JEE-Main-24-06-2022-Shift-2 (Memory Based)

Physics

Question: A proton, deuteron and an alpha particle enter a magnetic field with same energy find the ratio of their radii of curvature

Options:

(a) $1:\sqrt{2}:\sqrt{2}$

(b) $1:\sqrt{2}:1$

(c) $\sqrt{2}:1:1$

(d) $\sqrt{2}:\sqrt{2}:1$

Answer: (b)

Solution: Correct option is B

$\frac{mv^2}{R} = qvB$

For proton, $R_p = \frac{mv}{Bq} = \frac{\sqrt{2M_pE}}{q_pB}$

Similarly for deuteron and α – particle

$R_d = \frac{\sqrt{2M_dE}}{q_dB}$ and $R_\alpha = \frac{\sqrt{2M_\alpha E}}{q_\alpha B}$

According to the question

$\therefore R_p : R_d : R_\alpha$

Or, $\frac{\sqrt{M_p}}{q_p} : \frac{\sqrt{M_d}}{q_d} : \frac{\sqrt{M_\alpha}}{q_\alpha}$

$\therefore \frac{\sqrt{1}}{1} : \frac{\sqrt{2}}{1} : \frac{\sqrt{4}}{2}$

Or $1:\sqrt{2}:1$.

Question: Read the assertion and reason carefully to mark the correct option out of the options given below.

Assertion: An ac circuit can be created with 0 reactance.

Reason: An ac circuit without power is not possible.

Options:

(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.

(b) If both assertion and reason are true, but the reason is not the correct explanation of the assertion.

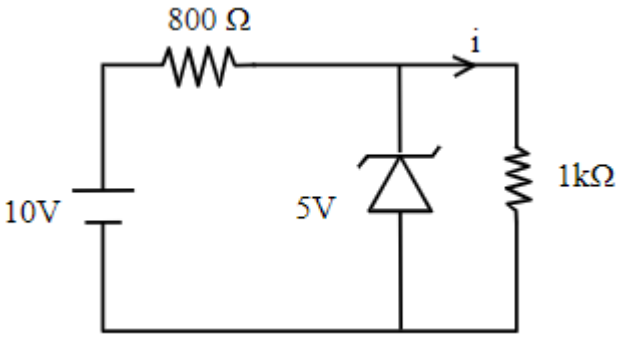
(c) If assertion is true, but reason is false.

(d) If both the assertion and reason are false.

Answer: (c)

Solution: In a circuit, reactance can be zero either if there are no inductors and capacitors in the circuit, or the individual reactance of inductors and capacitors cancel each other, making net reactance zero. So, statement 1 is true.
But an ac circuit will always consume some power.
So, Reason is false.

Question: For the circuit diagram shown below, calculate the load current across $1k\Omega$ resistor.
$i = ?$



Answer: 5.00
Solution:
Voltage across $1k\Omega$ resistor

$V = 5v$

$$I = \frac{5}{1\times10^{3}} = 5\times10^{-3}\,A = 5mA$$

Question: Two intensities are in ratio 9:4. Find ratio of $I_{max} : I_{min}$
Answer: 25.00
Solution:
Given $\frac{I_2}{I_1} = \frac{9}{4}$

Maximum intensity $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

Minimum Intensity $I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

Thus ratio of maximum to minimum intensities $\frac{I_{max}}{I_{min}} = \frac{\left(1+\frac{\sqrt{I_2}}{\sqrt{I_1}}\right)}{\left(1-\frac{\sqrt{I_2}}{\sqrt{I_1}}\right)^2}$

$$\therefore \frac{I_{max}}{I_{min}} = \frac{\left(1+\frac{3}{2}\right)^2}{\left(1-\frac{3}{2}\right)^2}$$

Or $\frac{I_{max}}{I_{min}} = \frac{(2+3)^2}{(2-3)^2} = \frac{25}{1}$

Question: A 100 gram nail is being hit by a hammer of 1.5kg with velocity of 60 m/s, if one fourth of the energy is being utilized to melt the nail, then find the change in temperature. (Specific heat of nail = 0.42 J per gram per degree Celsius)

Answer: 2.00

Solution: Heat given $\Delta Q = Q$

$$\Delta\omega = \frac{Q}{4}$$

From FLOT

$$\Delta Q = \Delta\omega + \Delta U$$

$$\Rightarrow \Delta U = Q - \frac{Q}{4} = \frac{3Q}{4}$$

For mono atomic gas

$$\Delta U = \frac{3}{2}nR\partial T = \frac{3Q}{4}$$

$$\Rightarrow Q = 2nR\partial T$$

So, $2nR\partial T = nC_p\partial T$

$$C_p = 2R$$

Question: A particle of mass 5kg is thrown upwards air friction is 10N find ratio of time of flight of ascent and descend

Options:

