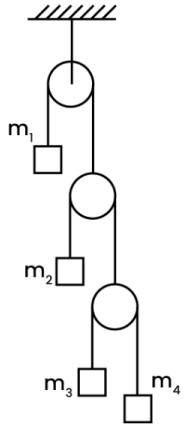
JEE-Main-26-06-2022-Shift-2 (Memory Based)

Physics

Question: Find the relation between $a_1\ a_2\ a_3\ a_4$

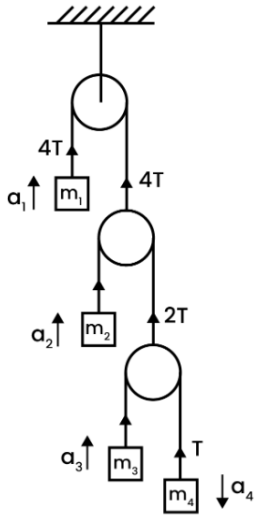


Options:

(a) 3
(b) 2
(c) 1
(d) 0

Answer: (d)

Solution:



$4a_1 + 2a_2 + a_3 - a_4 = 0$

Question: What is the expression of Reynold's number?

Options:

(a) 3
(b) 2
(c) 1
(d) 0
Answer:
Solution:

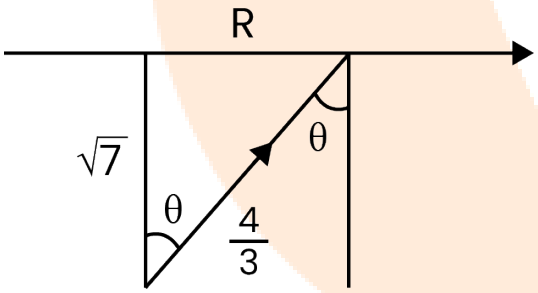
$Re = \frac{\rho u L}{\mu}$

Re = reynolds number
ρ = density of the fluid
u = flow speed
L = characteristic linear dimension
μ = dynamic viscosity of the fluid

Question: A bulb is placed at depth $\sqrt{7}$ from the surface in water of $\mu = 4/3$ .The area through which light comes out is $\pi x \text{m}^2$ find x

Options:

(a) 11
(b) 9
(c) 7
(d) 3
Answer: (b)
Solution:



$\frac{4}{3}\sin\theta = 1\sin 90^{\circ}$

$\sin\theta = \frac{3}{4}$

$\frac{R}{\sqrt{7}} = \tan\theta = \frac{\sin\theta}{\sqrt{1-\sin^2\theta}}$

$= \frac{3/4}{\sqrt{1-9/16}} = \frac{3/4}{\sqrt{7}/4} = \frac{3}{\sqrt{7}}$

$R = 3m$

$\therefore$ Area through which light comes out $= \pi(3)^2 = 9\pi \text{m}^2$

$\therefore x = 9$

Question: 20 tuning forks are arranged in increasing order of frequency such that every tuning fork produces 4 beats with previous one. If frequency of last tuning fork is double the first, then the frequency of first tuning fork is?

Options:

(a) 46
(b) 56
(c) 76
(d) 86
Answer:

Solution:
20 tuning forks =19 intervals

Each produces a beat of

$4 \therefore N_0$ of beats producod
$= 19 \times 4 = 76$

If first tuning fork has freq. f then acc.to qn. last one has frequency. 2 f.

$|2f - f| = 76$
$\therefore f = 76\,\text{Hz}$

Question: Temp of cold reservoir was 324k and heat given by the hot reservoir was 300J and heat given to the sink was 180 find the temp of hot reservoir.

Options:

(a) 540
(b) 335
(c) 232
(d) 457
Answer: 540

Solution:
$\frac{Q_H}{Q_L} = \frac{T_H}{T_L}$
$\frac{300}{180} = \frac{T_H}{324}$
$\Rightarrow T_H = 540\,\text{K}$

Question: Find the ratio of rotational kinetic energy to the total kinetic energy of a rolling solid sphere?

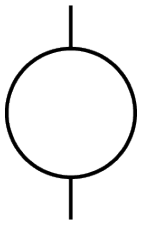
Options:

(a) 7/5
(b) 2/5
(c) 2/7

(d) 5/7

Answer: (c)

Solution:



Let m be the mass, r the radius of the sphere and let v and ω be the linear and angular velocities in rolling down.

Thus total kinetic energy = linear kinetic energy + rotational kinetic energy

$= \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

where I is the moment of inertia ie, $I = \frac{2}{5}mr^2$

Hence, total kinetic energy $= \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mr^2\right)\frac{v^2}{r^2}$

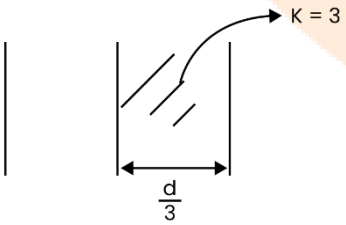
$\frac{1}{2}mv^2 + \frac{1}{5}mv^2$

$\frac{7}{10}mv^2$

So, the ratio $= \frac{\text{Rotational } KE}{\text{Total } KE}$

$= \frac{\frac{1}{5}mv^2}{\frac{7}{10}mv^2} = 2:7$

Question: If C is capacity of empty capacitor, then what's capacity of



Options:

(a) 2/3

(b) 3/5

(c) 2/6

(d) 3/2

Answer: (d)

Solution:

For partially filled capacitor

$$C = \frac{\varepsilon_0 A}{[d - t + t/k]}$$

Here $t = \frac{d}{2}$ and $k = 3$

$$\therefore C = \frac{\varepsilon_0 A}{\left[d - \frac{d}{2} + \frac{d}{6}\right]} = \frac{\varepsilon_0 A}{2/3d}$$

$$= \frac{3}{2}\varepsilon_0 \frac{A}{d} = \frac{3}{2}C$$

Question: Find the impulse given to the ball when batsman hits the ball of mass 0.5kg in the direction of the bowler with the same speed bowler throws it at him which is 15m/s.

