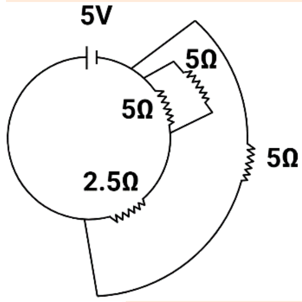
JEE-Main-25-06-2022-Shift-1 (Memory Based)

Physics

Question: Statement1: An ac circuit can be created with 0 reactance.
Options: Statement 2: An ac circuit without power is not possible.
(a) If both Statement 1 and Statement 2 are true and the Statement 2 is the correct explanation of the Statement 1.
(b) If both Statement 1 and Statement 2 are true, but the Statement 2 is not the correct explanation of the Statement 1.
(c) If Statement 1 is true, but Statement 2 is false.
(d) If both the Statement 1 and Statement 2 are false.
Answer: (c)
Solution:
When there is only inductor and capacitor in the circuit with same reactance then net reactance of the circuit will be zero. Hence statement one is true.
If the angle between V(rms value) and I(rms value) is 90 degree then the power in the circuit will be zero hence statement 2 is wrong.



Question:
Find the current in battery?
Answer: 2A
Solution:
5Ω and 5Ω are in parallel So,

$$R' = \frac{5\times5}{5+5} = \frac{25}{10} = 2.5\Omega$$

Now 2.5Ω and 2.5Ω are in series

$$R'' = 2.5\Omega + 2.5\Omega$$

$$R'' = 5\Omega$$

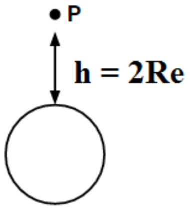
Now 5Ω and 5Ω are in parallel.

$$Req = \frac{5\times5}{5+5} = 2.5\Omega$$

Hence current in battery

$$I = \frac{V}{R} = \frac{5}{2.5} = 2A$$

Question: Find g at a height equal to the diameter of earth.



Answer: $(\frac{g}{9})$

Solution:

$$g(h) = \frac{G\ Me}{(Re + h)^2}$$

$$= \frac{G\ Me}{(Re + 2\,Re)^2}$$

$$= \frac{G\ Me}{9\ Re^2}$$

$$= \frac{g}{9}$$

Question: Suitable cable for 10THz frequency?

Options:

(a) Co-axial

(b) Copper wire

(c) Optical cable

(d) None of these.

Answer: (c)

Solution:

Such high frequency can travel in optical cables.

Question: 1. Transformer $\rightarrow$ A. measuring small current

2. Metal detector $\rightarrow$ B. Resonance in AC circuit

3. Galvanometer $\rightarrow$ C. Increase or Decrease voltage

Options:

(a) 1-B, 2-C, 3-A

(b) 1-C, 2-B, 3-A

(c) 1-A, 2-B, 3-C

(d) None of these.

Answer: (b)

Solution: Transformer $\rightarrow$ Increase or Decrease voltage

Metal Detector $\rightarrow$ Resonance in AC circuit
Galvanometer $\rightarrow$ Measure small current

Question: $I_1 = 9$ and $I_2 = 1$ at point p the phase difference is $\frac{\pi}{2}$ and at a point q is π. Then the difference between intensities.

Answer: (6I)

Solution:

$$I_P = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos 90^\circ$$

$$I_P = 9I + 1I + 2\sqrt{9} \times 0$$

$$I_P = 10I \text{ ....(1)}$$

$$I_q = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos 180^\circ$$

$$= 9I + I + 2I\sqrt{9} \times (-1)$$

$$= 10I - 6I$$

$$= 4I \quad \text{.......(2)}$$

$$I_P - I_q = 10I - 4I$$

$$= 6I$$

Question: If dielectric constant is 4 and relative permeability is 1. The find the angle for total internal reflection.

Answer: ($\theta_c = 30^\circ$)

Solution:

$$R.I = \sqrt{\frac{\mu}{\mu_0} \times \frac{\varepsilon}{\varepsilon_0}}$$

$$V = \frac{1}{\sqrt{\mu_m \varepsilon_m}}$$

$$C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$RI\ (n) = \frac{C}{V} = \frac{\frac{1}{\sqrt{\mu_0 \varepsilon_0}}}{\frac{1}{\sqrt{\mu_m \varepsilon_m}}}$$

$$n = \sqrt{\frac{\mu_m \varepsilon_m}{\mu_0 \varepsilon_0}}$$

$$n = \sqrt{\mu_r \varepsilon_r}$$

$$n = \sqrt{4 \times 1}$$

$$n = 2$$

For TIR

$\theta_c = \sin^{-1}\left(\frac{1}{n}\right)$

$\theta_c = \sin^{-1}\left(\frac{1}{2}\right)$

$\theta_c = 30^o$

Question: When wattless current flows in AC circuit, then the circuit is

Options:

(a) L only

(b) RC only

(c) RLC

(d) R only

Answer: (a)

Solution:

If there is only inductor in the circuit then current will be perpendicular to Voltage hence $\cos(\pi)$ will be zero for this combination. So the answer is option (a).

Question: $z = \frac{a^2b^3}{c^4}$ If % a = 2, % b = 1, % c = 3. Find % z.