(a) 1:1

(b) $B\sqrt{\frac{2}{3}}$

(c) $\frac{2}{3}$

(d) $\sqrt{\frac{3}{2}}$

Answer: (b)

Solution:

Correct option is (B)

$$v = u + at$$

$$0 = u + -(g+2)t$$

$$t = \frac{u}{g+2}$$

$$0 - u^2 = 2as - u^2 = 2 \times -(g+2)s$$

$$s = u^2 / 2(g+2)$$

$$s = \frac{1}{2}aT^2$$

$$\frac{u^2}{2(g+2)} = \frac{1}{2} \times (g-2)T^2$$

$$\frac{u}{\sqrt{(g^2-4)}} = T$$

$$\frac{t}{T} = \frac{\sqrt{g}-2}{\sqrt{g}+2} = \sqrt{\frac{8}{12}} = \sqrt{\frac{2}{3}}$$

Question: A Carnot engine absorbs 5000 kcal from reservoir at 727°C and rejects to the sink at 127°C. Find the work done by engine

Answer: 12.6

Solution:

$Q_1 = 5000\text{kcal}$

$T_1 = 727 + 273 = 1000\,\text{K}$

$T_2 = 127 + 273 = 400\,\text{K}$

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{1000-400}{1000} = 0.6$$

Also,

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{5000 - Q_2}{3000} = 0.6$$

$\Rightarrow Q_2 = 5000 - 3000 = 2000\text{kcal}$

$W = Q_1 - Q_2 = 5000 - 2000 = 3000\text{kcal}$

$\Rightarrow W = 3 \times 4.2 \times 10^6\,\text{J} = 12.6 \times 10^6\,\text{J}$

Question: Two massless springs, with spring constants 2k and 9k respectively having 50g and 100g attached at free and, both have same Vmax, then find ratio of amplitudes of vibrations.

Answer: 3:1

Solution:

Masses are executive SHM

Given,

$V_1 = V_2$

$A_1\omega_1 = A_2\omega_2$

$$\frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} \ldots\text{(i)}$$

we know that

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\Rightarrow \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

$\Rightarrow \omega = \sqrt{\frac{k}{m}}$

$\omega = \sqrt{\frac{2k}{50g}}$ and $\omega_2 = \sqrt{\frac{9k}{100g}}$

From…(i)

$\frac{A_1}{A_2} = \sqrt{\frac{9k}{100} \times \frac{50}{2k}} = \frac{3}{2}$

Question: Photon of 3.2 ev and 1.4 ev are bombarded on metal with work function 0.8 ev. Then the ratio of maximum velocities of ejected electrons are

Answer: 2:1

Solution:

$k_{max} = E - \phi$

$k_1 = 3.2 - 0.8 = 2.4\text{eV}$

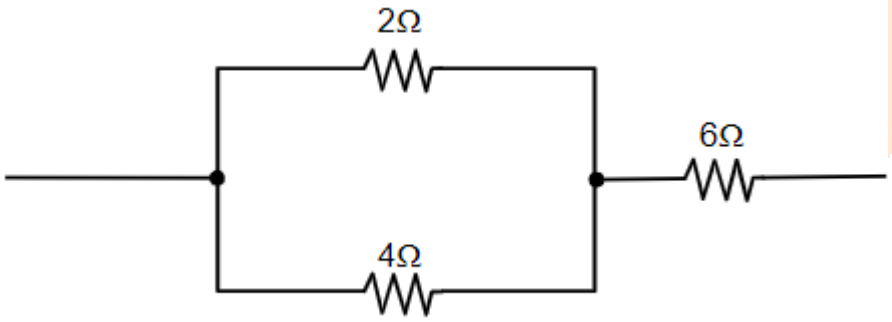
$k_2 = 1.4 - 0.8 = 0.6\text{eV}$

$\frac{k_1}{k_2} = \frac{2.4}{0.6} = \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2}$

$\Rightarrow \left(\frac{v_1}{v_2}\right)^2 = \frac{4}{1}$

$\Rightarrow \frac{v_1}{v_2} = \frac{2}{1}$

Question: If $A = 2\Omega, B = 4\Omega 8C = 6\Omega.$ Arrange such that $\text{Req} = \frac{22}{3}\Omega$



Answer: 22:3

Solution:

$\frac{1}{R'} = \frac{1}{2} + \frac{1}{4}$

$R' = \frac{8}{6} = \frac{4}{3}\Omega$

$R_{eq} = \frac{4}{3} + 6 = \frac{4+18}{3} = \frac{22}{3}\Omega$

Question: The number of turns in a coil are 1000 with the area as 1 m^2 present in a uniform magnetic field of 0.07T. It is rotating along its vertical diameter with one revolution per second, find the maximum emf generated.