Answer:

Solution:

Impulse = Change in momentum

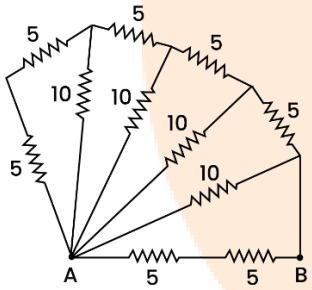
$|I| = |\Delta p| = m\vec{v}_f - m\vec{v}_i$

Here vel. only changes direction

$\therefore |\Delta p| = 2mV = 2(0.5)(15)$

$= 15\,\text{N} - 5$

Question: Find equivalent resistance across AB.



Options:

(a) 3.33 Ω

(b) 15 Ω

(c) 5 Ω

(d) 10 Ω

Answer: (a)

Solution:

$R' = 5 + 5 = 10\Omega$

$R'' = \frac{10 \times 10}{10 + 10} = 5\Omega$

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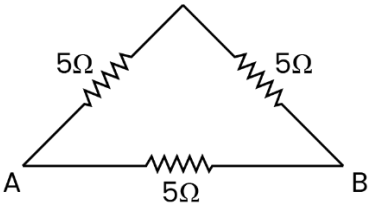
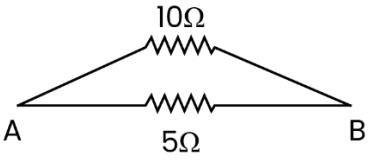




$$R_{AB} = \frac{10 \times 5}{10 + 5} = \frac{50}{15} = 3.33\Omega$$

Question: Dimensional formula of mutual inductance is

Options:

(a) $MLT^{-2}A^{-2}$

(b) $ML^{3}T^{-2}A^{-2}$

(c) $ML^{2}T^{-2}A^{-2}$

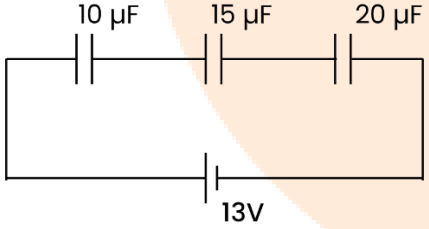
(d) $MLT^{-3}A^{-1}$

Answer: (c)

Solution:

Dimensional formula for mutual inductance is $ML^{2}T^{-2}A^{-2}$.

Question: What is the charge on 15 μF Capacitor



Options:

(a) 30 μC

(b) 60 μC

(c) 39 μC

(d) 45 μC

Answer: (b)

Solution:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{10} + \frac{1}{15} + \frac{1}{20}$$

$$\frac{1}{C_{eq}} = \frac{6+4+3}{60}$$

$C_{eq} = \frac{60}{13}\mu F$

Charge will be same on each capacitor in series.

$Q = \frac{60}{13} \times 13 = 60\mu C$

Question: A body vertically projected at t = 0 sec and another body is project at t = 2 sec with same initial velocity 50 m/s, then At what time t = _____ sec, they meet.

Options:

(a) 12

(b) 4

(c) 6

(d) 8

Answer: (c)

Solution:

$y_1 = 50t - \frac{1}{2} \times 10 \times t^2$

$y_2 = 50(t-2) - \frac{1}{2} \times 10 \times (t-2)^2$

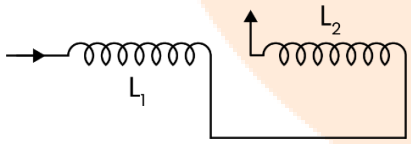
At $y_1 = y_2$ bodies will meet

$50t - 5t^2 = 50t - 100 - 5(t^2 + 4 - 4t)$

$20t - 20 - 100 = 0$

t = 6 sec.

Question: The equivalent relation for given circuit will be



Options:

(a) $\frac{1}{L_1} + \frac{1}{L_2}$

(b) $L_1 + L_2 - 2M$

(c) $L_1 + L_2 + 2M$

(d) $L_1 + L_2 + M$

Answer: (c)

Solution:

For inductors in series

$L_{eq} = L_1 + L_2$

Due to mutual inductance

$L_{eq} = L_1 + L_2 + 2M$

Question: 64 small balls each of radius 2 cm having surface charge density $5\mu C/m^2$ each merge and form a bigger sphere. Find the ratio of surface charge density of bigger sphere by small sphere

Options:

(a) 4 : 1

(b) 8 : 1

(c) 16 : 1

(d) 64 : 1

Answer: (a)

Solution:

$\sigma_{small} = 5\mu C / m^2$

$Q_{total} = 64 \times 5 \times 4 \times \pi (2 \times 10^{-2})^2 \mu C$

Let radius of bigger sphere is R then

$\frac{4}{3}\pi R^3 = 64 \times \frac{4}{3} \times \pi \times (2)^3$

$R = 4 \times 2 = 8$ cm

$A = 4 \times \pi \times (8 \times 10^{-2})^2$

$\sigma_{Big} = \frac{64 \times 5 \times 4 \times \pi (2 \times 10^{-2})^2}{4 \times \pi (8 \times 10^{-2})^2}$

