25/07/2022
Morning

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Memory Based
Answers & Solutions

Time : 3 hrs. *for* **M.M. : 300**

JEE (Main)-2022 (Online) Phase-2

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

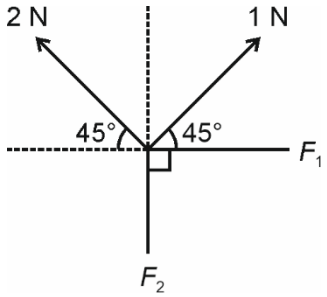
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. The ratio of $F_1 : F_2 = 1 : x$. Then find the value of x when net force is zero.



(1) 3 (2) 2
(3) 1 (4) 4

Answer (1)

Sol. As the net force is zero

$\Rightarrow \Sigma F_x = 0$ and $\Sigma F_y = 0$

$$\left(\sqrt{2} - \frac{1}{\sqrt{2}}\right) = F_1$$

$$\Rightarrow F_1 = \frac{1}{\sqrt{2}}$$

and $F_2 = \frac{3}{\sqrt{2}}$

$\Rightarrow F_1 : F_2 = 1 : 3$

$\Rightarrow x = 3$

2. Two positive charged particles A and B having same kinetic energy move inside a transverse uniform magnetic field. The ratio of radius of path of charge A to that of charge B is $\frac{3}{5}$, while mass of particle A is $\frac{4}{9}$ times that of particle B. The ratio of charge on particle A to that of B is

(1) $\frac{9}{10}$ (2) $\frac{10}{9}$
(3) $\frac{2}{5}$ (4) $\frac{5}{3}$

Answer (2)

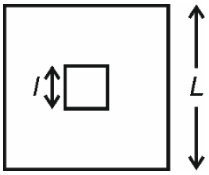
Sol. $r = \frac{mv}{qB} = \frac{\sqrt{2mKE}}{qB}$

$$\Rightarrow \frac{r_A}{r_B} = \sqrt{\frac{m_A}{m_B}} \times \frac{q_B}{q_A}$$

$$\Rightarrow \frac{q_A}{q_B} = \sqrt{\frac{m_A}{m_B}} \times \frac{r_B}{r_A}$$

$$= \frac{2}{3} \times \frac{5}{3} = \frac{10}{9}$$

3. A square loop of side length l is placed at centre of another square loop of side length L ($L >> l$). Mutual inductance of loops is equal to



(1) $\frac{2\sqrt{2}\mu_0 l^2}{\pi L}$ (2) $\frac{\mu_0 l^2}{\pi L}$
(3) $\frac{1}{2\sqrt{2}} \frac{\mu_0 l^2}{\pi L}$ (4) $\frac{\sqrt{2}\mu_0 l^2}{\pi L}$

Answer (1)

Sol. If I current flows inside the bigger conducting loop then magnetic field at the centre of bigger loop is

$$4 \times \frac{\mu_0 I}{4\pi \frac{L}{2}} \left(\sqrt{2}\right)$$

$$= \frac{\mu_0 I 2\sqrt{2}}{\pi L}$$

$\Rightarrow$ magnetic flux through smaller loop

$$= \frac{\mu_0 I 2\sqrt{2}}{\pi L} \times l^2$$

$\Rightarrow$ mutual inductance $= \frac{2\sqrt{2}\mu_0 l^2}{\pi L}$

4. A body of mass 0.5 kg has velocity $3x^2 + 5$. What is the work done in moving it from $x = 0$ m to $x = 2$ m?

(1) 60 J (2) 64 J
(3) 66 J (4) 72 J

Answer (3)

Sol. $v = 3x^2 + 5$

$v_i = 5$ m/s $\quad v_f = (3\,(2)^2 + 5)$

$= 17$ m/s

$\Delta K.E. = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$

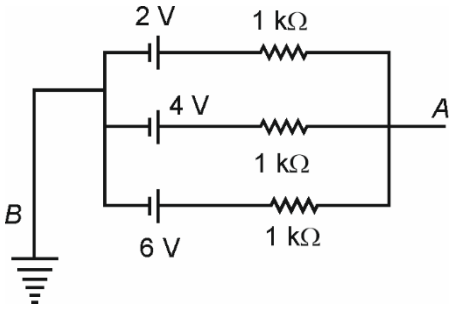
$= \frac{1}{2}\times\frac{1}{2}\times(289-25)$

