BOARD QUESTION PAPER : MARCH 2018

Note:
i. All questions are compulsory.
ii. Figures to the right indicate full marks.
iii. Graph of L.P.P. should be drawn on graph paper only.
iv. Use of logarithmic table is allowed.
v. Answers to the questions of Section - I and Section - II should be written in only one answer book.
vi. Answer to every new question must be written on a new page.

SECTION – I

Q.1. (A) Select and write the appropriate answer from the given alternatives in each of the following sub-questions: **(6) [12]**

i. If A = $\begin{bmatrix} 2 & -3 \\ 4 & 1 \end{bmatrix}$, then adjoint of matrix A is _______.

(A) $\begin{bmatrix} 1 & 3 \\ -4 & 2 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & -3 \\ -4 & 2 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 3 \\ 4 & -2 \end{bmatrix}$ (D) $\begin{bmatrix} -1 & -3 \\ -4 & 2 \end{bmatrix}$

ii. The principal solutions of sec $x = \frac{2}{\sqrt{3}}$ are

(A) $\frac{\pi}{3}, \frac{11\pi}{6}$ (B) $\frac{\pi}{6}, \frac{11\pi}{6}$

(C) $\frac{\pi}{4}, \frac{11\pi}{4}$ (D) $\frac{\pi}{6}, \frac{11\pi}{4}$

iii. The measure of acute angle between the lines whose direction ratios are 3, 2, 6 and –2, 1, 2 is _______.

(A) $\cos^{-1}\left(\frac{1}{7}\right)$ (B) $\cos^{-1}\left(\frac{8}{15}\right)$

(C) $\cos^{-1}\left(\frac{1}{3}\right)$ (D) $\cos^{-1}\left(\frac{8}{21}\right)$

(B) Attempt any THREE of the following: **(6)**

i. Write the negations of the following statements:
   a. All students of this college live in the hostel.
   b. 6 is an even number or 36 is a perfect square.

ii. If a line makes angles α, β, γ with the co-ordinate axes, prove that $\cos 2\alpha + \cos 2\beta + \cos 2\gamma + 1 = 0$.

iii. Find the distance of the point (1, 2, –1) from the plane $x - 2y + 4z - 10 = 0$.

iv. Find the vector equation of the line which passes through the point with position vector $4\hat{i} - \hat{j} + 2\hat{k}$ and is in the direction of $-2\hat{i} + \hat{j} + \hat{k}$.

v. If $\bar{a} = 3\hat{i} - 2\hat{j} + 7\hat{k}$, $\bar{b} = 5\hat{i} + \hat{j} - 2\hat{k}$ and $\bar{c} = \hat{i} + \hat{j} - \hat{k}$, then find $\bar{a} \cdot (\bar{b} \times \bar{c})$.

Q.2. (A) Attempt any TWO of the following: **(6)[14]**

i. Using vector method prove that the medians of a triangle are concurrent.

ii. Using the truth table, prove the following logical equivalence:

$p \leftrightarrow q \equiv (p \wedge q) \vee (\sim p \wedge \sim q)$.

iii. If the origin is the centroid of the triangle whose vertices are A(2, p, –3), B(q, –2, 5) and R(–5, 1, r), then find the values of p, q, r.

(B) Attempt any TWO of the following: **(8)**

i. Show that a homogeneous equation of degree two in x and y, i.e. $ax^2 + 2hxy + by^2 = 0$ represents a pair of lines passing through the origin if $h^2 - ab \geq 0$.

ii. In ΔABC, prove that $\tan\left(\frac{C-A}{2}\right) = \left(\frac{c-a}{c+a}\right)\cot\frac{B}{2}$

iii. Find the inverse of the matrix, $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$ using elementary row transformations.

Q.3. (A) Attempt any TWO of the following: **(6)[14]**

i. Find the joint equation of the pair of lines passing through the origin, which are perpendicular to the lines represented by $5x^2 + 2xy - 3y^2 = 0$.

ii. Find the angle between the lines $\frac{x-1}{4} = \frac{y-3}{1} = \frac{z}{8}$ and $\frac{x-2}{2} = \frac{y+1}{2} = \frac{z-4}{1}$

iii. Write converse, inverse and contrapositive of the following conditional statement:
If an angle is a right angle then its measure is 90°.