Answer: 439.6V

Solution:

$\varepsilon = 2\pi fNAB\sin\omega t$

$\varepsilon_{max} = 2\pi fNAB$

$\varepsilon_{max} = 2\pi \times 1 \times 1000 \times 1 \times 0.07$

$\varepsilon_{max} = 439.6\,V$

Question: A glass slab shows the lateral displacement of $4\sqrt{3}$ cm when a light is incident at an angle of 60 degrees, the refractive index of slab is $\sqrt{3}$, the light emerges parallel to its original path, then the thickness of slab would be __ (Integer type)

Answer: 12.00

Solution:

Lateral shift $d = \dfrac{t\sin(i-r)}{\cos r}$

Given $i = 30^\circ$ and $n = \sqrt{3}$

$n_1 \sin i = n_2 \sin r$

$\sin 60^\circ = \sqrt{3}\sin r$

$\dfrac{\sqrt{3}}{2} = \sqrt{3}\sin r$

$r = 30^\circ$

$d = u\sqrt{3} = \dfrac{t\sin\left(60^\circ - 30^\circ\right)}{\cos 30^\circ}$

$\Rightarrow u\sqrt{3} = t\tan 30^\circ = \dfrac{1}{\sqrt{3}}$

$\Rightarrow t = 4\times 3 = 12\,cm$

Question: A particle is thrown from ground such that at t = 2 sec its velocity is 20 ms-1 & it makes an angle of 45° with horizontal. Find max height?

Answer: 58.28

Solution:

At t = 2 sec

$V_x = 20\cos 45^\circ = \dfrac{20}{\sqrt{2}}\ m/s$

$V_y = V_y - gt$

$U_y = \dfrac{20}{\sqrt{2}} + 20 = 10\sqrt{2} + 20$

$H_{max} = \dfrac{v_y^2}{2g} = \dfrac{\left(10\sqrt{2}+20\right)^2}{20} = 58.28m$

Question: If the distance between the sun and earth is 3 times the present distance then find the new time of revolution of earth around sun in years.

Answer: $3\sqrt{3}$

Solution:

$$T_1^2 = r^3$$

$$T_2^2 = (3r)^3$$

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{3r}{r}\right)^3$$

$$\frac{T_2}{1} = (3)^{3/2}$$

$$T_2 = 3\sqrt{3} \text{ years}$$

Question: A string tied to an object of mass m on one end is rotated in a vertical circle with uniform velocity v, then tension in the string is

Options:

(a) Same throughout

(b) Maximum at top

(c) Minimum at top

(d) Minimum at bottom

Answer: (a, b)

Solution:

For bottom position

$$T = Mg + \frac{mv^2}{r}$$

For Top position

$$T = Mg + \frac{mv^2}{r} - mg$$

So Tension is minimum at top.

Question: Charge on capacitor is increased by 2C and energy stored becomes 44%. Find initial charge

Answer: 10 C

Solution:

$$\text{We know that } E = \frac{Q^2}{2C}$$

Increase in energy

$$= \frac{\frac{Q_f^2}{2C} - \frac{Q_1^2}{2C}}{\frac{Q_i^2}{2C}} \times 100 = 44 \qquad \text{....(i)}$$

Given

$Q_f = Q_i = 2$ .....(ii)

$2(Q_f + Q_i) = 0.44Q_1^2$

$2(2 + 2Q_i) = 0.44Q_1^2$

$11Q_i^2 - 100Q_i - 100 = 0$

$11Q_i^2 - 110Q_j + 10Q_j - 100 = 0$

$11Q_i(Q_i - 10) + 10(Q_i - 10) = 0$

$(Q_i - 10)(10Q_i + 10) = 0$

$Q_i = 10C$

Question: Q heat is given to a system containing monatomic gas. Q/4 work is done by the gas, then molar heat capacity of the gas is?

Answer: 2R

Solution:

Heat given $\Delta Q = Q$

$\Delta\omega = \frac{Q}{4}$

From FLOT

$\Delta Q = \Delta w + \Delta V$

$\Rightarrow \Delta V = Q = \frac{Q}{4} = \frac{3Q}{4}$

For monoatomic gas

$\Delta V = \frac{3}{2}nRdT = \frac{3Q}{4}$

$\Rightarrow Q = 2nRdT$

So,

$2nRdT = nC_pdT$

$C_P = 2R$

JEE-Main-24-06-2022-Shift-2 (Memory Based)

Chemistry

Question: Which gas is not green House gas?
Options:
(a) H_2O vapour
(b) O_3
(c) N_2
(d) CH_4
Answer: (c)
Solution: Besides carbon dioxide, other greenhouse gases are methane, water vapour, nitrous oxide, CFCs and ozone. Methane is produced.

Question: Which metal gives blue flame test?
Options:
(a) Cesium
(b) Lithium
(c) Barium
(d) Strontium
Answer: (a)
Solution: Cesium gives blue flame test.