$= \frac{1}{4}(264)$

= 66 J

$\Rightarrow$ Work done = 66 J

5. For the circuit shown below potential difference across point *A* and *B* is

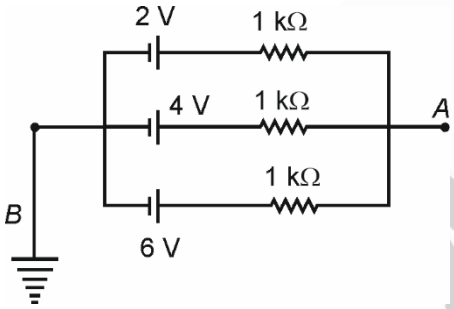


(1) 4 V (2) 3 V

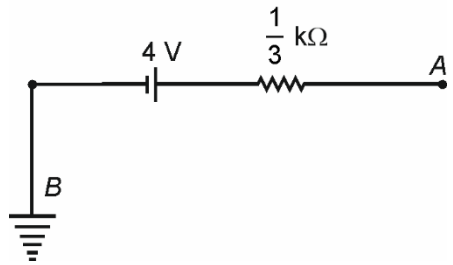
(3) 2 V (4) 1 V

Answer (1)

Sol.



As the circuit is incomplete so no current will be flowing from *A* point to *B* point. The combination of three cells can be reduced down to an equivalent cell between *A* and *B* as shown below.



as $E_{AB} = \frac{\frac{6}{1}+\frac{4}{1}+\frac{2}{1}}{3} = 4$ V

Now as the circuit from *A* to *B* is open so potential difference between *A* and *B* is equal to emf of the equivalent battery in between i.e., 4 V

6. Work function of a photosensitive material is 4.0 eV. The longest wavelength of light that can cause photoemission from the substance is

(1) 310 nm (2) 470 nm

(3) 3100 nm (4) 955 nm

Answer (1)

Sol. ϕ = 4 eV

At largest wavelength energy of photon capable to do photoelectric emission is equal to 4 eV.

$\lambda_{photon} = \frac{1240}{E(\text{eV})}$ nm

$= \frac{1240}{4} = 310$ nm

7. For a given velocity, stopping distance is 27 m. If velocity is reduced to $\frac{1}{3}$rd of initial velocity, the stopping distance (in m) will be

(1) 6 (2) 9

(3) 3 (4) 18

Answer (3)

Sol. In both cases, final velocity is zero.

$\Rightarrow 0^2 - u^2 = -2a(27)$ …(i)

And $0^2 - \left(\frac{u}{3}\right)^2 = -2a(S')$ …(ii)

From (i) and (ii)

$2a(27) = 9(2aS')$

Or $S' = 3$

8. The rms value of current flowing through capacitor is 6 A and voltage across it is 230 V. If angular frequency is 60 rad/s, then capacitance of capacitor is

(1) 435 μF (2) 576 μF

(3) 176 μF (4) 783 μF

Answer (1)

Sol. I_{rms} = 6 A

V_{rms} = 230 V

ω = 60 rad/s

$\frac{1}{\omega C} = \frac{V_{rms}}{I_{rms}} = \frac{230}{6}$

$\Rightarrow C = \frac{6}{60\times 230}$

$C = \frac{100}{23}\times 10^{-4}$

$= 43.5 \times 10^{-5}$ F

9. A solid cylinder and solid sphere are rolled down a inclined plane from same height. If masses are same then velocity of cylinder to sphere is

(1) $\sqrt{\frac{4}{5}}$ (2) $\sqrt{\frac{3}{5}}$

(3) $\sqrt{\frac{14}{15}}$ (4) $\sqrt{\frac{5}{7}}$

Answer (3)

Sol. $a = \frac{g\sin\theta}{1+\frac{K^2}{R^2}}$

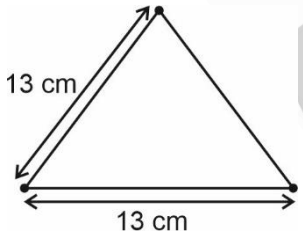
$\Rightarrow$ For solid cylinder $a_c = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2g\sin\theta}{3}$

For solid sphere $a_s = \frac{g\sin\theta}{1+\frac{2}{5}} = \frac{5g\sin\theta}{7}$

Now as distance moved by both the bodies is same on incline.

$\Rightarrow \frac{v_c}{v_s} = \sqrt{\frac{a_c}{a_s}} = \sqrt{\frac{\frac{2}{3}}{\frac{5}{7}}} = \sqrt{\frac{14}{15}}$

10. Three identical particle each of masses 100 g are placed at vertices of an equilateral triangle of side length 13 cm. Find the force on one of the mass



(1) $\frac{100\sqrt{3}}{169}G$ (2) $\frac{150\sqrt{3}}{169}G$

(3) $\frac{110\sqrt{2}}{189}$ (4) $\frac{170\sqrt{2}}{229}G$

Answer (1)

Sol. If F is the force applied by one particle on another then net force experienced by one particle = $\sqrt{3}F$

$\Rightarrow F_{net} = \sqrt{3}\times\frac{G\times m_1m_2}{r^2}$

$= \sqrt{3}\times\frac{G\times10^{-2}}{169\times10^{-4}}$

$= \frac{100\sqrt{3}}{169}G$

11. In a YDSE setup, red light of wavelength 700 nm is used to keep slit separation as 4 mm. If screen is placed 2 m away then find the width of fringe.