Answer: (19)

Solution:

% Z = 2%.a + 3% b + 4% c

= 2(2) + 3(1) + 4(3)

= 4 + 3 + 12

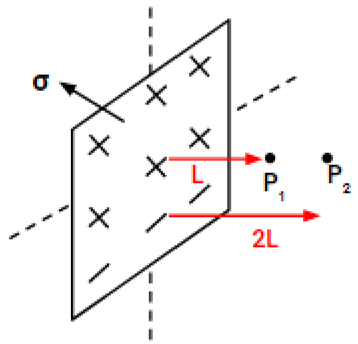
= 19

Question: Find the value of electric field due to non-conducting infinite sheet at turn given points.

Answer: ($E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$)

Solution:

$E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$



Question: If a current of 2A is flowing in the circuit for 15 sec then the energy is 300 watt. What will be the value of energy (in joules) if the current of 3A is flowing in the circuit for 10 sec?

Answer: (450)

Solution:

$$H = i^2Rt$$

$$300 = 2^2 \times R \times 15$$

$$R = 5\Omega$$

$$H' = 3^2 \times 5 \times 10$$

$$= 450$$

Question: $\vec{F} = 10\hat{i} + 5\hat{j}$

M = 100 g

If particle starts from rest and displacement vector at t = 2 sec is

$\vec{5} = a\hat{i} + b\hat{j}$

Find a/b

Answer: (2)

Solution:

$$F = 10\hat{i} + 5\hat{j}$$

$$M = 100 \text{ g} = 100 \times 10^{-3} \text{ kg}$$

$$\vec{a} = \frac{\vec{F}}{M} = 100\hat{i} + 50\hat{j}$$

$$u = 0,\ t = 2\text{sec},\ S = a\hat{i} + b\hat{j}$$

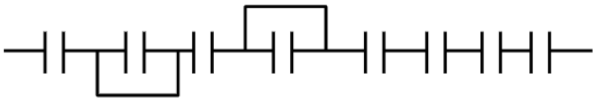
$$S = 0 + \frac{1}{2}at^2$$

$$a\hat{i} + b\hat{j} = \frac{1}{2}(100\hat{i} + 50\hat{j}) \times (2)^2$$

$$a\hat{i} + b\hat{j} = 200\hat{i} + 100\hat{j}$$

$$a/b = \frac{200}{100} = 2$$

Question: Find equivalent capacitance if capacitance of each capacitor is $8\mu F$.



Options:
(a) 48μF
(b) $\frac{4}{3}\mu$F
(c) 2μF
(d) 1μF

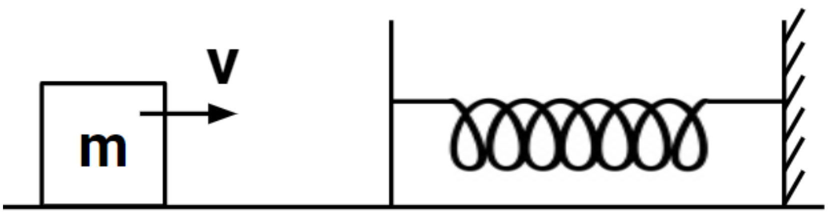
Answer: (b)
Solution:
2 capacitors are short circuited, and 6 capacitors are in series.
$C_{eq} = \frac{C}{6} = \boxed{\frac{4}{3}\mu\text{F}}$

Question: Find the ratio of speed of e^- in the third orbit of Bohr's model of H and He?

Options:
(a) 1 : 2
(b) 2 : 1
(c) 4 : 1
(d) 1 : 4

Answer: (a)
Solution:
$V \propto \frac{z}{n}$
$\frac{V_H}{V_{He}} = \frac{z_H}{z_{\text{He}}} \frac{n}{n} = \frac{1}{2}$

Question: A block is moving with velocity 12 m/s and compresses the spring by 30 cm such that velocity becomes half. Find spring constant if mass of block is 500 gram.



Answer: (600)

Solution:

Loss in $kE =$ Gain in PE of spring

$$\Rightarrow \frac{1}{2}mv^2 - \frac{1}{2}m\left(\frac{v}{2}\right)^2 = \frac{1}{2}kx^2$$

$$\frac{3}{8}mv^2 = \frac{1}{2}kx^2$$

$$\Rightarrow k = \frac{3}{4}\frac{mv^2}{x^2} = \frac{3}{4}\times\frac{1}{2}\times\frac{(12)^2}{(0.3)^2} = 600\,\mathrm{N/m}$$

Question: Terminal velocity of a sphere is directly proportional to.

Options:

(a) $\frac{r}{2}$

(b) r^0

(c) r^1

(d) r^2

Answer: (d)

Solution:

$$\mathrm{v} = \frac{2\mathrm{R}^2(\sigma - \rho)\mathrm{g}}{9\eta}$$

So, $v \propto r^2$

Question: A is a non-zero vector if

Options:

(a) $\vec{A}\cdot\vec{A} = 0$

(b) $\vec{A}\times\vec{A} = 0$

(c) $\vec{A}\times\vec{A} > 0$

(d) $\vec{A}\times\vec{A} < 0$

Answer: (b)

Solution:

$$\vec{A}\cdot\vec{A} = \left|\vec{A}\right|\times\left|\vec{A}\right|\times\cos 0^\circ = \left|\vec{A}\right|^2$$

$$\vec{A}\cdot\times\vec{A} = \left|\vec{A}\right|\times\left|\vec{A}\right|\times\sin 0^\circ = 0$$

Question: Velocity of light in two mediums is V_A and V_B. $V_A - V_B = 2.6\times10^7$. Reflective index of B is 1.43. Find refractive index of A.