$\sigma_{Big} = \frac{64 \times 5 \times 4 \times 10^{-4}}{64 \times 10^{-4}} = 20\mu C / m^2$

$\frac{\sigma_{Big}}{\sigma_{small}} = \frac{20}{5} = 4$

Question: The energy of emitted photoelectrons from a metal is 0.9 eV and energy of incident photon is 3.1 eV then the work function of a metal is

Options:

(a) 4.0 eV

(b) 2.2 eV

(c) 3.0 eV

(d) 3.1 eV

Answer: (b)

Solution:

$E = \phi + KE_{Max}$

$\phi = 3.1 - 0.9$

$\phi = 2.2$ eV

Question: Apparent wavelength is 670.7 nm and the original wavelength is 670 nm. Find the Relative speed of planet .

Options:

(a) 2.12×10^5 m/s

(b) 3.13×10^5 m/s

(c) 4.14×10^5 m/s

(d) 6.0×10^5 m/s

Answer: (b)

Solution:

We know that $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

$$\frac{670.7 - 670}{670} = \frac{v}{3 \times 10^8}$$

$$\Rightarrow v = \frac{0.7}{670} \times 3 \times 10^8 = 3.13 \times 10^5 \, m/s$$

Question: Arrange wavelengths of gamma, X Rays, Visible & Microwave in ascending order.

Options:

(a) Gamma < X Rays < Visible < Microwave

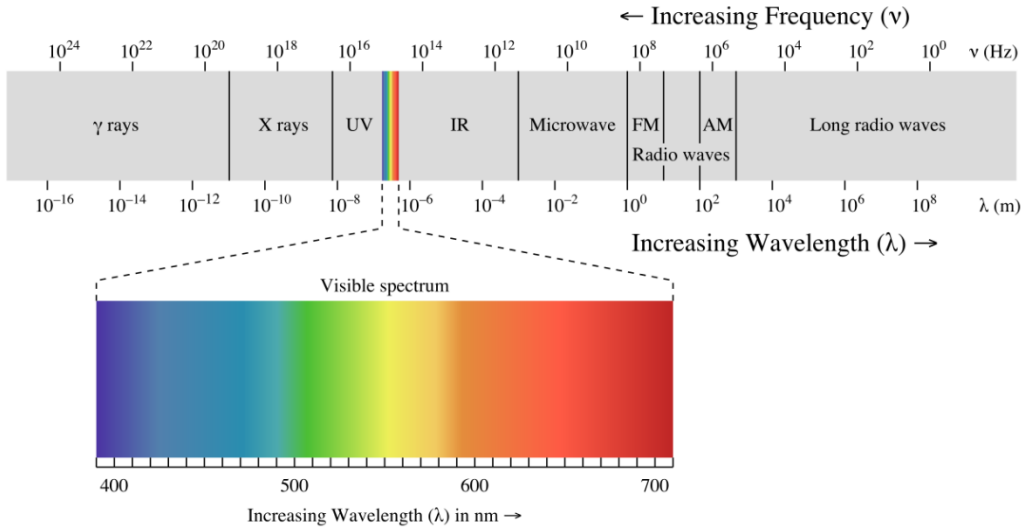
(b) Gamma > X Rays > Visible > Microwave

(c) Gamma > Visible > X Rays > Microwave

(d) Microwave < Gamma < X Rays < Visible

Answer: (a)

Solution:



JEE-Main-26-06-2022-Shift-2 (Memory Based)

Chemistry

Question: Oxides of nitrogen having an odd electrons is

Options:

(a) N_2O_5

(b) N_2O_3

(c) NO_2

(d) N_2O

Answer: (c)

Solution:

Question: $PhNO_2 + Sn/HCl \rightarrow A$

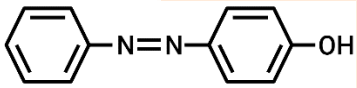
$A + NaNO_2/HCl \rightarrow B$

$B + \beta$-Naphthol $\rightarrow C$

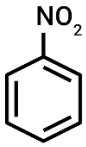
What is C ?

Options:

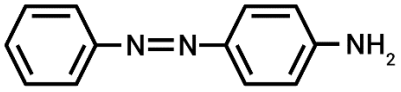
(a)



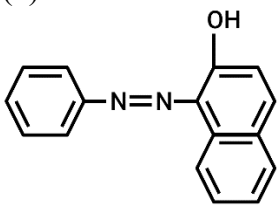
(b)



(c)