(1) 0.2 mm (2) 0.75 mm

(3) 1 mm (4) 0.35 mm

Answer (4)

Sol. $\beta = \frac{\lambda D}{d}$

$\beta = \frac{700\times10^{-9}\times2}{4\times10^{-3}}$

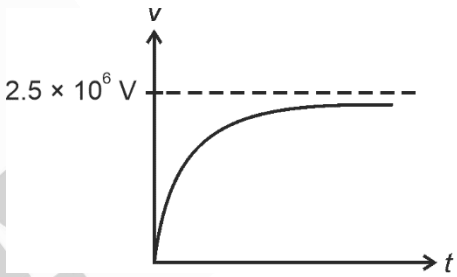
$\beta = \frac{14}{4}\times10^{-4}$

$= 0.35 \times 10^{-3}$ m

or 0.35 mm

12. A 2 μF capacitor is being charged from 0 to a maximum of 5C. Voltage across the capacitor varies as

(1)



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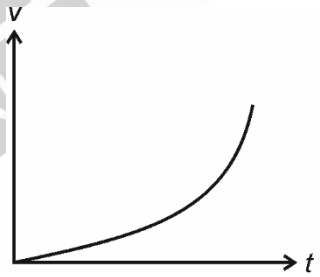

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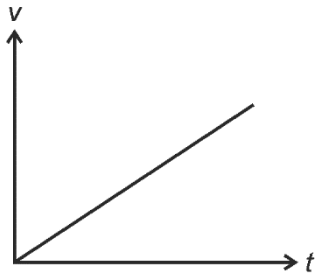

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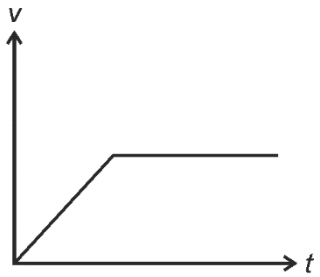


Answer (1)

Sol. As the charge stored on capacitor increases the potential difference across capacitor increases

$$\Rightarrow V_{max} = \frac{Q_{max}}{C} = \frac{5}{2\times10^{-6}}\ V = 2.5\times10^{6}\ V$$

Now the voltage across capacitor follows the relation.

$$V = V_{max}\left(1 - e^{-t/RC}\right)$$

$\Rightarrow$ Graph given in option 1 satisfies the relation.

13. If wavelength of incident wavelength is changed from 800 nm to 500 nm the velocity of electron doubles. The work function of metal is

(1) 1.24 eV

(2) 1.6 eV

(3) 1.00 eV

(4) 2.1 eV

Answer (1)

Sol. If velocity of maximum energy electron doubles then its kinetic energy becomes four times

$$K.E'_{max} = 4\ K.E_{max}$$

$$\Rightarrow \frac{1240}{800} = \phi + K.E_{max} \quad ...(1)$$

$$\frac{1240}{500} = \phi + 4\ K.E_{max} \quad ...(2)$$

$$\Rightarrow 3\phi = \left(\frac{1240}{200} - \frac{1240}{500}\right)\ eV$$

$$= (6.2 - 2.48)\ eV$$

$$= 3.72\ eV$$

$$\Rightarrow \phi \cong 1.24\ eV$$

14. Energy gap between conduction band, so that the light emitted is violet (4000 Å)

(1) 3.0 eV

(2) 4.0 eV

(3) 5.0 eV

(4) 2.0 eV

Answer (1)

Sol. For the light emitted to the violet

$$h\nu = E_{gap}$$

$$\Rightarrow \text{Energy gap} = \frac{1240}{400}\ eV$$

$$= 3.10\ eV$$

15. Which of the following statement is correct about the rainbow?

(1) Primary rainbow has red light of the bottom

(2) Primary rainbow has violet light at the top

(3) Secondary rainbow has red light at the bottom

(4) Secondary rainbow has violet light at the top

Answer (3)

Sol. Primary rainbow is caused by single internal reflection by water droplets causing red light to be visible at top while violet light visible at bottom of the primary rainbow.

Whereas secondary rainbow is caused by two internal refractions causing the violet light to be visible at bottom of the secondary rainbow.