Options:

(a) 1.27

(b) 1.43

(c) 1.62

(d) 1.15

Answer: (a)

Solution:

$$\mu_B = \frac{C}{V_B} \Rightarrow 1.43 = \frac{3\times10^8}{V_B} \Rightarrow V_B = \frac{3}{1.43}\times10^8 \text{ m/s}$$

$$\text{So}, V_A = 2.6\times10^7 + \frac{3}{1.43}\times10^8 = 2.36\times10^8 \text{ m/s}$$

$$\text{Hence } \mu_A = \frac{C}{V_A} = \frac{3\times10^8}{2.36\times10^8} = 1.27$$

Question:

$$\vec{F} = 3\hat{i} + 4\hat{j} + 2\hat{k}$$
$$\vec{r} = 2\hat{i} + \hat{j} + 2\hat{k}$$

Find $\vec{\tau}$ about origin?

Options:

(a) $-3\hat{i} + 5\hat{j} + 2\hat{k}$

(b) $3\hat{i} - 5\hat{j} - 2\hat{k}$

(c) $6\hat{i} - 2\hat{j} - 5\hat{k}$

(d) $-6\hat{i} + 2\hat{j} + 5\hat{k}$

Answer: (d)

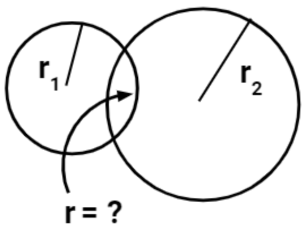
Solution:

$$\vec{\tau} = \vec{r}\times\vec{F}$$

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 3 & 4 & 2 \end{vmatrix}$$

$$= \hat{i}(2-8) - \hat{j}(4-6) + \hat{k}(8-3)$$
$$= -6\hat{i} + 2\hat{j} + 5\hat{k}$$

Question: Two soap bubbles of radius 6 cm and 9 cm respectively placed in contact with each other. Find the radius of curvature of interface.



Options:

(a) 18 cm

(b) 16 cm

(c) 12 cm

(d) 14 cm

Answer: (a)

Solution:

$$\frac{4T}{r_1} - \frac{4T}{r_2} = \frac{4T}{r}$$

$$\frac{1}{r_1} - \frac{1}{r_2} = \frac{1}{r}$$

$$\frac{47}{r_1} - \frac{47}{r_2} = \frac{47}{r}$$

$$\frac{1}{6} - \frac{1}{9} = \frac{1}{r}$$

$$\Rightarrow r = \frac{6 \times 9}{9 - 6} = 18cm$$

Question: For a long cylindrical wire of uniform cross section of radius R, find proportionality between r and B. Given r<<R.

Options:

(a) $B \propto r$

(b) $B \propto \frac{1}{r}$

(c) $B \propto r^2$

(d) $B \propto \frac{1}{r^2}$

Answer: (a)

Solution:

$$B = \frac{\mu_0}{2\pi} \cdot \frac{ir}{R^2}$$

$$B \propto r$$

Question: In calculating resistance of a galvanometer by half deflection method a student takes only one third of deflection of galvanometer. Which of the following will occur because of this?

Options:

(a) We can't calculate the value of resistance of galvanometer by this method

(b) We will get 1/3 times the value of resistance

(c) We will get 3 times the value of resistance.

Answer: (a)

Solution:

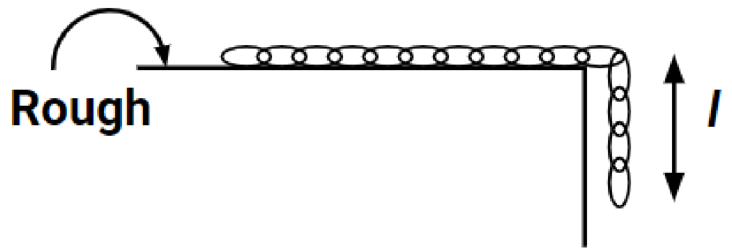
Resistance from half deflection method in given by

$$R_g = \frac{RS}{R-S}$$

if we do 1/3rd of deflection then $R_g' = 2\frac{RS}{R-2S}$

which is not related of actual resistance of galvanometer.

Question: A chain of length 6 m is hanged from the table. With $\mu = 0.5$ what is the maximum value of length l in meter that can be hanged?



Answer: (2)

Solution:

Let M be the mass of the chain and L its length.

If a length l hangs over the edge of the table, the force pulling the chain down is $\frac{M}{L}lg$.

The force of friction between the rest of the chain of length (L-l) and the table is $\frac{\mu M(L-l)}{L}g$.