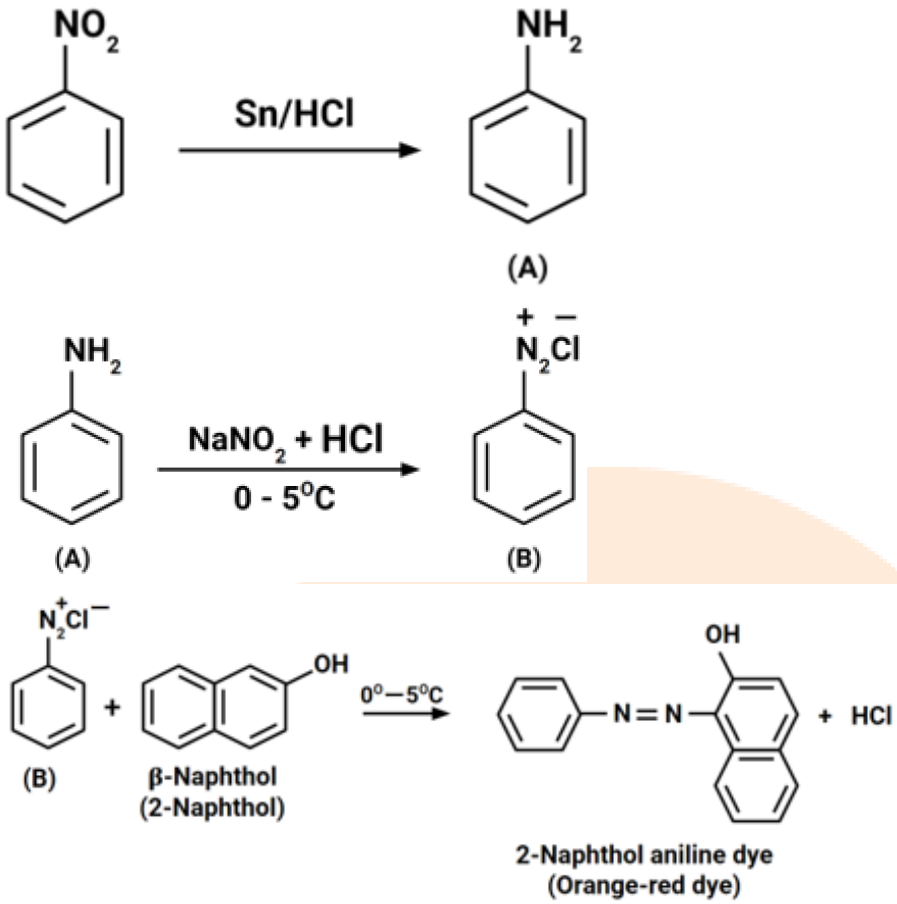


(d)



Answer: (d)

Solution:



Question: Identify the s-block element which do not give flame Test?
Options:
(a) Na
(b) K
(c) Be
(d) Ca
Answer: (c)
Solution: Be and Mg are the s-block elements that do not give flame test

Question: Which is not correct with respect to p-toluenesulphonychloride?
Options:
(a) It is hinsberg's reagent
(b) It forms a ppt which is soluble with alkali
(c) Used to distinguish primary and secondary amines
(d) Tertiary amines do not react with it
Answer: (b)
Solution: It forms a ppt which is soluble with alkali only in case of primary amines.

Question: Which of the following is a Metalloid?
Options:
(a) Bi
(b) Sc

(c) Te
(d) Hg
Answer: (c)
Solution: Bi, Hg and Sc are metals but Te is metalloid

Question: Which of the following is BOD value of polluted water?
Options:
(a) 4 ppm
(b) 17 ppm
(c) 8 ppm
(d) 5 ppm
Answer: (b)
Solution: Polluted water has BOD value > 10 ppm

Question: Which one of the following does not usually show +3 Oxidation state?
Options:
(a) La
(b) Lu
(c) Ce
(d) Gd
Answer: (c)
Solution: Cerium shows +4 oxidation state as it obtains noble gas configuration.

Question: Which of the following water soluble vitamin cannot be excreted easily?
Options:
(a) B1
(b) B2
(c) B12
(d) B6
Answer: (c)
Solution: Water soluble vitamin must be supplied regularly in diet because they are readily excreted in urine and cannot be stored (except vitamin B_{12}) in our body

Question: The correct order of nucleophilicity is
Options:
(a) $H_2O > OH^-$
(b) $NH_2^- > NH_3$
(c) R-OH >RO^-
(d) H – F – F^-
Answer: (b)
Solution: Conjugate anion of a species acts as a better nucleophile. Hence, NH_2^- is more nucleophilic than NH_3

Question: A nucleus has 2 types of radioactive decays. The half life of first is 3 hours and for the second is 4.5 hours. Calculate the correct half life of nucleus.
Options:

(a) 0.56 hours
(b) 3.75 hours
(c) 2.23 hours
(d) 1.80 hours
Answer: (d)
Solution:

$$\lambda = \frac{0.693}{t_{1/2}}$$

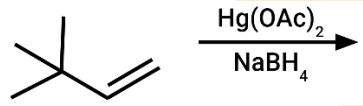
$$\lambda = \lambda_1 + \lambda_2$$

$$\frac{1}{t_{1/2}} = \frac{1}{(t_{1/2})_1} + \frac{1}{(t_{1/2})_2} = \frac{1}{3} + \frac{1}{4.5} = \frac{7.5}{3 \times 4.5}$$

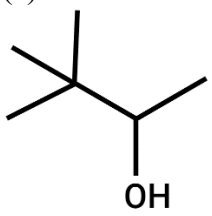
$$t_{1/2} = \frac{9}{5} = 1.8$$

Question: How many angular and radial nodes are in 4d orbital?
Options:
(a) 1, 3
(b) 1, 2
(c) 2, 1
(d) 1, 0
Answer: (c)
Solution:
Angular nodes = l
Radial nodes = n – l – 1
For 4d, l = 2 and n = 4
Angular nodes = 2
Radial nodes = 4 – 2 – 1 = 1

Question:



Options:
(a)



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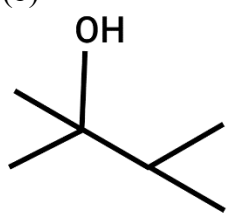

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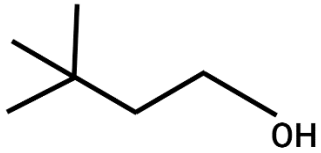

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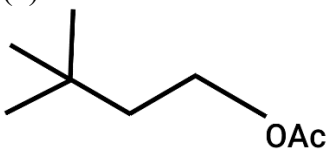
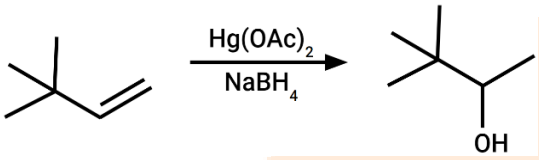


Answer: (a)

Solution:



Question: Toluene can be easily converted into benzaldehyde by which of the following reagents?