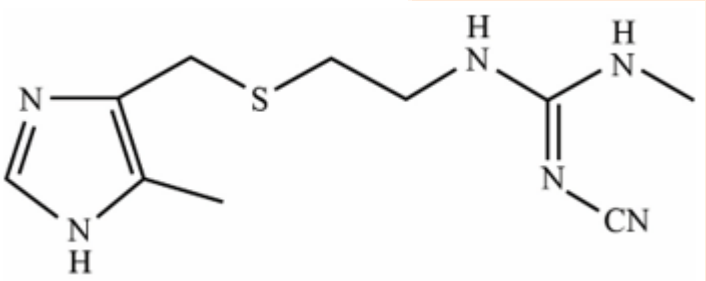
Question: Which of the following from doesn't exist in enamel?
Options:
(a) F^-
(b) Ca^{+2}
(c) P^{+3}
(d) P^{+5}
Answer: (c)
Solution: The F^- ions make the enamel on teeth much harder by converting hydroxyapatite, $[3(Ca_3(PO_4)_2.Ca(OH)_2]$, the enamel on the surface of the teeth, into much harder fluorapatite, $[3(Ca_3(PO_4)_2.CaF_2]$.

Question: Correct order of bond order of C_2^{2-}, N_2^{2-}, O_2^{2-}
Options:
(a) $C_2^{2-} > N_2^{2-} > O_2^{2-}$
(b) $N_2^{2-} > C_2^{2-} > O_2^{2-}$
(c) $C_2^{2-} > O_2^{2-} > N_2^{2-}$
(d) $O_2^{2-} > N_2^{2-} > C_2^{2-}$
Answer: (a)
Solution: Bond order of
$C_2^{2-} = 3$

$N_2^{2-} = 2$
$O_2^{2-} = 1$

Question: Which of the following has highest melting point?
Options:
(a) Ag
(b) Cs
(c) Hg
(d) Ga
Answer: (a)
Solution: Factual

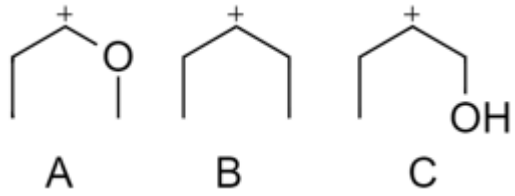
Question: Compound in the given structure is



Options:
(a) Codeine
(b) Morphine
(c) Ranitidine
(d) Cimetidine
Answer: (d)
Solution: The drug cimetidine was designed to prevent the interaction of histamine with receptors present in the stomach wall.

Question: H_2 gas is produced in the preparation of:
Options:
(a) Na_2CO_3
(b) NaOH
(c) Na metal
(d) $NaHCO_3$
Answer: (b)
Solution: NaOH is prepared by electrolysis of brine solution
$NaCl \rightarrow Na^+ + Cl^-$
$H_2O \rightarrow H^+ + OH^-$
On cathode $2H^+ + 2e^- \rightarrow H_2$
On anode $2Cl^- \rightarrow Cl_2 + 2e^-$

Question: Compare the stability of carbocations



Options:
(a) A > B > C
(b) B > C > A
(c) A > C > B
(d) C > A > B
Answer: (a)
Solution: In A lone pairs of oxygen causes extra stability of carbocation through resonance
In B +I effect of two alkyl groups increase the stability of carbocation
In C –I effect of OH decreases the stability of carbocation

Question: The CFSE is maximum for:
Options:
(a) $[Mo(H_2O)_6]^{3+}$
(b) $[Cr(H_2O)_6]^{3+}$
(c) $[Mn(H_2O)_6]^{3+}$
(d) $[Fe(H_2O)_6]^{3+}$
Answer: (a)
Solution: In all the given 4 compounds oxidation number of central metal atom and ligands are same. In such case CFSE depends on the electronic configuration of central metal atom. CFSE increases as we move downwards in a group in greater extent than the increase in period from left to right.

Question: Which of these chemicals are used in fire extinguishers?
Options:
(a) Soda ash
(b) Caustic Soda
(c) Baking soda
(d) Washing soda
Answer: (c)
Solution: Sodium hydrogen carbonate is known as baking soda because it decomposes on heating to generate bubbles of carbon dioxide (leaving holes in cakes or pastries and making them light and fluffy). Sodium hydrogen carbonate is made by saturating a solution of sodium carbonate with carbon dioxide. The white crystalline powder of sodium hydrogen carbonate, being less soluble, gets separated out. $Na_2CO_3 + H_2O + CO_2 \rightarrow 2NaHCO_3$
Sodium hydrogen carbonate is a mild antiseptic for skin infections. it is used in lire extinguishers.