For equilibrium the two forces must be equal:

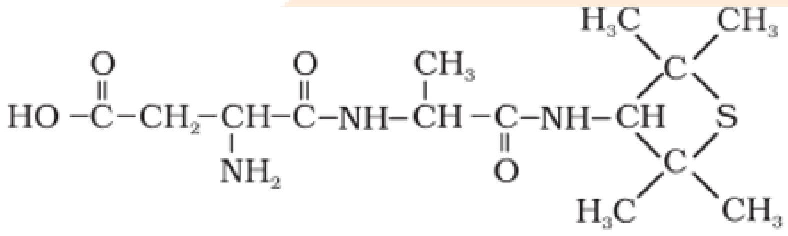
$$\Rightarrow \frac{Ml}{L}g = \frac{\mu M(L-l)}{L}g$$
$$\Rightarrow l = \mu(L-l)$$
$$\Rightarrow l = \frac{\mu L}{1+\mu}$$

$$l = \frac{0.5\times 6}{(1+0.5)} = 2\,\text{m}$$

JEE-Main-25-06-2022-Shift-1 (Memory Based)

Chemistry

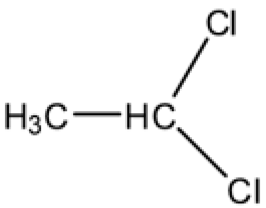
Question: Which of the following is artificial sweetener?
Options:
(a) Bithional
(b) Alitame
(c) Lactose
(d) Salvarsan
Answer: (b)
Solution:



Sweetness value of Alitame is 2000

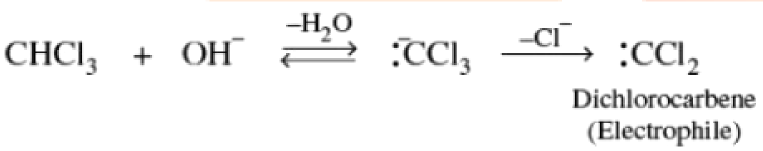
Question: Which has highest ionic mobility in aqueous solution?
Options:
(a) Be^{2+}
(b) Mg^{2+}
(c) Ba^{2+}
(d) Sr^{2+}
Answer: (c)
Solution: Due to the high heat of hydration of small cations, ionic mobility for the small cations is low. Order of ionic mobility is $Be^{2+} < Mg^{2+} < Sr^{2+} < Ba^{2+}$

Question: IUPAC of ethylidene chloride
Options:
(a) 1,1-Dichloroethane
(b) 1,2- Dichloromethane
(c) 1,2- Trichloroethane
(d) None of these
Answer: (a)
Solution:

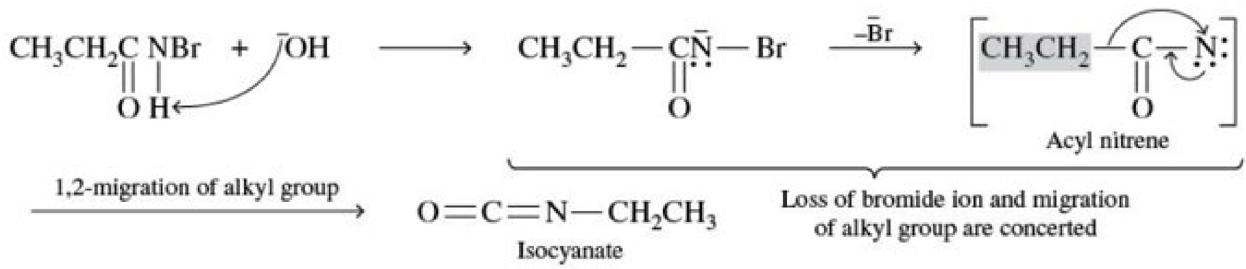


1,1-Dichloroethane

Question: Entropy change = 550 Jk^{-1} mol^{-1}
Enthalpy change = -165 kJ mol^{-1}
Find temperature at equilibrium.
Options:
(a) 300 K
(b) 500 K
(c) 600 K
(d) 700 K
Answer: (a)
Solution:
$\Delta S = 550\ JK^{-1}$
$\Delta H = -165\ kJ\ mol^{-1}$
$\Delta G = \Delta H - T\Delta S$
At equilibrium, $\Delta G = 0$
$0 = \Delta - 165 \times 100 - T(550)$
$550\ T = 165000$
$T = \dfrac{165000}{550} = 300\ K$

Question: Intermediate in Reimer tiemann reaction
Options:
(a) Carbene
(b) Carbanion
(c) Carbocation
(d) None of these
Answer: (a)
Solution: Generation of electrophile


Question: Intermediate in Hoffmann Bromamide reaction
Options:
(a) Nitrene
(b) Carbanion
(c) Carbocation
(d) None of these
Answer: (a)
Solution:



Question: Free radical polymerisation can take place in ___

Options:

(a) Terylene

(b) Melamine

(c) Nylon 6, 6

(d) Teflon

Answer: (d)

Solution: Teflon is synthesized using a free radical polymerization technique.

Question: Which of the following is incorrect about Tyndall effect?

Options:

(a) Greater difference in refractive index

(b) Diameter of dispersed particles is less than the wavelength of incident light

(c) Use to differentiate two colloidal solutions

(d) Suspensions show Tyndall effect

Answer: (b)

Solution: The diameter of the dispersed particles is not much smaller than wavelength of the light used. The refractive indices of the dispersed phase and the dispersion medium differ greatly in magnitude.

Question: In 681 g of $C_7H_5N_3O_6$ the number of nitrogen atoms are $x \times 10^{21}$. What is the value of x?