Options:

(a) CO, HCl, Anhyd. $AlCl_3$

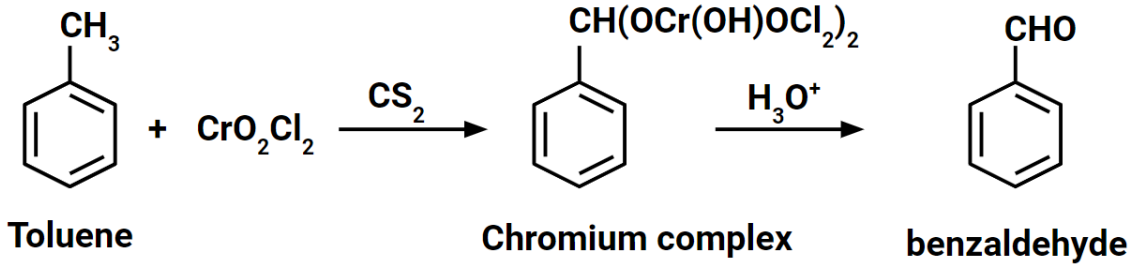
(b) Acetic acid, CS_2

(c) (i) CS_2 Chromyl chloride, (ii) H_3O^+

(d) H_2, $Pd/BaSO_4$

Answer: (c)

Solution:



Question: Boiling of Hard water produces

Options:

(a) $CaCO_3$ and $Mg(OH)_2$

(b) $Ca(OH)_2$ and $MgCO_3$

(c) $CaCO_3$ and $MgCO_3$

(d) $Ca(OH)_2$ and $Mg(OH)_2$

Answer: (a)

Solution: Boiling of hard water causes the precipitation of calcium carbonate and magnesium hydroxide.

$$Mg(HCO_3)_2 \xrightarrow{\text{Heating}} Mg(OH)_2 \downarrow + 2CO_2 \uparrow$$

$$Ca(HCO_3)_2 \xrightarrow{\text{Heating}} CaCO_3 \downarrow + H_2O + CO_2 \uparrow$$

Question: Match the Following.

Enzyme	Function
I) Invertase	(A) Starch to maltose
II) Maltase	(B) Maltose to Glucose
III) Zymase	(C) Sugar to ethanol
IV) Diastase	(D) Inversion of Cane Sugar

Options:

(a) I → D; II → B; III → C; IV → A

(b) I → A; II → B; III → C; IV → D

(c) I → D; II → C; III → B; IV → A

(d) I → C; II → B; III → A; IV → D

Answer: (a)

Solution:

I) Invertase ⇒ Inversion of Cane Sugar

II) Maltase ⇒ Maltose to Glucose

III) Zymase ⇒ Sugar to ethanol

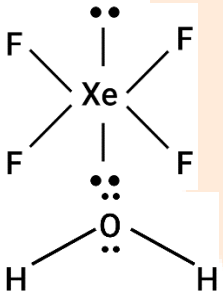
IV) Diastase ⇒ Starch to maltose

Question: Number of molecules having two lone pairs on the central atom among the following is:

CH_4, H_2O, XeF_4, SF_4

Answer: 2.00

Solution: XeF_4 and H_2O



Question: Number of electrons in t_{2g} orbital of compound formed by reacting $[Co(H_2O)]^{2+}$ with excess NH_3 in the presence of air is :

Answer: 6.00

Solution: $[Co(H_2O)_6]^{2+} + 6NH_3 \rightarrow [Co(NH_3)_6]^{3+} + 6H_2O + e^-$

NH_3 with Co^{3+} acts as a strong ligand.

Hence, all 6 electrons of Co will be present in t_{2g} orbital.

Question: E_{cat}-E_{uncat} = 10, T = 300 K, prexponential factor is given. Find ratio of K_{cat} to K_{uncat}.

Answer: 1.00

Solution:

E_{cat}-E_{uncat} = 10, T = 300K

$K = Ae^{-E_a/RT}$

$$\frac{K_{cat}}{K_{uncat}} = e^{\frac{-(Ea_{cat} - Ea_{uncat})}{RT}}$$

$$\frac{K_{cat}}{K_{uncat}} = e^{-\left(\frac{10}{8.314\times300}\right)} = e^{-\left(\frac{1}{249.42}\right)}$$

$$\frac{K_{cat}}{K_{uncat}} = 0.995 \approx 1$$

Question: 6.1 g of CNG is supplied with 208 g of oxygen. CO_2 and H_2O is produced with a lot of heat. How much CO_2 is produced? [Consider CNG as methane]
Answer: 17.00
Solution:
$CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$
6.1g 208 g

No. of moles $CH_4 = \frac{6.1}{16} = 0.38$

No. of moles $O_2 = \frac{208}{32} = 6.5$

CH_4 (CNG) is limiting reagent
0.38 moles CH_4 will produce 0.38 moles of CO_2
Amount of $CO_2 = 0.38 \times 44 = 16.72 \simeq 17$ g

JEE-Main-26-06-2022-Shift-2 (Memory Based)