16. In *LC* oscillation, to increase resonant frequency

(1) *L* Increases, *C* Remains same

(2) *L* Decreases, *C* Remains same

(3) *C* Increases, *L* Increases

(4) *C* Increases, *L* Remains same

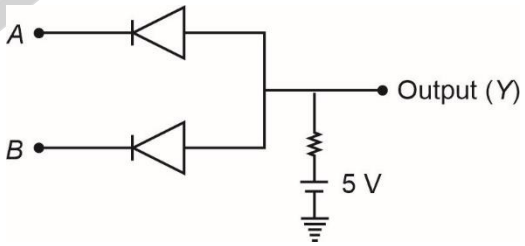
Answer (2)

Sol. $\omega = \frac{1}{\sqrt{LC}}$

$$f_{res} = \frac{1}{2\pi\sqrt{LC}}$$

to increase the resonance frequency either *L* or *C* or both should be decreased.

17. What is the truth table for following



(1)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

(2)

A	B	Y
0	1	1
1	0	1
1	1	0
0	0	1

(3)

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

(4)

A	B	Y
1	0	1
0	1	1
1	1	1
0	0	0

Answer (3)

Sol. The shown circuit in the question resemble to that of an AND gate configuration so truth table will be as given below

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

18. Which of the following have same dimensions?

(1) Electric displacement $(\vec{D})$ and charge

(2) Displacement current and electric field

(3) Electrostatic pressure and magnetic field energy density

(4) Magnetic moment and magnetic flux

Answer (3)

Sol. Dimensions of electrostatic pressure = $[M^1L^{-1}T^{-2}]$

Dimensions of magnetic field energy density

$$= \left[\frac{B^2}{2\mu_0}\right] = [M^1L^{-1}T^{-2}]$$

i.e. Dimensions of magnetic field energy density is same as that of electrostatic pressure.

19.

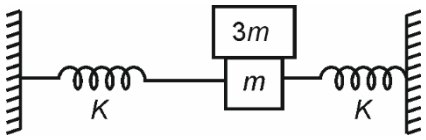
20.

SECTION - B

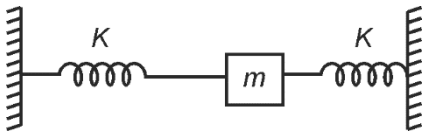
Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. For the system shown in figure, ratio of time period of oscillation T_1/T_2 is (Assume no relative slipping between blocks)

(1)



(2)



Answer (2)

Sol. In case 1

$$T_1 = 2\pi\sqrt{\frac{m+3m}{K_{eq}}}$$

$$= 2\pi\sqrt{\frac{4m}{2K}}$$

In case 2

$$T_2 = 2\pi\sqrt{\frac{m}{2K}}$$

$$\Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{4}{1}} = \frac{2}{1}$$

22.

23.

24.

25.

26.

27.

28.

29.

30.

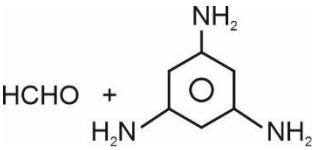
CHEMISTRY

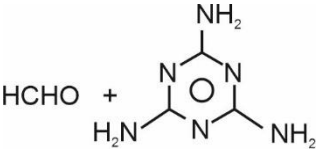
SECTION – A

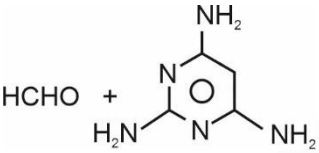
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

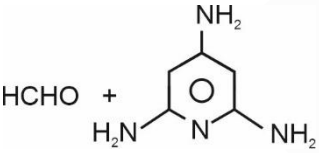
Choose the correct answer :

1. Melamine polymer is made up of

(1) HCHO + 

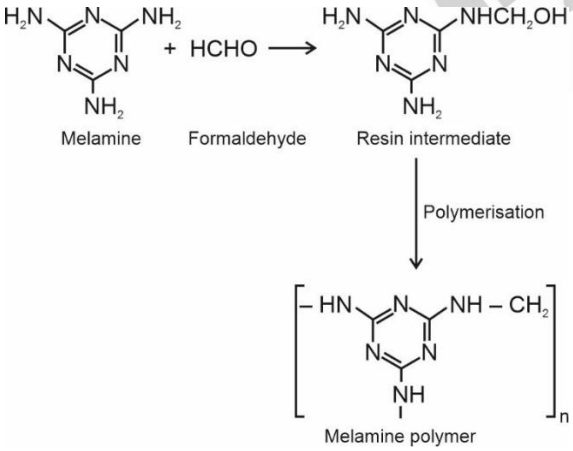
(2) HCHO + 

(3) HCHO + 

(4) HCHO + 

Answer (2)

Sol.



2. Which of the following compound is not a photochemical smog?

(1) NO (2) NO_2

(3) SO_2 (4) PAN

Answer (3)

Sol. The common components of photochemical smog are ozone, nitric oxide, acrolein, formaldehyde and peroxyacetyl nitrate (PAN)

SO_2 is present in classical smog.