Options:

(a) 2271.2

(b) 3429.7

(c) 4217.5

(d) 5419.8

Answer: (d)

Solution: Molar mass of $C_7H_5N_3O_6 = 84 + 5 + 42 + 96 = 227\ g\ mol^{-1}$

No. of moles $= \dfrac{681}{227} = 3$ moles

No. of N atoms $= 3 \times 3 \times 6.022 \times 10^{23}$

$= 5.4198 \times 10^{24}$

$= 5419.8 \times 10^{21}$

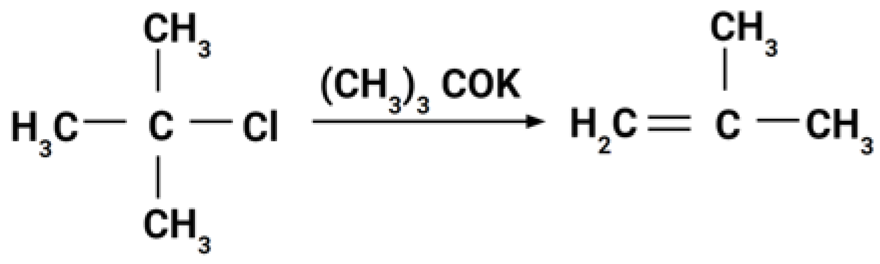
$x = 5419.8$

Question: Which of the following strongest oxidizing agent?

Options:

(a) Mn^{3+}
(b) Cr^{3+}
(c) Ti^{3+}
(d) Fe^{3+}
Answer: (a)
Solution: Mn^{3+} is strong oxidizing agent because Mn^{3+} can easily be converted into Mn^{2+} having d^5 configuration which is stable half-filled configuration.

Question: $(CH_3)_3CCl + (CH_3)_3COK$ gives?
Options:
(a) SN_1
(b) SN_2
(c) E_1
(d) E_2
Answer: (d)
Solution: Bulky base and bulky substrate prefers E_2 mechanism



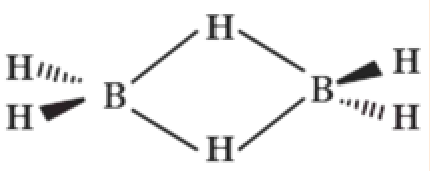
Question: What is eutrophication?
Options:
(a) Loss in biodiversity
(b) Increase in biodiversity
(c) Break down of Organic matter
(d) High biodegradability
Answer: (a)
Solution: It results in loss of oxygen and in subsequent loss of biodiversity known as Eutrophication.

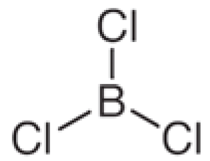
Question: Phenol reacted with nitric acid which gives two products. Which is the best method to separate the products?
Options:
(a) Steam distillation
(b) Fractional crystallization
(c) Sublimation
(d) Chromatographic separation
Answer: (a)
Solution: o-nitrophenol and p-nitro-phenol can be separated by steam distillation. Ortho nitro-phenol is steam volatile (intra molecular hydrogen bonding) whereas p-nitro phenol is least steam volatile when compared to the ortho nitro-phenol due to inter molecular hydrogen bonding.

Question: In the leaching of gold when NaCN is added the complex formed is X which when on addition of Zn forms Y. X and Y are respectively
Options:
(a) $[Au(CN)_2]^-$ and $[Zn(CN)_4]^{2-}$
(b) $[Au(CN)_4]^{2-}$ and $[Zn(CN)_4]^{2-}$
(c) $[Au(CN)_2]^-$ and $[Zn(CN)_4]^-$
(d) None
Answer: (a)
Solution: $4Au + 8CN^- + 2H_2O + O_2 \rightarrow 4\underset{(X)}{\left[Au(CN)_2\right]^-} + 4OH^-$

$2[Au(CN)_2]^- + Zn \rightarrow \underset{(Y)}{\left[Zn\,(CN)_4\right]^{2-}} + 2Au$

Question: e^- deficient species are PH_3, B_2H_6, CCl_4, NH_3, LiH, BCl_3 =?
Options:
(a) PH_3, B_2H_6
(b) CCl_4, NH_3
(c) BCl_3, B_2H_6
(d) LiH, BCl_3
Answer: (c)
Solution: In BCl_3, Boron has 3 valence electrons, after forming bonds with chlorine electrons around it increases to 6, but it still is short of 2 electrons. In B_2H_6, B forms 3C -2e^- bond and has less number of electrons. Hence, these are electron deficient species.





Question: Which of the following are isoelectronic?
Options:
(a) HF and H_2O
(b) CH_4 and SF_6
(c) O_2 and O_3
(d) H_2 and F_2
Answer: (a)
Solution:
HF = 1 + 9 = 10
H_2O = 2 + 8 = 10

Question: Product formed on reaction of AgCl with aq NH_3
Options:

(a) $[Ag(NH_3)_4]Cl$
(b) $[Ag(NH_3)_2]Cl_2$
(c) $[Ag(NH_3)_2]Cl$
(d) $[Ag(NH_3)_2]Cl$
Answer: (c)
Solution: $AgCl(s) + 2NH_3(aq) \rightarrow [Ag(NH_3)_2Cl](aq)$

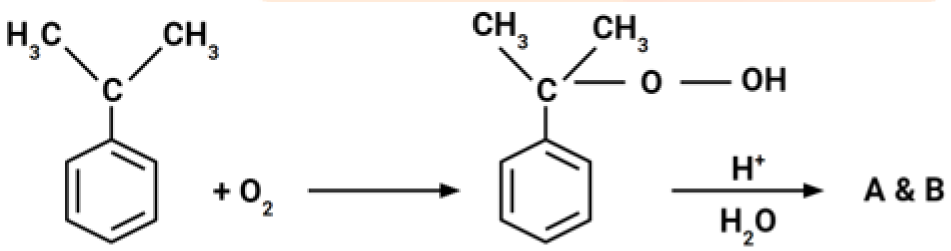
Question: The ratio of speed of electron in the 3rd orbit of helium to the speed of 3rd orbit of halogen
Options:
(a) 1:1
(b) 2:1
(c) 1:2
(d) 4:1
Answer: (b)
Solution: $V \propto \frac{Z}{n}$