MATHEMATICS

Question: Find $\cos^{-1}\left(\frac{3}{10}\cos\left(\tan^{-1}\frac{4}{3}\right)+\frac{2}{5}\sin\left(\tan^{-1}\frac{4}{3}\right)\right)$

Options:

(a) 0

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{6}$

(d) $\frac{\pi}{2}\cos\frac{\pi}{4}$

Answer: (b)

Solution:

$\cos^{-1}\left(\frac{3}{10}\times\frac{3}{5}+\frac{2}{5}\times\frac{4}{5}\right)$

$\cos^{-1}\left(\frac{9}{50}+\frac{8}{25}\right)$

$\cos^{-1}\left(\frac{1}{2}\right)=\frac{\pi}{3}$

Question: If function $f(x)=x-1$ and $g(x)=\frac{x^2}{x^2+1}$ then fog is:

Options:

(a) One-one and onto

(b) One-one but not onto

(c) Onto but not one-one

(d) Neither one-one nor onto

Answer: (d)

Solution:

$$f(g(x))=g(x)=\frac{x^2}{x^2+1}-1$$

$$=\frac{-1}{x^2+1}$$

$f(g(x))$ is even function $\Rightarrow$ many-one

$f(g(x))$ is always negative $\Rightarrow$ into

$\Rightarrow f\left(g\left(x\right)\right)$ is neither one-one nor onto

Question: $\lim\limits_{x\to 0}\dfrac{\cos\left(\sin x\right)-\cos x}{x^4}=?$

Answer: $\dfrac{1}{6}$

Solution:

$\lim\limits_{x\to 0}\dfrac{\cos\left(\sin x\right)-\cos x}{x^4}$

$=\lim\limits_{x\to 0}\dfrac{2\sin\left(\dfrac{x+\sin x}{2}\right)\sin\left(\dfrac{x-\sin x}{2}\right)}{x^4}$

As $x\to 0\Rightarrow \sin x\to 0$

$=\lim\limits_{x\to 0}\left(\dfrac{x+\sin x}{2}\right)\left(\dfrac{x-\sin x}{2}\right)\dfrac{2\sin\left(\dfrac{x+\sin x}{2}\right)\sin\left(\dfrac{x-\sin x}{2}\right)}{x^4\left(\dfrac{x+\sin x}{2}\right)\left(\dfrac{x-\sin x}{2}\right)}$

$=\lim\limits_{x\to 0}\dfrac{x^2-\sin^2 x}{2x^4}$

It is of the form $\dfrac{0}{0}$, so applying L-Hospital's rule

$=\lim\limits_{x\to 0}\dfrac{2x-2\sin x\cos x}{8x^3}$

$=\lim\limits_{x\to 0}\dfrac{2x-\sin 2x}{8x^3}$

Again applying L-Hospital's rule

$=\lim\limits_{x\to 0}\dfrac{2-2\cos 2x}{24x^2}$

Again applying L-Hospital's rule

$=\lim\limits_{x\to 0}\dfrac{4\sin 2x}{48x}$

Again applying L-Hospital's rule

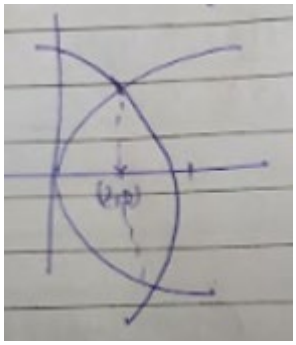
$=\lim\limits_{x\to 0}\dfrac{8\cos 2x}{48}$

$=\dfrac{1}{6}$

Question: Find area bounded by $y^2=8x$ & $y^2=16\left(3-x\right)$

Answer: 16.00

Solution:



$y^2 = 8x;\ y^2 = 16(3-x)$

$8x = 16(3-x)$

$\Rightarrow x = 6-2x$

$\Rightarrow x = 2,\ y = \pm 4$

$$\text{Area } = 2\left[\int_0^4 \left(3-\frac{y^2}{16}\right)-\left(\frac{y^2}{8}\right)dy\right]$$

$$= 2\left[\int_0^4 3-\frac{3y^2}{16}dy\right]$$

$$= 6\left[4-\frac{4}{3}\right]$$

$$= 16$$

Question: If $g(x) = \int \frac{1}{x} \times \sqrt{\frac{1-x}{1+x}}\,dx,\ g(0) = 1$ then $g(1) = ?$

Answer: $\frac{\pi}{3} + \ln\left|2-\sqrt{3}\right|$

Solution:

Given, $g(x) = \int \frac{1}{x} \times \sqrt{\frac{1-x}{1+x}}\,dx$

Put $x = \cos 2\theta$

$dx = -2\sin 2\theta\, d\theta$

$$\Rightarrow g(x) = \int \frac{-2\sin 2\theta}{\underset{\text{II}}{\cos 2\theta}} \underset{\text{I}}{\tan\theta}\, d\theta$$

$$= 2\int\left(\frac{1-2\sin^2\theta}{1-2\sin^2\theta} - \frac{1}{1-2\sin^2\theta}\right)d\theta$$

$$= 2\left[\int 1\,d\theta - \int \frac{\sec^2\theta}{\sec^2\theta - 2\tan^2\theta}d\theta\right]$$

$$= 2\theta - 2\int \frac{\sec^2\theta\, d\theta}{1-\tan^2\theta}$$

$$g(x)=2\theta-2\frac{1}{2}\ln\left|\frac{1+\tan\theta}{1-\tan\theta}\right|+C$$

$$g(x)=\cos^{-1}(x)-\ln\left|\frac{1+\sqrt{\frac{1-x}{1+x}}}{1-\sqrt{\frac{1-x}{1+x}}}\right|+C$$

