Total No. of Printed Pages—12

HS/XII/A. Sc. Com/M/NC/21

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MATHEMATICS

(New Course)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

General Instructions :

(i) All questions are compulsory.

(ii) This question paper contains 36 questions divided into four Sections A, B, C and D. Section—A comprises of 20 questions of *1* mark each, Section—B comprises of 6 questions of *2* marks each, Section—C comprises of 6 questions of *4* marks each and Section—D comprises of 4 questions of *6* marks each.

(iii) There is no overall choice. However, internal choice has been provided in 9 questions of Section—A, 5 questions of Section—B, 5 questions of Section—C and 2 questions of Section—D. You have to attempt only one of the alternatives in all such questions.

(iv) Use of calculator is not permitted.

SECTION—A

1. If $R=\{(1,-1),(2,-2),(3,-1)\}$ is a relation, then find the domain and range of R. 1

Or

Find the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$. 1

2. If $f:\mathbb{R}\to\mathbb{R}$ is a function defined by $f(x)=x^2$, $\forall x\in\mathbb{R}$, then show that f is not one-one. 1

3. Construct a 2×2 matrix $A=[a_{ij}]$, whose elements are given by

$$a_{ij}=\frac{(i+j)^2}{2}$$ 1

Or

Find the value of AB when $A=[1\,2\,3\,4]$ and

$$B=\begin{bmatrix}1\\2\\3\\4\end{bmatrix}$$ 1

4. Use determinant to find the value of K for which the points $A(3,-2)$, $B(K,2)$ and $C(8,8)$ are collinear. 1

Or

Find the value of λ so that the matrix

$$\begin{bmatrix} 5-\lambda & \lambda+1 \\ 2 & 4 \end{bmatrix}$$

is singular. 1

5. If

$$\begin{vmatrix} 4 & m \\ -3 & 5 \end{vmatrix} = 8$$

find the value of m. 1

Or

If

$$\begin{vmatrix} x-2 & -3 \\ 3x & 2x \end{vmatrix} = 3$$

find the value of x. 1

6. Show that the matrix

$$A = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}$$

is skew-symmetric. 1

7. If $y = e^{3\log x}$, then find $\frac{dy}{dx}$. 1

Or

Find the value of $\frac{d}{dx}(\sin^2 x^4 + \cos^2 x^4)^4$. 1

8. Find the value of $\int_2^3 |x|\, dx$. 1

Or

Find the value of $\int_0^{\pi/2} \cos 2x \; dx$. 1

9. Find the slope of the tangent to the curve

$$y = 2x^2 + 3\sin x$$

at $x = 0$. 1

10. What is the order and the degree of the following differential equation? 1

$$\left(\frac{d^2y}{dx^2}\right)^3 + 2\left(\frac{dy}{dx}\right)^5 + 9y = \sin x$$

11. If $\bar{a}$ and $\bar{b}$ are two vectors such that $|\bar{a}| = \sqrt{2}$, $|\bar{b}| = 2$ and $\bar{a} \cdot \bar{b} = \sqrt{6}$, find the angle between $\bar{a}$ and $\bar{b}$. 1

Or

Find the dot product of the vectors $\bar{a} = \hat{i} - \hat{j} + \hat{k}$ and $\bar{b} = \hat{i} - \hat{k}$. 1

12. If $P(1, 3, 4)$ and $Q(2, 5, 3)$ be two points in space, find the unit vector along $\overrightarrow{PQ}$. 1

13. A and B appear for an interview for two vacancies in a company. The probability of A's selection is $\frac{1}{5}$ and that of B's selection is $\frac{1}{6}$. What is the probability that both of them got selected? 1

14. If A and B are events such that

$$P(A) = \frac{5}{11},\ P(B) = \frac{6}{11} \text{ and } P(A \cup B) = \frac{7}{11}$$

find $P\left(\frac{B}{A}\right)$. 1

15. Find $\bar{a} \times \bar{b}$ where $\bar{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ and $\bar{b} = \hat{i} + 2\hat{j} - \hat{k}$. 1

Or

Find the vector equation of the straight line joining the points (1, 2, 3) and (2, 1, 4). 1

Choose the correct answer :

16. The value of $\int 2^x dx$ is

(a) $\frac{2^{x+1}}{x+1} + C$

(b) $2^x \log 2 + C$

(c) $\frac{2^x}{\log 2} + C$

(d) None of the above 1

17. The derivative of a constant function is

(a) a non-zero constant

(b) zero

(c) the function itself

(d) None of the above 1

Or

The second-order derivative of $\log x$ with respect to x is

(a) $\frac{1}{x}$

(b) $\frac{1}{x^2}$

(c) $-\frac{1}{x^2}$

(d) 1 1

18. If $f(x) = 2$, then the value of $f(2)$ is

(a) 2

(b) x

(c) x^2

(d) 2^x 1

19. If $\bar{a} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\bar{b} = \hat{i} - 9\hat{j} - 3\hat{k}$, then $\bar{a}$ and $\bar{b}$ are

(a) perpendicular vectors

(b) parallel vectors

(c) equal vectors

(d) None of the above 1

20. The maximum value of $Z = 4x + 3y$, subject to the constraints $x + y \leq 4,\ \ x \geq 0,\ \ y \geq 0$ is

(a) 8

(b) 10

(c) 12

(d) 16 1

SECTION—B

21. Show that the relation R in the set of real numbers $\mathbb{R}$ defined by $R = \{(a, b) : a \leq b\}$ is transitive but not symmetric. 2

