## 2019

## STATISTICS

Full Marks : 100
Time : 3 hours
The figures in the margin indicate full marks for the questions
General Instructions :
(i) Write all the answers in the Answer Script.
(ii) Attempt Part-A (Objective Questions) serially.
(iii) Attempt all parts of a question together at one place.
( PART : A—OBJECTIVE )
(Marks: 50 )

## SECTION-I

(Marks: 20 )

1. Choose and write the correct answer : $1 \times 10=10$
(a) If $X$ is a random variable and $E(X)=\frac{3}{2}$, then the value of $E(7 X+2)$ is
(i) $\frac{24}{2}$
(ii) $\frac{25}{2}$
(iii) $\frac{2}{25}$
(iv) $\frac{2}{24}$

## (2)

(b) If $X$ is a random variable, then
(i) $E\left(X^{2}\right) \geq\{E(X)\}^{2}$
(ii) $E\left(X^{2}\right)=\{E(X)\}^{2}$
(iii) $E\left(X^{2}\right) \leq\{E(X)\}^{2}$
(iv) None of the above
(c) In Poisson distribution
(i) variance < mean
(ii) variance $=$ mean
(iii) variance > mean
(iv) None of the above
(d) A binomial variate $X$ has mean 6 and variance 4 . Then the values of $n$ and $p$ are
(i) $n=28$ and $p=\frac{2}{3}$
(ii) $n=8$ and $p=1$
(iii) $n=18$ and $p=\frac{1}{3}$
(iv) None of the above

## ( 3 )

(e) In an index number formula, the Time Reversal Test is
(i) $P_{01} \times P_{10}=1$
(ii) $P_{01} \times P_{10}=0$
(iii) $P_{01}+P_{10}=1$
(iv) None of the above
(f) If the value of all commodities in the base year is $\sum p_{0} q_{0}$ and the value of all commodities in the current year is $\sum p_{1} q_{1}$, then the value index is defined as
(i) $V_{01}=\frac{\sum p_{1} q_{1}}{\sum p_{0} q_{0}} \times 100$
(ii) $V_{01}=\frac{\sum p_{0} q_{0}}{\sum p_{1} q_{1}} \times 100$
(iii) $\quad V_{01}=\left(\sum p_{1} q_{1}\right)\left(\sum p_{0} q_{0}\right) \times 100$
(iv) None of the above

## ( 4 )

(g) Decrease in death rate due to advancement in medical science can be classified under the head
(i) random
(ii) cyclical
(iii) seasonal
(iv) trend
(h) Time series consists of
(i) one component
(ii) two components
(iii) three components
(iv) four components

## ( 5 )

(i) In SRSWOR, $E(\bar{x})=$ sample mean and $\bar{X}=$ population mean. Then
(i) $E(\bar{x})>\bar{X}$
(ii) $E(\bar{x})=\bar{X}$
(iii) $E(\bar{x})<\bar{X}$
(iv) None of the above
(j) If $\sigma^{2}$ is the population variance, then which of the following is correct in case of SRSWR?
(i) $V(\bar{x})>\frac{\sigma^{2}}{n}$
(ii) $V(\bar{x})<\frac{\sigma^{2}}{n}$
(iii) $V(\bar{x})=\frac{\sigma^{2}}{n}$
(iv) None of the above

## ( 6 )

2. Fill in the blanks :
(a) If $X$ is a random variable and $a$ is constant, then $V(a X)=$ $\qquad$ —.
(b) If $X$ follows binomial distribution with parameters $n$ and $p$, then $E\left(\frac{X}{n}\right)=$ $\qquad$ .
(c) Fisher's index number is the $\qquad$ of Laspeyres' and Paasche's indices.
(d) Recession is associated with $\qquad$ variation of time series.
(e) The relation between $S_{1}^{2}$ and $\sigma^{2}$ is $(N-1) S_{1}^{2}=$ $\qquad$ $\sigma^{2}$.
3. State whether the following statements are True or False :
(a) If $f(x)$ is a Probability Density Function, then $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x) d x=1$.
(b) In normal distribution, mean < median < mode.