3. Drugs that inhibit natural function are

(1) Agonist (2) Antagonist

(3) Allosteric (4) Antihistamines

Answer (2)

Sol. Drugs that bind to the receptor site and inhibit its natural function are called antagonists.

These are useful when blocking of a message is required.

Agonists mimic the natural messenger by switching on the receptor.

4. After denaturation protein contain which type of structure?

(1) Primary (2) Secondary

(3) Tertiary (4) Quarternary

Answer (1)

Sol. After denaturation protein contains only the primary structure.

5. Arrange the following alkali metals in the increasing order of their density.

(1) Li < Na < Rb < K < Cs

(2) Li < Na < K < Rb < Cs

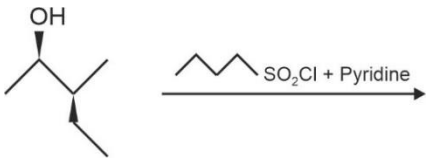
(3) K < Li < Na < Cs < Rb

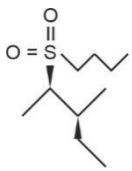
(4) Li < K < Na < Rb < Cs

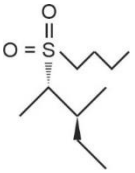
Answer (4)

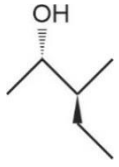
Sol. The correct order of density of alkali metals is

Li < K < Na < Rb < Cs

6. 

(1) 

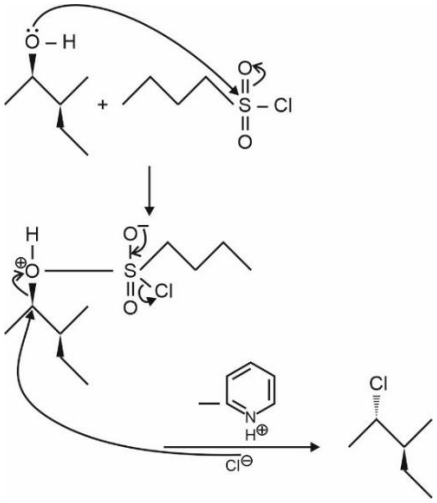
(2) 

(3) 

(4) 

Answer (4)

Sol.



7. IUPAC name of [Rn] $5f^{14}\ 6d^1\ 7s^2$ is

(1) Un nil unium (2) Un nil bium

(3) Un nil trium (4) Un nil septium

Answer (3)

Sol. Atomic number of the element is 103 and its IUPAC name is Un nil trium

8. The pK_a value of weak acid HA is 4.8 and pK_b value of weak base BOH is 4.78. The pH of corresponding salt BA will be

(1) 8.58 (2) 7.01

(3) 4.79 (4) 9.22

Answer (2)

Sol. BA is a salt of weak acid and weak base. pH of an aqueous solution of BA is given by

$$pH = 7 + \frac{1}{2}pK_a - \frac{1}{2}pK_b$$

$$= 7 + \frac{1}{2}(4.8) - \frac{1}{2}(4.78)$$

$= 7 + 2.4 - 2.39 = 7.01$

9. $SO_2Cl_2 + 2H_2O \longrightarrow 2HCl + H_2SO_4$

If 16 moles of NaOH are required to neutralise acid formed. How many moles of SO_2Cl_2 are required

(1) 16 (2) 8

(3) 4 (4) 2

Answer (3)

Sol. $SO_2Cl_2 + 2H_2O \longrightarrow 2HCl + H_2SO_4$

Let x moles of SO_2Cl_2 are required

$\therefore$ Moles of $H^{\oplus}$ formed = $4x$

Moles of $OH^{\ominus}$ from 16 moles of NaOH = 16

$\therefore\ 4x = 16$ [For complete neutralisation]

$x = 4$

10. Which of the following compound absorbs shortest wavelength of light?

(1) $[Co(NH_3)_6]^{3+}$

(2) $[Co(NH_3)_5H_2O]^{3+}$

(3) $[Co(NH_3)_5Cl]^{2+}$

(4) $[Co(CN)_6]^{3-}$

Answer (4)

Sol. Among the given complexes, the most stable complex will absorb the shortest wavelength. Higher oxidation state of the metal ion and strong field ligand will form the most stable complex. $[Co(CN)_6]^{3-}$ is the most stable complex and hence will absorb shortest wavelength.

11. $S_1 \rightarrow$ Acrolein can be obtained from glycerol by treatment with $KHSO_4$.

$S_2 \rightarrow$ Acrolein has a fruity smell and presence of glycerol can be identified with acrolein.