(B) Attempt any TWO of the following: **(8)**

i. Prove that: $\sin^{-1}\left(\frac{3}{5}\right) + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1}\left(\frac{56}{65}\right)$

ii. Find the vector equation of the plane passing through the points A(1, 0, 1), B(1, –1, 1) and C(4, –3, 2).

iii. Minimize $Z = 7x + y$ subject to
$5x + y \geq 5, x + y \geq 3, x \geq 0, y \geq 0$

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SECTION – II

Q.4. (A) Select and write the appropriate answer from the given alternatives in each of the following sub-questions: (6)[12]

i. Let the p.m.f. of a random variable X be –

$P(x) = \frac{3-x}{10}$ for $x = -1, 0, 1, 2$

$= 0$ otherwise

Then E(X) is _______.

(A) 1 (B) 2
(C) 0 (D) −1

ii. If $\int_0^k \frac{1}{2+8x^2}\,dx = \frac{\pi}{16}$, then the value of k is _______.

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$
(C) $\frac{1}{4}$ (D) $\frac{1}{5}$

iii. Integrating factor of the linear differential equation $x\frac{dy}{dx} + 2y = x^2 \log x$ is _______.

(A) $\frac{1}{x^2}$ (B) $\frac{1}{x}$
(C) x (D) x^2

(B) Attempt any THREE of the following: (6)

i. Evaluate: $\int e^x \left[\frac{\cos x - \sin x}{\sin^2 x}\right] dx$

ii. If $y = \tan^2(\log x^3)$, find $\frac{dy}{dx}$.

iii. Find the area of ellipse $\frac{x^2}{1} + \frac{y^2}{4} = 1$.

iv. Obtain the differential equation by eliminating the arbitrary constants from the following equation: $y = c_1e^{2x} + c_2e^{-2x}$

v. Given X ~ B (n, p)
If n = 10 and p = 0.4, find E(X) and Var. (X).

Q.5. (A) Attempt any TWO of the following: **(6)[14]**

i. Evaluate: $\int \frac{1}{3+2\sin x+\cos x}\,dx$

ii. If $x = a\cos^3 t$, $y = a\sin^3 t$,

show that $\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$

iii. Examine the continuity of the function:

$$f(x) = \frac{\log 100 + \log(0.01+x)}{3x}, \text{ for } x \neq 0$$

$$= \frac{100}{3} \qquad \text{for } x = 0; \text{ at } x = 0$$

(B) Attempt any TWO of the following: **(8)**

i. Find the maximum and minimum value of the function:
$f(x) = 2x^3 - 21x^2 + 36x - 20$.

ii. Prove that: $\int \frac{1}{a^2 - x^2}\,dx = \frac{1}{2a}\log\left|\frac{a+x}{a-x}\right| + c$

iii. Show that: $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$, if $f(x)$ is an even function.

$= 0$, if $f(x)$ is an odd function.

Q.6. (A) Attempt any TWO of the following: **(6)[14]**

i. If $f(x) = \frac{x^2-9}{x-3} + \alpha$, for $x > 3$

$= 5$, for $x = 3$

$= 2x^2 + 3x + \beta$, for $x < 3$

is continuous at $x = 3$, find α and β.

ii. Find $\frac{dy}{dx}$ if $y = \tan^{-1}\left(\frac{5x+1}{3-x-6x^2}\right)$

iii. A fair coin is tossed 9 times. Find the probability that it shows head exactly 5 times.

(B) Attempt any TWO of the following: **(8)**

i. Verify Rolle's theorem for the following function:
$f(x) = x^2 - 4x + 10$ on $[0, 4]$

ii. Find the particular solution of the differential equation:

$$y(1+\log x)\frac{dx}{dy} - x\log x = 0$$

when $y = e^2$ and $x = e$

iii. Find the variance and standard deviation of the random variable X whose probability distribution is given below:

x	0	1	2	3
$P(X = x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$