of $A\left(\frac{3}{\sqrt{a}},\sqrt{a}\right)$ in y-axis is B & image of B is x-axis is C. Point $D(3\cos\theta,a\sin\theta)$ lies in 4th quadrant. If maximum area of $\Delta ACD=12$ sq. units, then find a.

Answer: 8.00

Solution:

Image of $A\left(\frac{3}{\sqrt{a}},\sqrt{a}\right)$ in y-axis will be $\left(-\frac{3}{\sqrt{a}},\sqrt{a}\right)B,$

Also, image of B in x-axis will be $\left(-\frac{3}{\sqrt{a}},-\sqrt{a}\right)C$

Given, $D(3\cos\theta,a\sin\theta)$ and IVth quadrant

$$\text{Area of } \Delta ACD=\frac{1}{2}\begin{vmatrix}\frac{3}{\sqrt{a}} & \sqrt{a} & 1\\ -\frac{3}{\sqrt{a}} & -\sqrt{a} & 1\\ 3\cos\theta & a\sin\theta & 1\end{vmatrix}$$

$$=\frac{1}{2}\begin{vmatrix} 0 & 0 & 2 \\ -\frac{3}{\sqrt{a}} & -\sqrt{a} & 1 \\ 3\cos\theta & a\sin\theta & 1 \end{vmatrix}\left(R_1 \to R_1 + R_2\right)$$

$$12=\frac{1}{2}\left|2\left(-3\sqrt{a}\sin\theta+3\sqrt{a}\cos\theta\right)\right|$$

$$12=3\sqrt{a}\left(\cos\theta-\sin\theta\right)$$

Now, maximum value of $\cos\theta-\sin\theta=\sqrt{2}$

$$\therefore 12=3\sqrt{a}\left(\sqrt{2}\right)$$

$$\sqrt{a}=\frac{12}{3\sqrt{2}}$$

$$a=\frac{16}{2}=8$$