Question: What is energy of one mole of photons of wavelength 300 nm?
Options:
(a) $3 \times 10^{-19}\ N_A$
(b) $5 \times 10^{-19}\ N_A$

(c) $6 \times 10^{-19}\ N_A$
(d) $7 \times 10^{-19}\ N_A$
Answer: (a)
Solution:

$$E = N_A \times \frac{hc}{\lambda}$$

$$E = \frac{N_A \times 20 \times 10^{-26}}{300 \times 10^{-9}}$$

$$E = \frac{N_A \times 2 \times 10^{-25}}{3 \times 10^{-7}}$$

$E = N_A \times 0.6 \times 10^{-18}$
$E = 6 \times 10^{-19}\ N_A$

Question: For the equilibrium, A(g) ⇋ B(g), ΔH = -40 kJ/mol. The ratio of the activation energies of the forward (E_f) and reverse (E_b) reactions is 2/3 then what is E_f
Options:
(a) 40 kJ/mol
(b) 80 kJ/mol
(c) 120 kJ/mol
(d) 60 kJ/mol
Answer: (b)
Solution:
$\Delta H = E_f - E_b$
$E_f - E_b = -40$

$$\frac{E_f}{E_b} = \frac{2}{3}$$

$$E_b = \frac{3}{2}\ E_f$$

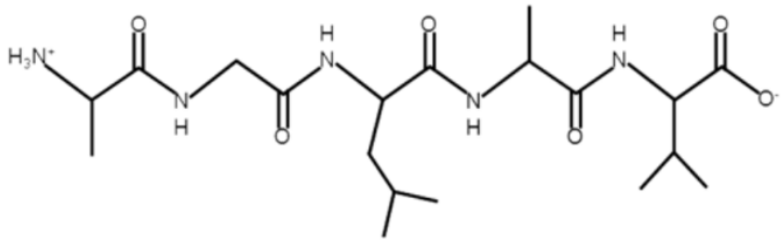
$$E_f - \frac{3}{2} E_f = -40$$

$$\frac{1}{2}\ E_f = 40$$

$E_f = 80$ kJ/mol

Question: Which of the ores have sulphide in them? (Integer based)
1. Baryte
2. Galena
3. Zinc Blende
4. Copper Pyrites
Answer: 3.00
Solution:
1. Baryte ⇒ $BaSO_4$
2. Galena ⇒ PbS
3. Zinc Blende ⇒ ZnS
4. Copper Pyrites ⇒ $CuFeS_2$

Question: Alanylglycylleulalanylvaline. No. of peptide linkage (Integer type)
Answer: 4.00
Solution:



There are five amino acids and four peptide linkages in the given fragment

Question: Volume occupied by a 3g of gas A at 300 K is same as occupied by 0.2g of H_2 at 200K. What is molar mass of A (in g/mol).
Answer: 45.00
Solution:

$n_1T_1 = n_2T_2$

$$\frac{3}{m} \times 300 = \frac{0.2}{2} \times 200$$

$$\frac{900}{20} = m$$

$m = 45$ g/mol

JEE-Main-24-06-2022-Shift-2 (Memory Based)

MATHEMATICS

Question: Find the sum of roots of $\left(e^{2x}-4\right)\left(6e^{2x}-5e^{x}+1\right)=0$

Options:

(a) $\ln 6$

(b) $\ln 3$

(c) $-\ln 3$

(d) $\ln 2$

Answer: (c)

Solution:

Let $e^{x}=t \Rightarrow \left(t^{2}-4\right)\left(6t^{2}-5t+1\right)=0$

$\left(t^{2}-4\right)(3t-1)(2t-1)=0$

$\Rightarrow t=-2,\frac{1}{3},\frac{1}{2},2$

$\because t>0 \Rightarrow \therefore t=\frac{1}{3},\frac{1}{2},2 \Rightarrow e^{x}=\frac{1}{3},\frac{1}{2},2$

$x=\ln\left(\frac{1}{3}\right),\ln\left(\frac{1}{2}\right),\ln(2)$

$\therefore \text{ sum } =\ln\left[\frac{1}{3}\times\frac{1}{2}\times 2\right]=\ln\left(\frac{1}{3}\right)=-\ln 3$

Question: $\lim\limits_{n\to\infty}\sum\limits_{r=1}^{n}\frac{n^{2}}{\left(n^{2}+r^{2}\right)(n+r)}$:

Options:

(a) $\frac{1}{4}\ln 2+\frac{\pi}{8}$

(b) $\frac{1}{4}\ln 2-\frac{\pi}{8}$

(c) $-\frac{1}{4}\ln 2-\frac{\pi}{8}$

(d)

Answer: (a)