## ( 7 )

(c) Fisher's index number does not satisfy 'Factor Reversal Test'.
(d) Natural cause such as changes in the climate and weather conditions are associated with Seasonal Variations.
(e) The total number of samples when sampling is done without replacement from a population of size $N$ is ${ }^{N} C_{n}$.

$$
\begin{aligned}
& \text { SECTION-II } \\
& \text { ( Marks : } 30 \text { ) }
\end{aligned}
$$

4. Answer the following questions : $3 \times 10=30$
(a) If a random variable $X$ takes the values $1,2,3$ with probability $P(X=r)=\frac{r}{6}: r=1,2,3$, then find $E(X)$ and $P(X \geq 2)$.
(b) If a die is thrown and let $X$ denotes the point on the uppermost face, then find $E(X)$.
(c) If $X$ follows the Poisson distribution and $P(X=1)=P(X=2)$, then find the value of $P(X=4)$.

## ( 8 )

(d) If $X$ is a normal variate with mean 5 and standard deviation 4, then find (i) $P(X<13)$ and (ii) $P(X>1)$. Given $\Phi(2)=0.9772$ and $\Phi(1)=0.8413$.
(e) Write any three uses of Cost of Living Index Number.
(f) Explain the term 'Factor Reversal Test'.
(g) Enumerate the objective of analysis of timeseries.
(h) Write down the properties of normal distribution.
(i) Write any three advantages of sample survey over census.
(j) Explain the term 'Principle of Statistical Regularity' in Sample Survey.

## ( 9 )

( PART : B—DESCRIPTIVE )
(Marks: 50)
Answer four questions, taking at least one from each Group

## GROUP—A

5. (a) If $1 \%$ residents of a city are colour-blind, find the probability that out of 100 persons selected at random, at most one is colour-blind.

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(b) Prove that $E\left[(X-c)^{2}\right]=\operatorname{Var}(X)+[E(X)-c]^{2}$, where $c$ is a constant.
(c) Write down the mean and variance of the following :

$$
f\left(x ; 5, \frac{1}{2}\right)={ }^{5} C_{x}\left(\frac{1}{2}\right)^{x}\left(\frac{1}{2}\right)^{5-x}, x=0,1,2,3,4,5
$$

6. (a) Prove that $V(a X+b)=a^{2} V(X)$, where $a$ and $b$ are constants.
(b) For a normal distribution, mean $=57.9765$ and 3 rd quartile $=60$. Then find standard deviation.
(c) In binomial distribution, prove that $P(X=x+1)=\frac{n-x}{x+1} \cdot \frac{p}{q} \cdot P(X=x)$.

## ( 10 )

## GROUP-B

7. (a) Given the following data, compute Fisher's price index number. Also interpret the result : $5+1 \frac{112}{2}=61 / 2$

|  | Base Year |  | Current Year |  |
| :---: | :---: | :---: | :---: | :---: |
| Commodity | Price | Quantity | Price | Quantity |
| $A$ | 5 | 7 | 7 | 4 |
| $B$ | 3 | 2 | 4 | 3 |
| $C$ | 1 | 5 | 1 | 5 |
| $D$ | 4 | 4 | 6 | 3 |

(b) Show that Laspeyres' price index number and Paasche's index number do not satisfy Factor Reversal Test.
8. (a) Define a Time Series. What are its components? Explain any one of them.
(b) Describe the method of moving average for measurement of trend.

## ( 11 )

## GROUP-C

9. (a) Write a note on the difference between census and sample survey.
(b) In SRSWOR, show that $E(\bar{x})=\bar{X}$, i.e., the sample mean is an unbiased estimator of population, where $\bar{x}$ and $\bar{X}$ have usual meanings.
10. (a) Write short notes on the following:

$$
2 \times 3=6
$$

(i) Sampling error
(ii) Non-sampling error
(iii) Random sampling
(b) From the following data on stratified random sampling, estimate the population mean :

| Stratum size | Sample size | Sample mean |
| :---: | :---: | :---: |
| 440 | 10 | 96.8 |
| 400 | 10 | 86.2 |
| 110 | 5 | 221.0 |

(c) What do you mean by stratified random sampling?