$V = 2.188 \times 10^6 \frac{Z}{n} m$

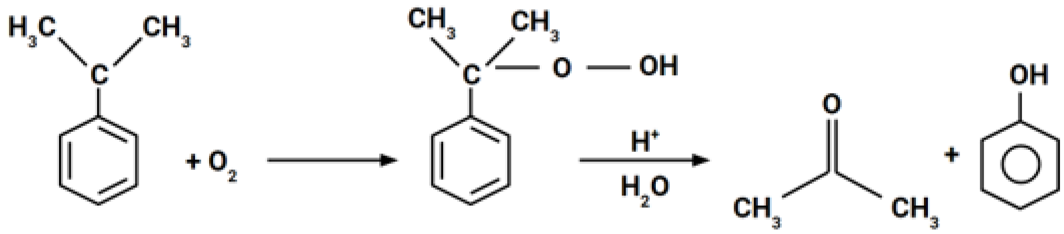
2:1

Question: Value of BOD of clean and polluted water
Options:
(a) Clean water < 5ppm, Polluted water < 10ppm
(b) Clean water > 5ppm, Polluted water < 10ppm
(c) Clean water > 5ppm, Polluted water > 10ppm
(d) Clean water > 10ppm, Polluted water > 20ppm
Answer: (a)
Solution: BOD of clean water is 3–5 ppm and polluted water is 6–9 ppm.

Question:



Options:
(a) Acetone-A, Phenol-B
(b) Acetaldehyde-A, Phenol-B
(c) Acetophenone-A, Ethanal-B
(d) Benzaldehyde-A, Ethanal -B
Answer: (a)
Solution:



Question: How many will be strongly bonded when one electron is removed?
NO, O_2, N_2, B_2
Answer: 2.00
Solution: NO and O_2

Bond order of NO $=\dfrac{N_B - N_A}{2}=\dfrac{10-5}{2}=2.5$

Bond order of NO^+ $=\dfrac{N_B - N_A}{2}=\dfrac{10-4}{2}=3$

Bond order of O_2 $=\dfrac{N_B - N_A}{2}=\dfrac{10-6}{2}=2$

Bond order of O_2^+ $=\dfrac{N_B - N_A}{2}=\dfrac{10-5}{2}=2.5$

JEE-Main-25-06-2022-Shift-1 (Memory Based)

MATHEMATICS

Question: For $\hat{a}, \hat{b}$, which is correct:

Options:

(a) $\left|\hat{a}+\hat{b}\right| = \left|\hat{a}-\hat{b}\right| \tan\frac{\theta}{2}$

(b) $\left|\hat{a}-\hat{b}\right| = \left|\hat{a}+\hat{b}\right| \tan\frac{\theta}{2}$

(c) $\left|\hat{a}+\hat{b}\right| = \left|\hat{a}-\hat{b}\right| \cos\frac{\theta}{2}$

(d) $\left|\hat{a}-\hat{b}\right| = \left|\hat{a}+b\right| \cos\frac{\theta}{2}$

Answer: (b)

Solution:

Given, $\hat{a}.\hat{b}$ are unit vectors

$\therefore \left|\hat{a}-\hat{b}\right|^2 = \left|\hat{a}\right|^2 + \left|\hat{b}\right|^2 - 2\,\hat{a}\cdot\hat{b} = 1+1-2\cos\theta$

$\left|\hat{a}+\hat{b}\right|^2 = \left|\hat{a}\right|^2 + \left|\hat{b}\right|^2 + 2\hat{a}\cdot\hat{b} = 1+1+2\cos\theta$

$\Rightarrow \left|\hat{a}-\hat{b}\right|^2 = 2(1-\cos\theta) = 4\sin^2\frac{\theta}{2}$

and $\left|\hat{a}+\hat{b}\right|^2 = 2(1+\cos\theta) = 4\cos^2\frac{\theta}{2}$

$\therefore \frac{\left|\hat{a}-\hat{b}\right|^2}{\left|\hat{a}+\hat{b}\right|^2} = \tan^2\frac{\theta}{2}$

$\Rightarrow \left|\hat{a}-\hat{b}\right| = \left|\hat{a}+\hat{b}\right| \tan\frac{\theta}{2}$

Question: $(x+1)\frac{dy}{dx} - y = e^{3x}(1+x)^2$. $y(0) = \frac{1}{3}$ then $x = \frac{-3}{y}$ at $x=-4$ is point of

Options:

(a) minimum

(b) maximum

Answer: (a)

Solution:

Given, $\frac{dy}{dx} - \frac{y}{x+1} = e^{3x}(1+x)$

$$\text{I.F.} = e^{\int -\frac{dx}{(x+1)}} = e^{\ln(x+1)^{-1}}$$

$$\text{I.F.} = \frac{1}{x+1}$$

$$\Rightarrow \frac{y}{x+1} = \int e^{3x} dx$$

$$\frac{y}{x+1} = \frac{e^{3x}}{3} + c$$

$$y = \frac{e^{3x}(x+1)}{3} + c(x+1)$$

$$y(0) = \frac{1}{3}$$

$$\Rightarrow \frac{1}{3} = \frac{1}{3} + c$$

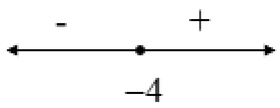
$$\Rightarrow c = 0$$

$$\therefore y = \frac{e^{3x}}{3}(x+1)$$

$$\text{Now, } \frac{dy}{dx} = \frac{1}{3}\left[\frac{e^{3x}}{3}(x+1) + e^{3x}\right]$$

$$= \frac{e^{3x}}{9}(x+1+3)$$

$$= \frac{e^{3x}}{9}(x+4)$$



At $x = -4$, its will make minimum

Question: $f(x) = x^3 + x - 5$ and $f(g(x)) = x$ and $g'(63) =$

Answer: $\frac{1}{49}$

Solution:

Given, $f(g(x)) = x$

$$g(x) = f^{-1}(x)$$

$$\Rightarrow \frac{d}{dx} g(x) = \frac{d}{dx} f^{-1}(x)$$

$$\Rightarrow \frac{d}{dx} f^{-1}(x)_{\text{at } x=63} = \frac{1}{\frac{d}{dx} f(x)_{\text{at } x=4}}$$

$$= \frac{1}{(3x^2+1)_{\text{at } x=4}}$$

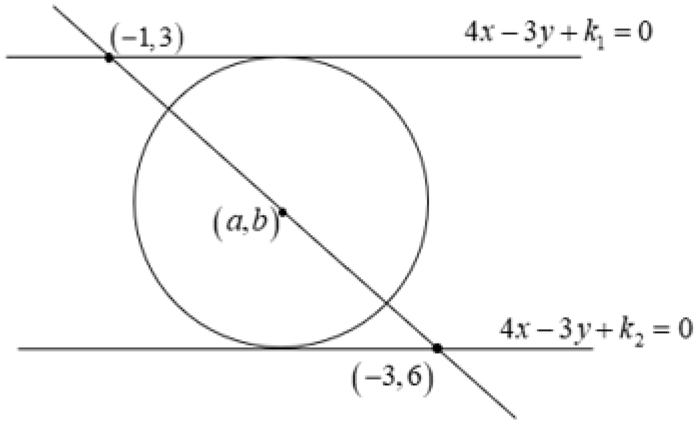
$=\frac{1}{49}$

Question: Circle is touched by lines $4x-3y+k_1=0$ and $4x-3y+k_2=0$. A line passing through centre touches lines respectively at $(-1,3)$ and $(-3,6)$. Equation of circles is

Answer: $(x+2)^2+\left(y-\frac{9}{2}\right)^2=\left(\frac{17}{10}\right)^2$

Solution:

Given,



$\therefore (-1,3)$ satisfy $4x-3y+k_1=0$

$\therefore -4-9+k_1=0$

$k_1=13$

$(-3,6)$ satisfy $4x-3y+k_2=0$

$\therefore -12-18+k_2=0$

$k_2=30$

Now, diameter of circle is distance between lines

$\therefore d=\left|\frac{k_2-k_1}{\sqrt{4^2+3^2}}\right|=\left|\frac{17}{5}\right|$

$\Rightarrow r=\frac{17}{10}$

Centre of circle $\left(\frac{-1-3}{2},\frac{3+6}{2}\right)=\left(-2,\frac{9}{2}\right)$

$\therefore$ Radius of circle

$(x+2)^2+\left(y-\frac{9}{2}\right)^2=\left(\frac{17}{10}\right)^2$

Question: $\frac{1}{2\times3^{10}}+\frac{1}{2^2\times3^9}+\frac{1}{2^3\times3^8}+...+\frac{1}{2^{10}\times3}=\frac{k}{2^{10}\times3^{10}}$. Find remainder when k is divided by 6.

Answer: 5.00

Solution:

Given, $\frac{1}{2\times3^{10}}+\frac{1}{2^2\times3^9}+\frac{1}{2^3\times3^8}+...+\frac{1}{2^{10}\times3}=\frac{k}{2^{10}\times3^{10}}$

It is G.P. series, with common ratio $\frac{3}{2}$

$$S_{10}=\frac{1}{2\times3^{10}}\left[\frac{\left(\frac{3}{2}\right)^{10}-1}{\frac{3}{2}-1}\right]$$

$$=\frac{1}{2\times3^{10}}\left[\frac{3^{10}-2^{10}}{2^{10}\left(\frac{1}{2}\right)}\right]$$

$$=\frac{1}{2^{10}\times3^{10}}\left(3^{10}-2^{10}\right)$$

$\therefore\ k=3^{10}-2^{10}=\left(9^5\right)-\left(4\right)^5$

$k=(9-4)\left(9^4+9^3\cdot4+9^2\cdot4^2+9\cdot4^3+4^4\right)$

$=5\left(9^4+4^4\right)+5(6\text{ Integer})$

$=5(6561+256)+6(\text{Integer})$

$=34085+6\text{ Integer}$

So remainder is 5 when k is divided by 6

Question: $\frac{a+b}{7}=\frac{b+c}{8}=\frac{c+a}{9}$. Find $\frac{R}{r}$.