Now, $g(1)=0$

$\Rightarrow 0=0-0+C$

$C=0$

$$\therefore g\left(\frac{1}{2}\right)=\frac{\pi}{3}-\ln\left|\frac{1+\sqrt{\frac{\frac{1}{2}}{\frac{3}{2}}}}{1-\sqrt{\frac{\frac{1}{2}}{\frac{3}{2}}}}\right|$$

$$=\frac{\pi}{3}+\ln\left|2-\sqrt{3}\right|$$

Question: If ${}^{40}C_0+{}^{41}C_1+{}^{42}C_2+....+{}^{60}C_{20}=\frac{n}{m}\,{}^{60}C_{20}$, and m,n are coprime, then $n+m=?$

Answer: 102.00

Solution:

${}^{41}C_0+{}^{41}C_1+{}^{42}C_2+....+{}^{60}C_{20}$

$\Rightarrow {}^{42}C_1+{}^{42}C_2+....+{}^{60}C_{20}$

$\Rightarrow {}^{43}C_2+{}^{44}C_3+....+{}^{60}C_{20}$

$\Rightarrow {}^{60}C_{19}+{}^{60}C_{20}$

$\Rightarrow {}^{61}C_{20}=\frac{m}{n}\,{}^{60}C_{20}$

$\frac{61!}{20!41!}=\frac{m}{n}\frac{60!}{20!40!}$

$\frac{m}{n}=\frac{61}{41}$

$\therefore m+n=102$

Question: If $p+q=3, p^4+q^4=369$ then $\left(\frac{1}{p}+\frac{1}{q}\right)^{-2}=?$

Answer: 4.00 or 64.00

Solution:

$p+q=3\Rightarrow p^2+q^2+2pq=9$

Also, $p^4+q^4=369$

$\left(p^2+q^2\right)^2-2p^2q^2=369$

$\left[(p+q)^2-2pq\right]^2-2(pq)^2=369$

$(16-2pq)^2-2(pq)^2=369$

Let $pq=t$

$81+4t^2-36t-2t^2=369$

$2t^2-36t-288=0$

$t^2-18t-144=0$

$t=\dfrac{18\pm\sqrt{324+576}}{2}$

$t=\dfrac{18+30}{2}$

$t=24,-6$

$\therefore\left(\dfrac{1}{p}+\dfrac{1}{q}\right)^{-2}=\left(\dfrac{p+q}{pq}\right)^{-2}=\left(\dfrac{pq}{3}\right)^2$

$=\left(\dfrac{24}{3}\right)^2$ or $\left(-\dfrac{6}{3}\right)^2$

$=4$ or 64

Question: Side lengths of a cuboid are $2x,5x$ & $4x$. There is closed hemisphere of radius r such that sum of surface area of cuboid & hemisphere is constant. Find ratio of x & r such that sum of volumes is maximum.

Answer: $\dfrac{45}{19}$

Solution:

Surface area $=76x^2+3\pi r^2=$ constant $=k$

$V=40x^3+\dfrac{2}{3}\pi r^3=40x^3+\dfrac{2}{3}\pi\left(\dfrac{k-76x^2}{3\pi}\right)^{\frac{3}{2}}$

$\dfrac{dV}{dx}=0\Rightarrow 120x^2+\dfrac{2}{3}\pi\cdot\dfrac{3}{2}\left(\dfrac{k-76x^2}{3\pi}\right)^{\frac{1}{2}}\cdot\left(\dfrac{-152x}{3\pi}\right)=0$

$120x^2=\dfrac{152x}{3}\left(\dfrac{k-76x^2}{3\pi}\right)^{\frac{1}{2}}$

$\left(\dfrac{45x}{19}\right)^2=\left(\dfrac{k-76x^2}{3\pi}\right)^{\frac{1}{2}}$

$\left(\dfrac{45}{19}\right)^2\cdot x^2=r^2$

$\Rightarrow\dfrac{r}{x}=\dfrac{45}{19}$

Question: A 3 digit number is randomly formed, find the probability that its common divisor with 36 is only 2.

Answer: $\frac{1}{6}$

Solution:

Total 3 digit numbers will be = 900

$n(s)=900$

Now, need 3 digit number whose common divisor with 36 is only 2

Such numbers will be

102,106,110,....998

$\therefore$ 225 numbers

But in these 102,114,126,....990

i.e., 75 numbers and to be deleted

$\therefore$ Probability $=\frac{225-75}{900}$

$=\frac{150}{900}$

$=\frac{1}{6}$

Question: If l_1 is the tangent to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$ and l_2 is a straight lines passing through (0, 0) and perpendicular to l_1. If the locus of the point of intersection of l_1 and l_2 is $\left(x^2+y^2\right)^2=\alpha x^2+\beta y^2$. Then, the values of $\alpha+\beta$ is ____.