Solution:

$$\lim_{n\to\infty}\frac{1}{n}\sum_{r=1}^{n}\frac{1}{\left[1+\left(\frac{r}{n}\right)^{2}\right]\left[1+\left(\frac{r}{n}\right)\right]}=\int_{0}^{1}\frac{dx}{\left(1+x^{2}\right)(1+x)}$$

$$= \frac{1}{2}\int_0^1 \frac{dx}{1+x} - \frac{1}{2}\int_0^1 \left(\frac{x-1}{x^2+1}\right)dx$$

$$= \frac{1}{2}\left[\log(1+x) - \frac{1}{2}\log\left(1+x^2\right) + \tan^{-1}(x)\right]_0^1$$

$$= \frac{1}{2}\left[\log 2 - \frac{1}{2}\log 2 + \frac{\pi}{4}\right]$$

$$= \frac{1}{4}\log 2 + \frac{\pi}{8}$$

Question: If $\Delta_r = \begin{vmatrix} 2^{r-1} & \dfrac{(r+1)!}{\left(1+\frac{1}{r}\right)} & 4r^3 - 2nr \\ a & b & c \\ 2^n - 1 & (n+1)! - 1 & n^3(n+1) \end{vmatrix}, n \in N$ then $\sum_{r=1}^{n} \Delta_r =$

Options:

(a) abc

(b) $(n+3)!$

(c) 0

(d) $a(n!) + b.2^n + c$

Answer: (c)

Solution:

$$\sum_{r=1}^{n} 2^{r-1} = 1 + 2 + 2^2 + \ldots + 2^{n-1} = 1\left(2^n - 1\right) = 2^n - 1$$

$$\sum_{r=1}^{n}\left(4r^3 - 2nr\right) = 4\left[\frac{n(n+1)}{2}\right]^2 - 2n\left[\frac{n(n+1)}{2}\right]$$

$$= n^2(n+1)^2 - n^2(n+1)$$

$$= n^3(n+1)$$

$$\sum_{r=1}^{n} \frac{(r+1)!}{1+\frac{1}{r}} = \sum_{r=1}^{n} r \times r! = \sum_{r=1}^{n}\left[(r+1)! - r!\right]$$

$$= (n+1)! - 1$$

$$\sum_{r=1}^{n}\Delta_r=\begin{vmatrix}\sum_{r=1}^{n}2^{r-1} & \sum_{r=1}^{n}\frac{(r+1)!}{\left(1+\frac{1}{r}\right)} & \sum_{r=1}^{n}\left(4^3-2nr\right)\\ a & b & c\\ 2^n-1 & (n+1)!-1 & n^3(n+1)\end{vmatrix}$$

$$=\begin{vmatrix}2^n-1 & (n+1)!-1 & n^3(n+1)\\ a & b & x\\ 2^n-1 & (n+1)!-1 & n^3(n+1)\end{vmatrix}$$

$$=0$$

Question: Find remainder when: $1+3+3^2+3^3+....+3^{2021}$ is divided by 50.

Answer: 4.00

Solution:

$$1+3+3^2+3^3+....+3^{2021}=\frac{3^{2022}-1}{2}=\frac{9^{1011}-1}{2}$$

$$=\frac{(10-1)^{1011}-1}{2}=\frac{{}^{1011}C_0(10)^{1011}+.....-{}^{1011}C_{1009}(10)^2+10110-1-1}{2}$$

$$=\frac{100k+10108}{2}=50k+5054=50p+4$$

$\therefore$ Remainder $=4$

Question: $1,2,3,4,5,6,9\rightarrow$ seven digit number multiple of 11?

Answer: 432.00

Solution:

Sum of digit at even place $=x$

Sum of digit at odd place $=y$

$\because x+y=30, \therefore x-y=0$

$x-y=\pm 11$

Only possibility is $x+y=30$ and $x-y=0$

$\therefore x=15,\ y=15$

So digit at even place is $\{1,5,9\},\{2,4,9\},\{4,5,6\}$

Odd place is $\{2,3,4,6\},\{1,3,5,6\},\{1,2,3,9\}$ respectively.

$\therefore$ Total number $=3(3!\cdot 4!)=18\times 24=144\times 3=432$

Question: $\cos\left(x+\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{3}\right)=\frac{1}{4}\cos^2(2x)$. Find number of solutions $[-3\pi,3\pi]$?

Answer: 7.00

Solution:

Given, $\cos\left(x+\frac{\pi}{3}\right)\cos\left(x-\frac{\pi}{3}\right)=\frac{1}{4}\cos^2(2x)$

$\Rightarrow \cos^2 x-\sin^2\frac{\pi}{3}=\frac{1}{4}\cos^2 2x$

$\Rightarrow \cos^2 x-\frac{3}{4}=\frac{1}{4}\cos^2 2x$

$\Rightarrow 4\cos^2 x-3=\cos^2 2x$

$\Rightarrow 4\cos^2 x-3=\left(2\cos^2 x-1\right)^2$

$\Rightarrow 4\cos^2 x-3=4\cos^4 x+1-4\cos^2 x$

$\Rightarrow 4\cos^2 x-8\cos^2 x+4=0$

$\Rightarrow \cos^4 x-2\cos^2 x+1=0$

$\Rightarrow \left(\cos^2 x-1\right)^2=0$

$\Rightarrow \cos^2 x=1$

$\cos x=\pm 1$

In $[-3\pi,3\pi]$

$\cos x=1$, at $x=0,-2\pi,2\pi$

$\cos=-1$, *at* $x=\pi,-\pi,-3\pi,3\pi$

So, total 7 solutions

Question: $x^7-2x+3=p(x)$. Find number of real roots.