(1) S_1 and S_2 both are correct

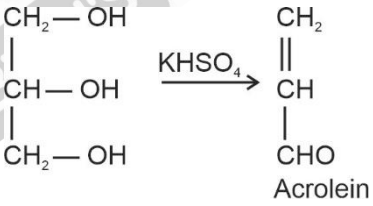
(2) S_1 and S_2 both are incorrect

(3) S_1 is correct S_2 is incorrect

(4) S_1 is incorrect S_2 is correct

Answer (3)

Sol. Glycerol reacts in presence of $KHSO_4$ to form acrolein



Acrolein

Acrolein has a pungent odour.

12. In the extraction of copper which of the following compound is used in slag formation?

(A) Al_2O_3 (B) NiO

(C) CaO (D) ZnO

(E) SiO_2

(1) Only A, C & E (2) Only A, D, E

(3) Only E (4) Only A and D

Answer (3)

Sol. In the extraction of copper, silica is used to remove iron oxide as impurity in the form of slag.

13. What happens to viscous force with changes in temperature (T) and area (A)?

(1) Increases with increase in temperature as well as area

(2) Decreases with decrease in temperature as well as area

(3) Increases with increase in temperature and decreases with increase in area

(4) Decreases with increase in temperature and increases with increase in area

Answer (4)

Sol. F_v (viscous force) = $\eta A \frac{dv}{dx}$

where η is coefficient of viscosity

$\therefore$ viscous force increases with increase in area of contact

It decreases with the increase in temperature because at high temperature, molecules have high kinetic energy and can overcome the intermolecular forces to slip past one another between the layers

14. Which of the following is correct option for A & B. According to following reaction

$BF_3 + NaH \rightarrow A + NaF$

$A + (Me)_3N \rightarrow B$

(1) (A) BH_3
(B) $Me_3N \cdot BF_3$

(2) (A) B_2H_6
(B) $Me_3N \cdot BH_3$

(3) (A) B_2H_6
(B) $B(CH_3)_3$

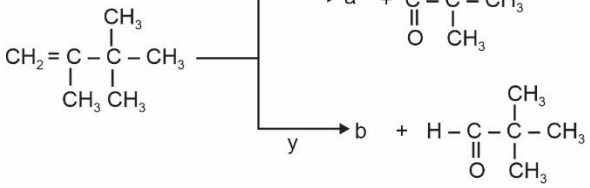
(4) (A) BH_3
(B) $(BN)_x$

Answer (2)

Sol.

$2BF_3 + 6NaH \rightarrow \underset{(A)}{B_2H_6} + 6NaF$

$B_2H_6 + 2Me_3N \rightarrow 2Me_3\overset{+}{N}\underset{(B)}{-}\overset{-}{B}H_3$

15. 

a $\rightarrow$ on oxidation gives a compound, produced from ant.

b $\rightarrow$ produced from combustion of natural gas identify the correct option

(1) x = (i) O_3
(ii) Zn/H_2O

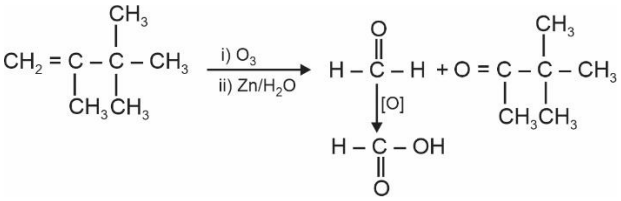
(2) x = (i) O_3
(ii) H_2O_2

(3) y = $KMnO_4/OH^-$

(4) $KMnO_4/H^+$

Answer (1)

Sol. Compound 'a' can be $H-\overset{O}{\overset{\|}{C}}-H$ which on oxidation gives H-COOH that is produced from ant



16. Which of the following is the correct decreasing order of ionic radii of

Al^{3+}, K^+, Na^+, Mg^{2+} ?

(1) $K^+ > Na^+ > Mg^{2+} > Al^{3+}$

(2) $Na^+ > Mg^{2+} > K^+ > Al^{3+}$

(3) $Mg^{2+} > Al^{3+} > Na^+ > K^+$

(4) $K^+ > Al^{3+} > Mg^{2+} > Na^+$

Answer (1)

Sol. The correct order of ionic radii is

$K^+ > Na^+ > Mg^{2+} > Al^{3+}$

17. Interhalogen compound formed by the reaction of bromine with excess of F_2 followed by hydrolysis gives Bromine containing compound.