Answer: 5.00

Solution:

Equation of tangent $l_1: y=mx\pm\sqrt{9m^2-4}$ ....(1)

Then $l_2: y=-\frac{1}{m}x$ ....(2)

For locus of point of intersection, eliminate m

$\Rightarrow y=-\frac{x^2}{y}\pm\sqrt{9\frac{x^2}{y^2}-4}$

$\left(x^2+y^2\right)^2=9x^2-4y^2$

$\Rightarrow \alpha=9,\ \beta=-4$

$\Rightarrow \alpha+\beta=5$

Question: $\int_0^{\sqrt{2}}\frac{\left(x^2-2\right)}{\left(x^2+2\right)\sqrt{x^4+4}}=?$

Answer: $\frac{-\pi}{8}$

Solution:

$$I=\int_0^{\sqrt{2}}\frac{\left(x^2-2\right)}{\left(x^2+1\right)\sqrt{x^4+4}}$$

$$I=\int_0^{\sqrt{2}}\frac{\left(1-\frac{2}{x^2}\right)dx}{\left(x+\frac{2}{x}\right)\sqrt{x^2+\frac{4}{x^2}}}$$

$$I=\int_0^{\sqrt{2}}\frac{\left(1-\frac{2}{x^2}\right)dx}{\left(x+\frac{2}{x}\right)\sqrt{\left(x+\frac{2}{x}\right)^2-4}}$$

Put $x+\frac{2}{x}=t$

$$\left(1-\frac{2}{x^2}\right)dx=dt$$

$$I=\int_0^{2\sqrt{2}}\frac{dt}{t\sqrt{t^2-4}}=\frac{1}{2}\sec^{-1}\left(\frac{t}{2}\right)\Big]_{\infty}^{2\sqrt{2}}$$

$$I=\frac{1}{2}\left[\frac{\pi}{4}-\frac{\pi}{2}\right]=\frac{-\pi}{8}$$

Question: $A=\sum_{n=1}^{\infty}\frac{1}{\left(3+(-1)^n\right)^n}$, $B=\sum_{n=1}^{\infty}\frac{(-1)^n}{\left(3+(-1)^n\right)^n}$. Find $\frac{A}{B}$.

Answer: $\frac{-11}{9}$

Solution:

$$A=\sum_{n=1}^{\infty}\frac{1}{\left[3+(-1)^n\right]^n}=\frac{1}{2}+\frac{1}{4^2}+\frac{1}{2^3}+\frac{1}{4^4}+\frac{1}{2^5}+\frac{1}{4^6}+....$$

$$=\frac{1}{2}\left[1+\frac{1}{2^2}+\frac{1}{2^4}+....\infty\right]+\frac{1}{4^2}\left[1+\frac{1}{4^2}+\frac{1}{4^6}+....\infty\right]$$

$$=\frac{1}{2}\left[\frac{1}{1-\frac{1}{4}}\right]+\frac{1}{16}\left[\frac{1}{1-\frac{1}{16}}\right]$$

$$=\frac{2}{3}+\frac{1}{15}=\frac{11}{15}$$

$$B=\sum_{n=1}^{\infty}\frac{(-1)^n}{\left[3+(-1)^n\right]^n}=\frac{-1}{2}+\frac{1}{4^2}-\frac{1}{2^3}-\frac{1}{4^4}+....\infty$$

$$=\frac{-2}{3}+\frac{1}{15}=\frac{-9}{15}$$

$$\therefore \frac{A}{B}=\frac{-11}{9}$$

Question: $16\sin 20\sin 40\sin 80=?$

Answer: $2\sqrt{3}$

Solution:

$$16\sin 20\cdot\sin(60-20)\cdot\sin(60+20)$$

$$=16\sin 20\cdot\left[\sin^2 60-\sin^2 20\right]$$

$$=16\sin 20\left[\frac{3}{4}-\sin^2 20\right]=4\left[3\sin 20-4\sin^3 20\right]$$

$$=4\sin(3\times 20)$$

$$=4\sin 60$$

$$=2\sqrt{3}$$

Question: Mean and standard deviation of 50 observations is 15 and 2. It was found that one observation was incorrect. Sum of correct and incorrect observation is 7. The mean of correct observation is 16. Find standard deviation of correct observation.

Answer: $\sqrt{43}$

Solution:

$n=50,\ \bar{x}=15,\ \sigma=2$

Let incorrect observation is x_1 and correct is x_1'

$$\therefore \bar{x}=\frac{x_1+x_2+x_3+...x_{50}}{50}=15$$

$$\Rightarrow \sum_{x=1}^{50}x_1{}^0=750$$

$$x_1+x_1'=70$$

$$\text{Now, } \bar{x}'=\frac{x_1'+x_2+x_3+....x_{50}}{50}=16$$

$$x_1'+\sum_{i=2}^{50}x_i=800$$

$$\therefore x_1'-x_1=50$$

$$\Rightarrow x_1'=60,\ x_1=10$$

$$\text{Now, } \sigma^2=4=\frac{x_1{}^2+x_2{}^2+x_3{}^2+...x_{50}{}^2}{50}-225$$

$\Rightarrow x_2^2 + x_3^2 + ... + x_{50}^2 = 11350$

$\therefore \left(\sigma^2\right)' = \dfrac{\left(x_1'\right)^2 + x_2^2 + x_3^2 + ...x_{50}^2}{50} - (16)^2$

$= 6.56 = \sqrt{43}$

Question: If $z^2 + z + 1 = 0,\ x \in C$. Find $\left|\sum_{k=1}^{15}\left(z^k + \dfrac{1}{z^k}\right)^2\right|$.

Answer: 30.00

Solution:

$z^2 + z + 1 = 0 \Rightarrow z = \omega$ (cube root of units)

$\left|\sum_{k=1}^{15}\left(\omega^k + \dfrac{1}{\omega^k}\right)^2\right| = \left|\sum_{k=1}^{15}\left(\omega^k + \omega^{2k}\right)^2\right|$

$= 5(2)^2 + 10(-1)^2$

$= 30$