Answer: 1.00

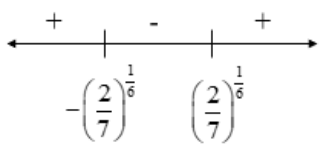
Solution:

$p(x)=x^7-2x+3$

$p'(x)=7x^6-2$

$p'(x)=0\Rightarrow x=\pm\left(\frac{2}{7}\right)^{\frac{1}{6}}$

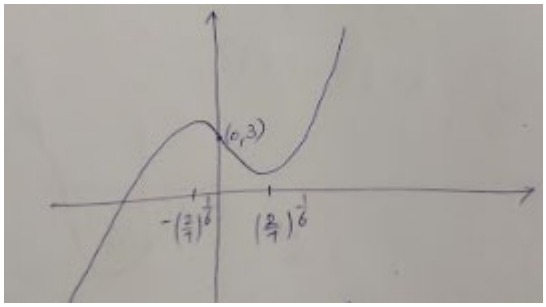
$p'(x)\rightarrow$



$p''(x)=42x^5$

$p''(x)>0$ for $x>0$ and $p''(x)<0$ for $x<0$

Graph of $p(x)$



$$p\left(\left(\frac{2}{7}\right)^{\frac{1}{6}}\right)=\left(\frac{2}{7}\right)^{\frac{7}{6}}-2\left(\frac{2}{7}\right)^{\frac{1}{6}}+3>0$$

$$p\left(-\left(\frac{2}{7}\right)^{\frac{1}{6}}\right)=-\left(\frac{2}{7}\right)^{\frac{7}{6}}+2\left(\frac{2}{7}\right)^{\frac{1}{6}}+3>0$$

Hence, number of real roots = 1

Question: Let a triangle of maximum area be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{4}=1,$ such that the area is $6\sqrt{3}.$ Find the eccentricity.

Answer: $\frac{\sqrt{3}}{2}$

Solution:

Maximum area of triangle inscribed in $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is $\frac{3\sqrt{3}}{4}ab$

$\therefore\ \frac{3\sqrt{3}}{4}a\cdot 2=6\sqrt{3}\Rightarrow a=4$

$\therefore\ e=\sqrt{\frac{a^2-b^2}{a^2}}=\sqrt{\frac{12}{16}}=\frac{\sqrt{3}}{2}$

Question: $(x-h)^2+(y-k)^2=r^2$ the arc touches x-axis at (1, 0) and $k>0$. $x+y=0$ intersect arc at 2 points chord length 2 unit. Find value of $h+k+r$?

Answer: 7.00

Solution:

$(x-h)^2+(y-k)^2=r^2$ touches x-axis at (1, 0)

$\therefore (1-h)^2+k^2=r^2\Rightarrow h^2+k^2-2h+1=r^2$

$\therefore h=1$ and $k=r$

Distance from $(1,k)$ on $x+y=0$ is $d=\left|\frac{1+k}{\sqrt{2}}\right|$

$\therefore r^2=1+d^2=1+\left(\frac{1+r}{\sqrt{2}}\right)^2$

$2\left(r^2-1\right)=r^2+1+2r$

$r^2-2r-3=0$

$\Rightarrow (r-3)(r+1)=0$

$\Rightarrow r=3=k$

$\therefore\ h+k+r=1+3+3=7$

Question: If S is given as $\{S:1,2,3,4....100\}$ then find the number of value of S such that {H.C.F of 24 & S is 1}

Answer: 1633.00

Solution:

Sum $=(1+2+....+100)-(2+4+....100)-(3+6+9.....99)+(6+12+18+....96)$

$=\left(\frac{100\times101}{2}\right)-\left(\frac{2\times50\times51}{2}\right)-\left(\frac{3\times33\times34}{2}\right)+6\left(\frac{16\times17}{2}\right)$

$=5050-2550-1683+816=1633$

Question: If it is given that $x*y=x^2+y^3$. Now if $(x*1)*1$ and $x*(1*1)$ both are equal then find the value of $2\sin^{-1}\left(\frac{x^4+x^2-2}{x^4+x^2+2}\right)$?