22. Show that

$$\tan^{-1}\left(\frac{x}{\sqrt{a^2 - x^2}}\right) = \sin^{-1}\left(\frac{x}{a}\right)$$ 2

Or

Evaluate $\sin\left\{\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right\}$. 2

23. If

$$A=\begin{bmatrix}3 & 1\\0 & 2\end{bmatrix} \text{ and } B=\begin{bmatrix}-2 & 1\\0 & 3\end{bmatrix}$$

find the matrix X such that $2A+X=B$. 2

Or

Find the cofactors of the elements of the second column of the determinant

$$\begin{vmatrix}8 & 4 & 2\\2 & 9 & 4\\1 & 2 & 8\end{vmatrix}$$ 2

24. Is the function defined by

$$f(x)=\begin{cases}2x+3 & , \text{ if } x\le 2\\2x-3 & , \text{ if } x>2\end{cases}$$

continuous at $x=2$? Justify. 2

Or

Find $\frac{dy}{dx}$, when $x=\sin t$ and $y=\cos 2t$. 2

25. By using the properties of definite integral, show that

$$\int_0^1 x(1-x)^5\,dx = \frac{1}{42}$$ 2

Or

Evaluate $\int \frac{(\tan^{-1} x)^2}{4+4x^2}\,dx.$ 2

26. If $y=\sqrt{x+\sqrt{x+\sqrt{x+\sqrt{x+\cdots}}}}$ to ∞, then prove that

$$\frac{dy}{dx} = \frac{1}{2y-1}$$ 2

Or

Solve the following equation : 2

$$(x^2+1)\frac{dy}{dx} = xy$$

SECTION—C

27. *(a)* Find the value of k, if the function defined by

$$f(x) = \begin{cases} kx+1 & , \text{ if } x \le 5 \\ 3x-5 & , \text{ if } x > 5 \end{cases}$$

is continuous at $x=5$. 2

(b) Use definition to find the derivative of x^2. 2

28. If $y = \sin^{-1} x$, then prove that

$$(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$ 4

Or

Find the interval in which the function

$$f(x) = 2x^3 - 3x^2 - 36x + 7$$

is

(a) strictly increasing;

(b) strictly decreasing. 4

29. Prove that

$$\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x} + \sqrt{\cos x}}\, dx = \frac{\pi}{4}$$ 4

Or

Find the equation of the tangent line to the curve $y = x^2 - 2x + 7$ which is parallel to the line $2x - y + 9 = 0$. 4

30. Find the Cartesian and vector equation of the line which passes through the point $(-2, 4, -5)$ and parallel to the line given by

$$\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$$ 4

Or

Find the vector equation of the plane passing through the intersection of the planes $\bar{r} \cdot (\hat{i} + \hat{j} + \hat{k}) - 6 = 0$ and $\bar{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) + 5 = 0$ and the point (1, 1, 1). 4

31. Find two positive numbers whose product is 49 and the sum is minimum. 4

Or

If $\bar{a} = 3\hat{i} - \hat{j}$ and $\bar{b} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\bar{b}$ in the form $\bar{b} = \bar{c} + \bar{d}$ where $\bar{c}$ is parallel to $\bar{a}$ and $\bar{d}$ is perpendicular to $\bar{a}$. 4

32. If A and B are two events such that

$$2P(A) = P(B) = \frac{5}{13} \text{ and } P\left(\frac{A}{B}\right) = \frac{2}{5}$$

find P(not A and not B). 4

Or

Solve the following LPP graphically : 4

Maximize $Z = 4x + y$

subject to the constraints

$$x + y \leq 50$$
$$3x + y \leq 90$$
$$x \geq 0,\ y \geq 0$$

SECTION—D

33. If

$$A = \begin{bmatrix} 4 & 5 & 3 \\ 1 & 0 & 6 \\ 2 & 7 & 9 \end{bmatrix}$$

verify that $A \cdot (\text{adj}\, A) = (\text{adj}\, A) \cdot A = |A| I_3$. 6

Or

Solve the system equations by matrix method : 6

$$5x - y + z = 4$$
$$3x + 2y - 5z = 2$$
$$x + 3y - 2z = 5$$

34. Integrate the following : 3×2=6

(i) $\int \frac{(x+1)e^x}{\cos^2(xe^x)} dx$

(ii) $\int e^x \left(\frac{1}{x} - \frac{1}{x^2} \right) dx$

35. Find the shortest distance between the lines

$$\bar{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$$
$$\bar{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$$ 6

36. State Bayes' theorem on probability. Use this theorem to solve the following [*(a)* or *(b)*] : 2+4=6

(a) An insurance company insured 2000 scooty drivers, 4000 taxi drivers and 6000 bus drivers in a particular year. The probability of their accidents are 0·01, 0·03 and 0·15 respectively. One of the insured drivers meets with an accident. What is the probability that the person drives a scooty?

Or

(b) First bag contains 3 red and 4 black balls, and second bag contains 5 red and 6 black balls. A ball is drawn at random from one of the bags and it is found to be red. Find the probability that it was drawn from the second bag.

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