Total No. of Printed Pages—7

HS/XII/A. Sc. Com/M/18

2018

MATHEMATICS

Full Marks : 100

Time : 3 hours

General Instructions :

(i) Write all the answers in the Answer Script.

(ii) The question paper consists of three Sections—A, B and C.

(iii) Section—A consists of 15 questions, carrying *2* marks each.

(iv) Section—B consists of 10 questions, carrying *4* marks each, out of which 2 questions have internal choices.

(v) Section—C has 5 questions, carrying *6* marks each, out of which 2 questions have internal choices.

SECTION—A

1. If $f(x) \quad \frac{4x \quad 3}{6x \quad 4}$, $x \quad \frac{2}{3}$, show that $(f \circ f)(x) \quad x$.

2. For what value of k the function

$$f(x) \quad \begin{cases} \frac{x^2 \quad 9}{x \quad 3}, & \text{when } x \quad 3 \\ k \quad , & \text{when } x \quad 3 \end{cases}$$

is continuous at $x \quad 3$?

3. Find the domain and range of the function $f : \mathbb{R} \quad \mathbb{R}$ such that $f(x) \quad x^2 \quad 1$.

4. Show that ' ' on Q defined by $a \quad b \quad ab \quad 1$ is commutative but not associative.

5. Find the principal value of $\tan^{1}(\sqrt{3})$.

6. Find the matrix X such that $2A \quad B \quad X \quad 0$, where

$$A \quad \begin{matrix} 3 & 1 \\ 0 & 2 \end{matrix} \quad \text{and} \quad B \quad \begin{matrix} 2 & 1 \\ 0 & 3 \end{matrix}$$

7. Evaluate :

$$x \log x \, dx$$

8. If A and B be two events such that $2P(A) \quad P(B) \quad \frac{5}{13}$ and $P(A|B) \quad \frac{2}{5}$, find $P(A \quad B)$.

9. Prove that the points $A(2, 0, 3)$, $B(3, 2, \quad 1)$ and $C(1, \quad 2, \quad 5)$ are collinear.

10. Find $\frac{dy}{dx}$, if $x^2 \quad y^2 \quad 3xy \quad 1$.

11. If $y \quad 2\sin x \quad 3\cos x$, show that

$$y \quad \frac{d^2y}{dx^2} \quad 0$$

12. If $\vec{a} \quad 5\hat{i} \quad \hat{j} \quad 7\hat{k}$ and $\vec{b} \quad \hat{i} \quad \hat{j} \quad \hat{k}$, find the value of for which $\vec{a} \quad \vec{b}$ and $\vec{a} \quad \vec{b}$ are perpendicular to each other.

13. Find the direction cosines of a line segment joining the points $A(2, 5, 7)$ and $B(3, 2, 9)$.

14. Verify that $y \quad A\cos 2x \quad B\sin 2x$ is a solution of the differential equation

$$\frac{d^2y}{dx^2} \quad 4y \quad 0$$

15. Using the properties of determinants, prove that

$$\begin{vmatrix} a & b & b & c & c & a \\ b & c & c & a & a & b \\ c & a & a & b & b & c \end{vmatrix} \quad 0$$

SECTION—B

16. Express the matrix

$$A = \begin{matrix} 1 & 5 & 1 \\ 2 & 3 & 4 \\ 7 & 0 & 9 \end{matrix}$$

as the sum of a symmetric and a skew-symmetric matrix.

17. Using vectors, find the area of ABC whose vertices are $A(1, 2, 3)$, $B(2, 5, \ 1)$ and $C(\ 1, 1, 2)$.

18. Find the equation of the plane passing through the intersection of the planes $2x \ 3y \ z \ 1 \ 0$ and $x \ y \ 2z \ 3 \ 0$ and perpendicular to the plane $3x \ y \ 2z \ 4 \ 0$.

Or

Find the image of the point (1, 2, 3) in the plane $x \ 2y \ 4z \ 38$.

19. Show that the function $f(x) \ |x \ 5|$ is continuous but not differentiable at $x \ 5$.

20. If $y \ \sqrt{x \ \sqrt{x \ \sqrt{x \ \ldots}}}$, prove that

$$\frac{dy}{dx} \ \frac{1}{2y \ 1}$$

21. Evaluate :

$$\frac{2x \quad 9}{(x \quad 2)(x \quad 3)^2}\,dx$$

22. Verify Rolle's theorem for the function $f(x) \quad x^3 \quad 7x^2 \quad 16x \quad 12$ in $[2, 3]$.

23. Using the properties of definite integrals, prove that

$$\int_0^1 x(1 \quad x)^5\,dx \quad \frac{1}{42}$$

24. The volume of a spherical balloon is increasing at the rate of 25 cubic centimeter per second. Find the rate of change of its surface at the instant when its radius is 5 cm.

Or

Show that $x \quad \frac{1}{x}$ has a maximum and a minimum, but the maximum value is less than the minimum value.

25. Solve :

$$(1 \quad x^2)\frac{dy}{dx} \quad 2xy \quad \cos x$$

SECTION—C

26. Solve the following system of equations using matrix method :

$$\begin{array}{rrrr} 2x & 3y & 5z & 16 \\ 3x & 2y & 4z & 4 \\ x & y & 2z & 3 \end{array}$$

27. Sketch the region common to the circle $x^2 \quad y^2 \quad 16$ and the parabola $x^2 \quad 6y$. Also find the area of the region using integration.

28. An insurance company insured 2000 scooters and 3000 motorcycles. The probability of an accident involving a scooter is 0·01 and that of a motorcycle is 0·02. An insured vehicle met with an accident. Find the probability that the accidented vehicle was a motorcycle.

29. A square piece of tin of side 18 cm is to be made into a box without the top, by cutting a square piece from each corner and fold up the flaps. What should be the side of a square to be cut off so that the volume of the box is maximum? Also find the maximum volume of the box.

Or

A wire of length 36 cm is cut into two pieces; one of the pieces is turned in the form of a square and the other in the form of an equilateral triangle. Find the length of each piece so that the sum of the areas of the two be minimum.

30. A company makes two types of toys, *A* and *B*. Type *A* requires 5 minutes each for cutting and 10 minutes each for assembling. Type *B* requires 8 minutes each for cutting and 8 minutes each for assembling. There are 3 hours available for cutting and 4 hours available for assembling in a day. The profit is ₹ 50 each on type *A* and ₹ 60 each on type *B*. How many toys of each type should the company make in a day to maximize the profit?

Or

A small firm manufactures gold rings and chains. The combined number of rings and chains manufactured per day is at most 24. It takes one hour to make a ring and half an hour for a chain. The maximum number of hours available per day is 16. If the profit on a ring is ₹ 300 and that on a chain is ₹ 190, how many of each should be manufactured daily so as to maximize the profit?

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