Answer: $\frac{\pi}{3}$

Solution:

$\left(x^2+1^3\right)^2+1^3=x\left(1^2+1^3\right)=x^2+2^3$

$x^4+2+2x^2=x^2+8$

$x^4+x^2-6=0$

$\Rightarrow \left(x^2+3\right)\left(x^2-2\right)=0$

$\Rightarrow x=\pm\sqrt{2}$

$\therefore 2\sin^{-1}\left[\frac{4+2-2}{4+2+2}\right]=2\sin^{-1}\left[\frac{4}{8}\right]=2\times\frac{\pi}{6}=\frac{\pi}{3}$

Question: pdf of x is

x	0	1	2	3	4
$P(x)$	k	$2k$	$3k$	$4k$	$5k$

Find $P\left(\frac{1<x<4}{x\le 2}\right)$

Answer: $\frac{1}{2}$

Solution:

$$P\left(\frac{1<x<4}{x\le 2}\right)=\frac{P(x=2)}{P(x\le 2)}$$

$$=\frac{P(x=2)}{P(x=0)+P(x=1)+P(x=2)}$$

$$=\frac{3k}{k+2k+3k}$$

$$=\frac{3k}{6k}=\frac{1}{2}$$

Question: Find minimum of $3x+4y;\ x,y\in R^+$ such that $x^3y^2=2^{13}$

Answer: 40.00

Solution:

$$x^3\cdot y^2=2^{13}$$

$$\therefore\ 3x+4y\Rightarrow\frac{x+x+x+2y+2y}{5}\ge\sqrt[5]{x^3.4y^2}$$

$$\Rightarrow\frac{3x+4y}{5}\ge\sqrt[5]{2^{15}}\ge 2^3\ge 8$$

$$\Rightarrow 3x+4y\ge 40$$

$$\Rightarrow(3x+4y)_{\min}=40$$

Question: $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{dx}{\left(e^x+1\right)\left(\sin^6 x+\cos^6 x\right)}$

Answer: π

Solution:

Given, $I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{dx}{\left(e^x+1\right)\left(\sin^6 x+\cos^6 x\right)}$ ....(1)

Applying property

$$I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{dx}{\left(e^{-x}+1\right)\left(\sin^6 x+\cos^6 x\right)}$$

$$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{e^x dx}{\left(e^{x}+1\right)\left(\sin^6 x+\cos^6 x\right)} \qquad ....(2)$$

Adding (1) and (2)

$$2I=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{\left(e^{x}+1\right)dx}{\left(e^{x}+1\right)\left(\sin^6 x+\cos^6 x\right)}$$

$$I=\int_{0}^{\frac{\pi}{2}}\frac{dx}{\sin^6 x+\cos^6 x}$$

$$=\int_{0}^{\frac{\pi}{2}}\frac{dx}{1-3\sin^2 x\cos^2 x}$$

$$=\int_{0}^{\frac{\pi}{2}}\frac{dx}{1-\frac{3}{4}\left(\sin 2x\right)^2}$$

Put $2x=t$

$2dx=dt$

$$I=\frac{1}{2}\int_{0}^{\pi}\frac{dt}{1-\frac{3}{4}\sin^2 t}$$

$$I=2\int_{0}^{\pi}\frac{dt}{4-3\sin^2 t}$$

$$I=2\int_{0}^{\pi}\frac{\sec^2 dt}{4\sec^2 t-3\tan^2 t}$$

$$=2\int_{0}^{\pi}\frac{\sec^2 t\,dt}{4+\tan^2 t}$$

$$I=2\cdot 2\int_{0}^{\frac{\pi}{2}}\frac{\sec^2 t\,dt}{4+\tan^2 t}$$

$$I=4\int_{0}^{\frac{\pi}{2}}\frac{\sec^2 t\,dt}{4+\tan^2 t}$$

$$I=4\left[\frac{1}{2}\tan^{-1}\left(\tan t\right)\right]_{0}^{\frac{\pi}{2}}$$

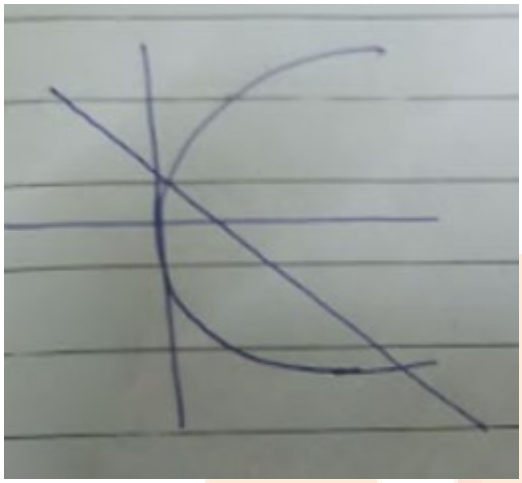
$$=2\left[\frac{\pi}{2}-0\right]$$

$= \pi$

Question: Find area bounded by $y^2 = 2x \ \& \ x + y = 4$

Answer: 18.00

Solution:



$y^2 = 2(4 - y) \Rightarrow y^2 + 2y - 8 = 0$

$(y + 4)(y - 2) = 0 \Rightarrow y = -4, 2$

$\therefore \text{ Area } = \int_{-4}^{2} \left((4 - y) - \frac{y^2}{2} \right) dy$

$= \left[4y - \frac{y^2}{2} - \frac{y^3}{6} \right]_{-4}^{2}$

$= 24 + 6 - 12$

$= 18$