2020
MATHEMATICS

Full marks: 100 Time: 3 hours

General instructions:

i) *Approximately 15 minutes is allotted to read the question paper and revise the answers.*
ii) *The question paper consists of 26 questions. All questions are compulsory.*
iii) *Marks are indicated against each question.*
iv) *Internal choice has been provided in some questions.*
v) *Use of simple calculators (non-scientific and non-programmable) only is permitted.*

N.B: Check that all pages of the question paper is complete as indicated on the top left side.

Section – A

1. **Choose the correct answer from the given alternatives:**

(a) Let R be the relation in the set **N** given by $\mathrm{R}=\{(a,b): a=b-2, b>6\}$.Then **1**

(i) $(2,4)\in \mathrm{R}$ (ii) $(3,8)\in \mathrm{R}$ (iii) $(6,8)\in \mathrm{R}$ (iv) $(8,7)\in \mathrm{R}$

(b) The value of $\tan^{-1}\sqrt{3}-\sec^{-1}(-2)$ is equal to **1**

(i) π (ii) $\frac{-\pi}{3}$ (iii) $\frac{\pi}{3}$ (iv) $\frac{2\pi}{3}$

(c) If A and B are symmetric matrices of same order, then AB – BA is **1**

(i) a zero matrix (ii) an identity matrix
(iii) a symmetric matrix (iv) a skew-symmetric matrix

(d) If $y=5\cos x-3\sin x$, then $\frac{d^2y}{dx^2}+y$ is equal to **1**

(i) 5 (ii) 2 (iii) 0 (iv) – 4

(e) The slope of the normal to the curve $y=2x^2+3\sin x$ at x = 0 is **1**

(i) $\frac{-1}{3}$ (ii) $\frac{1}{3}$ (iii) –3 (iv) 3

(f) $\int \frac{dx}{\sin^2 x\cos^2 x}$ is equal to **1**

(i) $\tan x+\cot x+C$ (ii) $\tan x\cot x+C$
(iii) $\tan x-\cot x+C$ (iv) $\tan x-\cot 2x+C$

(g) The order of the differential equation $2x^2\frac{d^2y}{dx^2}-3\frac{dy}{dx}+y=0$ is **1**

(i) 2 (ii) 1 (iii) 0 (iv) not defined

(h) The position vector of the point which divides the join of the points $2\vec{a}-3\vec{b}$ and $\vec{a}+\vec{b}$ in the ratio of 3 : 1 internally is **1**

(i) $\frac{3\vec{a}-2\vec{b}}{2}$ (ii) $\frac{7\vec{a}-8\vec{b}}{4}$ (iii) $\frac{3\vec{a}}{4}$ (iv) $\frac{5\vec{a}}{4}$

(i) If the vectors $\vec{a}$ and $\vec{b}$ be such that $|\vec{a}|=3,\ |\vec{b}|=\frac{\sqrt{2}}{3}$ and $\vec{a}\times\vec{b}$ is a unit vector, then the angle between $\vec{a}$ and $\vec{b}$ is **1**

(i) $\frac{\pi}{6}$ (ii) $\frac{\pi}{3}$ (iii) $\frac{\pi}{4}$ (iv) $\frac{\pi}{2}$

(j) The point through which the straight line $\frac{2x-3}{2}=\frac{2y+3}{-5}=\frac{z}{2}$ passes is **1**

(i) $\left(\frac{3}{2},\frac{-3}{2},0\right)$ (ii) $\left(\frac{-3}{2},\frac{3}{2},0\right)$ (iii) $\left(\frac{3}{2},\frac{3}{2},0\right)$ (iv) none of these

Section – B

2. Prove that $\tan^{-1}\sqrt{x}=\frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right),\ x\in[0,1]$ **2**

3. Prove that $\tan^{-1}\frac{2}{11}+\tan^{-1}\frac{7}{24}=\tan^{-1}\frac{1}{2}$. **2**

4. If $x\begin{bmatrix}2\\3\end{bmatrix}+y\begin{bmatrix}-1\\1\end{bmatrix}=\begin{bmatrix}10\\5\end{bmatrix}$, find the values of x and y. **2**

5. Determine the value of k so that the given function

$$f(x)=\begin{cases}\dfrac{x^2-9}{x-3}, & x\neq 3\\ k, & if\ x=3\end{cases}$$

is continuous at x = 3. **2**

6. Find $\frac{dy}{dx}$ if $2x+3y=\sin y$. **2**

7. Evaluate $\int\frac{(\log x)^2}{x}dx$. **2**

8. Solve $\frac{dy}{dx} = \frac{1+y^2}{y^3}$. **2**

9. Find $\left|\vec{a} \times \vec{b}\right|$ if $\vec{a} = \hat{i} - 7\hat{j} + 7\hat{k}$ and $\vec{b} = 3\hat{i} - 2\hat{j} + 2\hat{k}$ **2**