(1) Hypohalate (2) Bromate

(3) Perbromate (4) Bromite

Answer (2)

Sol. $Br_2 + \underset{\text{(Excess)}}{5F_2} \rightarrow 2BrF_5$

$BrF_5 + 3H_2O \rightarrow HBrO_3 + 5HF$

18. What is sum the degree of unsaturation of the following compounds

(a) $CH_3 - CH = CH - C \equiv N$

(b) $CH_3 - N = CH - C \equiv CH$

(c) $CH_2 = CH - NH - CH = CH_2$

(d) $CH_2 = CH - CH_2 - C \equiv N$

(1) 10 (2) 11

(3) 12 (4) 13

Answer (2)

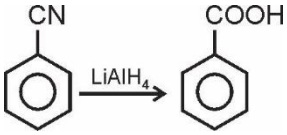
Sol. $CH_3 - CH = CH - C \equiv N$ DU = 3

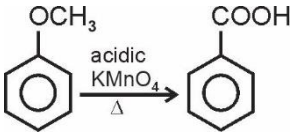
$CH_3 - N = CH - C \equiv CH$ DU = 3

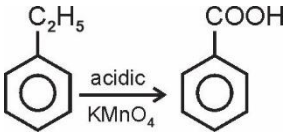
$CH_2 = CH - NH - CH = CH_2$ DU = 2

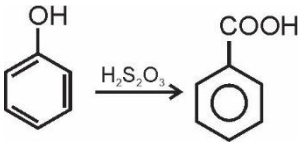
$CH_2 = CH - CH_2 - C \equiv N$ DU = 3

19. Which of the following reaction represents the correct product?

(1) 

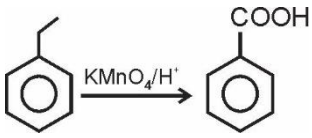
(2) 

(3) 

(4) 

Answer (3)

Sol.



20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. How many of the following compounds is/are diamagnetic?

$N_2, N_2^+, N_2^-, N_2^{2-}, O_2, O_2^+, O_2^-, O_2^{2-}$

Answer (02.00)

Sol. Species with no unpaired e^- are diamagnetic

N_2, and O_2^{2-} are diamagnetic

22. Consider a compound with molecular formula C_4H_5N. It is acyclic. How many sp^3 hybridised C-atoms are present in the given compound.

Answer (01.00)

Sol. Possible compound is

$CH_3 - CH = CH - C \equiv N$ or $CH_2 = CH - CH_2 - C \equiv N$

Only one sp^3 hybridised C-atom is present in given compound.

23. Bromine is reacted with excess of F_2. What is oxidation state of central atom in compound formed.

Answer (05.00)

Sol. When Bromine is heated with excess of fluorine BrF_5 is formed.

Oxidation state of central atom is +5 in BrF_5

$Br_2 + 5F_2 \longrightarrow 2BrF_5$

24. How many of the following ions will release H_2 gas on reacting with mineral acid?

$Ti^{2+}, V^{2+}, Cr^{2+}, Co^{3+}, Mn^{2+}, Zn^{2+}$

Answer (03.00)

Sol. The transition metal ion which reduces H^+ ions from mineral acids to H_2, itself gets oxidised. The stability of M^{3+} decreases from Mn^{2+} to Zn^{2+}. Based on the SRP values of the given metal ions only Ti^{2+}, V^{2+} and Cr^{2+} release H_2 gas on reaction with mineral acids.

$E^\circ_{Ti^{2+}/Ti^{3+}} = 0.37\ V;\ E^\circ_{V^{2+}/V^{3+}} = +0.26\ V;$

$E^\circ_{Cr^{2+}/Cr^{3+}} = +0.41\ V$

25. A gas is expanding isothermally and adiabatically in two different identical containers from same initial pressure. The ratio of final to initial volume is 2 : 1 in both the containers.

If $\gamma = 1.5$ & both the processes are reversible, find the ratio of final pressure in the containers having isothermal and adiabatic process respectively?

Answer (1.41)

Sol. For isothermal process

$p_i v = (p_f)_{isothermal}\ (2v)$ ...(i)

For adiabatic process

$p_i v^\gamma = (p_f)_{adiabatic}\ (2v)^\gamma$ ...(ii)

Dividing (i) and (ii), we get

$$v^{-0.5} = \frac{(p_f)_{iso}}{(p_f)_{ad}} \times (2)^{-0.5} \times v^{-0.5}$$

$$\therefore \quad \frac{(p_f)_{iso}}{(p_f)_{ad}} = \sqrt{2} \simeq 1.41$$

26.