Answer: $\frac{5}{2}$

Solution:

Given, $\frac{a+b}{7}=\frac{b+c}{8}=\frac{c+a}{9}=k$

$\therefore a+b=7k$

$b+c=8k$

$c+a=9k$

$2(a+b+c)=24k$

$a+b+c=12k$

$\Rightarrow c=5k,\ a=4k,\ b=3k$

$$S=\frac{a+b+c}{2}=6k$$

$$\Rightarrow \Delta=\sqrt{S(S-a)(S-b)(S-c)}=\sqrt{6k(2k)(3k)(k)}$$

$$\Delta=6k^2$$

Now, $\Delta=\dfrac{abc}{4R}$

$$6k^2=\frac{5k\cdot 4k\cdot 3k}{4R}$$

$$R=\frac{5}{2}k$$

Also, $r=\dfrac{\Delta}{S}=\dfrac{6k^2}{6k}=k$

$$\therefore \frac{R}{r}=\frac{\frac{5}{2}k}{k}=\frac{5}{2}$$

Question: $\displaystyle\int_0^{\pi}\frac{e^{\cos x}}{\left(1+\cos^2 x\right)}e^{\cos x}+e^{-\cos x}$

Answer: $\dfrac{\pi}{2\sqrt{2}}$

Solution:

$$I=\int_0^{\pi}\frac{e^{\cos x}}{\left(1+\cos^2 x\right)\left(e^{\cos x}+e^{-\cos x}\right)}dx \qquad ....(1)$$

Applying property

$$I=\int_0^{\pi}\frac{e^{-\cos x}}{\left(1+\cos^2 x\right)\left(e^{-\cos x}+e^{\cos x}\right)}dx \qquad ....(2)$$

(1) + (2)

$$2I=\int_0^{\pi}\frac{dx}{1+\cos^2 x}$$

$$I=\int_0^{\frac{\pi}{2}}\frac{dx}{1+\cos^2 x}$$

$$=\int_0^{\frac{\pi}{2}}\frac{\sec^2 x\,dx}{2+\tan^2 x}$$

$$=\frac{1}{\sqrt{2}}\left[\tan^{-1}(\tan x)\right]_0^{\frac{\pi}{2}}$$

$$=\frac{1}{\sqrt{2}}\left(\frac{\pi}{2}\right)$$

$$=\frac{\pi}{2\sqrt{2}}$$

Question: Solve: $y^2dx+\left(x^2-xy+y^2\right)dy=0$

Answer: $\ln\left(y\right)-\tan^{-1}\left(\frac{y}{x}\right)=c$

Solution:

Given, $y^2dx+\left(x^2-xy+y^2\right)dy=0$

$$\frac{dy}{dx}=\frac{-y^2}{x^2-xy+y^2}$$

Put $y=vx, \frac{dy}{dx}=v+x\frac{dv}{dx}$

$$v+x\frac{dv}{dx}=\frac{-v^2x^2}{x^2-vx^2+v^2x^2}$$

$$x\frac{dv}{dx}=\frac{-v^2-v+v^2-v^3}{1-v+v^2}$$

$$\int\frac{1-v+v^2}{v^3+v}dv=-\int\frac{dx}{x}$$

$$\int\left(\frac{v^2+1}{v\left(v^2+1\right)}-\frac{v}{v\left(v^2+1\right)}\right)dv=-\int\frac{dx}{x}$$

$$\int\left(\frac{1}{v}-\frac{1}{v^2+1}\right)dv=-\int\frac{dx}{x}$$

$$\ln v-\tan^{-1}v=-\ln x+c$$

$$\Rightarrow \ln y-\tan^{-1}\left(\frac{y}{x}\right)=c$$

Question: Number of 3 digit numbers which have sum of digits is 7

Answer: 28.00

Solution:

Let $x+y+z=7$

Here, x is in Hundred's place digit, y is ten's place and z is one's place digit

Using multinomial theorem

Coefficient of a^7 in $\left(a^1+a^2+\ldots+a^7\right)\left(a^0+a^1+\ldots+a^7\right)^2$

Coefficient of a^7 in $a\left(\frac{1-a^7}{1-a}\right)\left(\frac{1-a^8}{1-a}\right)^2$

Coefficient of a^6 in $\left(1-a^7\right)\left(1-a^8\right)^2\left(1-a\right)^{-3}$

Coefficient of a^6 in $\left(1-a^8\right)^2\left(1-a\right)^{-3}$

Coefficient of a^6 in $\left(1-a\right)^{-3} = {}^8C_6 = 28$

Question: $x^2-31x+228=0$, let $a_n=\alpha^n-\beta^n$. Then find the value of $\dfrac{a_{10}-31a_9}{57a_8}=$

Answer: 4.00

Solution:

Given, $x^2-31x+228=0$

$\therefore \alpha+\beta=31$

$\alpha\beta=228$

Also, $\alpha^2=31\alpha-228$ & $\alpha^2-31\alpha=228$

$\beta^2=31\alpha-228$ & $\beta^2-31\beta=228$

Now, $\dfrac{a_{10}-31a_9}{57a_8}=\dfrac{\left(\alpha^{10}-\beta^{10}\right)-31\left(\alpha^9-\beta^9\right)}{57a_8}$

$=\dfrac{\alpha^8\left(\alpha^2-31\alpha\right)-\beta^8\left(\beta^2-31\beta\right)}{57a_8}$

$=\dfrac{228\left(\alpha^8-\beta^8\right)}{57a_8}$

$=\dfrac{228}{57}$

$=4$