10. Find the equation of the line which passes through (1, 2, 3) and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$. **2**

11. Events A and B are such that $P(A) = \frac{1}{2}$, $P(B) = \frac{7}{12}$ and P (not A or not B) $= \frac{1}{4}$. Show that A and B are not independent. **2**

Section – C

12. Consider $f : \mathbf{R}_+ \to [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. Show that f is invertible with $f^{-1}(y) = \frac{\sqrt{y+6} - 1}{3}$. **4**

13. Using properties of determinants, show that

$$\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$$ **4**

14. **a.** Verify Rolle's Theorem for the function $f(x) = x^2 + 2x - 8,\ x \in [-4, 2]$

Or **4**

b. If $y = x^{\sin x} + (\sin x)^{\cos x}$, find $\frac{dy}{dx}$.

15. **a.** Find the equation of the tangent to the curve $y = \sqrt{3x-2}$ which is parallel to the line $4x - 2y + 5 = 0$.

Or **4**

b. A ladder 5 m long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall, at the rate of 2 cm/s. How fast is its height on the wall decreasing when the foot of the ladder is 4 m away from the wall?

16. **a.** Evaluate $\int_0^{\frac{\pi}{4}} \log(1 + \tan x)\, dx$.

Or **4**

b. Evaluate $\int \frac{x+2}{\sqrt{x^2+2x+3}}dx$.

17. Evaluate $\int \frac{e^x\left(x-1\right)}{\left(x+1\right)^3}dx$ **4**

18. **a.** Solve the differential equation $xdy-ydx=\sqrt{x^2+y^2}\,dx$

Or **4**

b. Solve the differential equation $\frac{dy}{dx}+2y\tan x=\sin x;\ y=0$ and $x=\frac{\pi}{3}$

19. By using vectors, find the area of the triangle with vertices A(1, 1, 2), B(2, 3, 5) and C(1, 5, 5). **4**

20. A bag contains 4 red and 4 black balls, another bag contains 2 red and 6 black balls. One of the two bags is selected at random and a ball is drawn from the bag which is found to be red. Find the probability that the ball is drawn from the first bag. **4**

21. **a.** From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

Or **4**

b. Two cards are drawn successively with replacement from a well-shuffled deck of 52 cards. Find the probability distribution of the number of aces.

Section – D

22. **a.** Solve the system of linear equations by using matrix method.

$$x-y+2z=7$$
$$3x+4y-5z=-5$$
$$2x-y+3z=12$$

Or **6**

b. If $A=\begin{bmatrix}2 & -3 & 5\\ 3 & 2 & -4\\ 1 & 1 & -2\end{bmatrix}$, find A^{-1}. By using A^{-1} solve the system of equations

$$2x-3y+5z=11$$
$$3x+2y-4z=-5$$
$$x+y-2z=-3$$

23. **a.** Show that the right circular cylinder of given surface and maximum volume is such that its height is equal to the diameter of the base.

Or **6**

b. Show that semi-vertical angle of right circular cone of given surface area and maximum volume is $\sin^{-1}\left(\frac{1}{3}\right)$.

24. **a.** Using the method of integration, find the area bounded by the curve $x^2 = 4y$ and the line $x = 4y - 2$.

Or **6**

b. Using the method of integration, find the area of the region bounded by the triangle whose vertices are (–1, 0), (1, 3) and (3, 2).

25. **a.** Find the Cartesian and vector equation of the plane through the line of intersection of the planes $x + y + z = 1$ and $2x + 3y + 4z = 5$ which is perpendicular to the plane $x - y + z = 0$.

Or **6**

b. Find the shortest distance between the lines whose vector equations are

$\vec{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and

$\vec{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$

26. **a.** Reshma wishes to mix two types of food P and Q in such a way that the vitamin contents of the mixture contain at least 8 units of vitamin A and 11 units of vitamin B. Food P costs `60/kg and food Q costs `80/kg. Food P contains 3 units/kg of vitamin A and 5 units/kg of vitamin B, while food Q contains 4 units/kg of vitamin A and 2 units/kg of vitamin B. Determine the minimum cost of the mixture.

Or **6**

b. A manufacturer produces nuts and bolts. It takes 1 hour of work on machine A and 3 hours on machine B to produce a package of nuts. It takes 3 hours on machine A and 1 hour on machine B to produce a package of bolts. He earns a profit of `17.50 per package on nuts and `7.00 per package on bolts. How many packages of each should be produced each day so as to maximize his profit, if he operates his machines for at the most 12 hours a day?

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