27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let PQ be a tower and R cuts the tower such that $RQ = 15$ cm. let S be any point on ground such that angle of elevation from R is 60° and PR makes an angle 15° on S. then height of tower is

(1) $10\left(\sqrt{3}+1\right)$

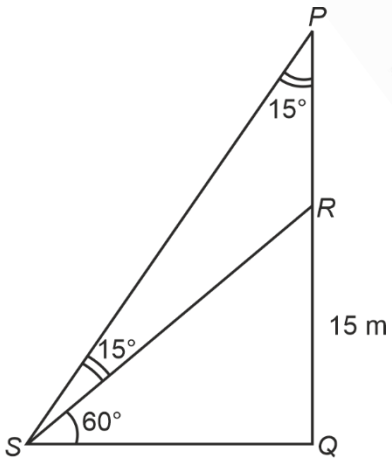
(2) $10\left(\sqrt{3}-1\right)$

(3) $5\sqrt{3}\left(2+\sqrt{3}\right)$

(4) $5\sqrt{3}\left(2-\sqrt{3}\right)$

Answer (3)

Sol. Refer to diagram



$\because$ $\angle PSR = \angle RPS = 15°$

So, $PR = RS$

$$\sin 60° = \frac{15}{RS} \Rightarrow RS = 10\sqrt{3}$$

Height of the tower $= 15 + 10\sqrt{3}$

$$= 5\sqrt{3}\left(2+\sqrt{3}\right)$$

2. If sum and product of mean and variance is 24 and 128 respectively, then the probability of no success is

(1) $\left(\frac{1}{4}\right)^{16}$ (2) $\left(\frac{2}{3}\right)^{16}$

(3) $\left(\frac{1}{2}\right)^{16}$ (4) $\left(\frac{1}{3}\right)^{16}$

Answer (1)

Sol. Let number of trials be n and probability of success be p. Let $q = 1 - p$

$\because$ Mean and variance are the roots of equation

$x^2 - 24x + 128 = 0$

So, mean $= np = 16$

and variance $= npq = 8$

Clearly $p = q = \frac{1}{2}$

and $n = 32$

Probability of no success $= {}^{32}C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^{32}$

$$= \left(\frac{1}{4}\right)^{16}$$

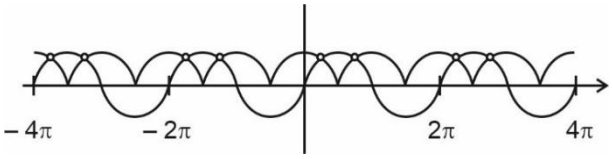
3. The number of solutions of x in $[-4\pi, 4\pi]$ of the equation $|\cos x| = \sin x$ is

(1) 2 (2) 4

(3) 6 (4) 8

Answer (4)

Sol. Sketching both $y = |\cos x|$ and $y = \sin x$ on same axis.



$\therefore$ Total 8 solutions.

4. Let α, β, γ and δ be the roots of $x^4 + x^3 + x^2 + x + 1 = 0$ then value of $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$ is

(1) 2021 (2) 1

(3) –2021 (4) –1

Answer (4)

Sol. $x^4 + x^3 + x^2 + x + 1 = 0$

$\Rightarrow \frac{x^5 - 1}{x - 1} = 0$

$\Rightarrow x^5 = 1$

So $\alpha, \beta, \gamma, \delta$ are the 5^{th} roots of unity except 1.

Clearly $\alpha^n + \beta^n + \gamma^n + \delta^n + 1^n = \begin{cases} 0 & \text{if } 5 \nmid n \\ 5 & \text{if } 5 \mid n \end{cases}$

So $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021} = -1$

5. Given curve $C_1 \equiv x^2 + (y-1)^2 = 1$, then locus of centre of circle which touches C_1 and x-axis is $C_2(x, y)$; $y > 0$ then area bounded by C_2 and line $y = 4$ is

(1) $\frac{64}{3}$ (2) $\frac{32}{3}$

(3) $\frac{16}{3}$ (4) $\frac{50}{3}$

Answer (2)

Sol. Let equation of circle be

$(x - \alpha)^2 + (y - \beta)^2 = \beta^2$

as if touches given circle $x^2 + (y - 1)^2 = 1$ we have

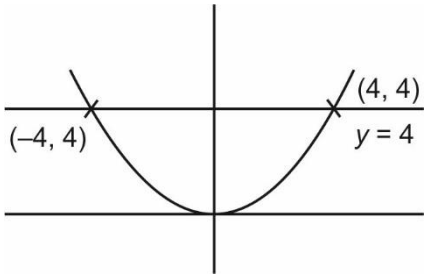
$d_{c_1 c_2} = r_1 + r_2$

$\Rightarrow \sqrt{\alpha^2 + (\beta - 1)^2} = |\beta + 1|$

$\Rightarrow \alpha^2 = 4\beta$

$\therefore$ $C_2(x, y) = x^2 = 4y$

Required area



$$A = 2\int_0^4 \left(4 - \frac{x^2}{4}\right) dx = 2\left(4x - \frac{x^3}{12}\right)\Big|_0^4$$

$$= 2\left(16 - \frac{32}{3}\right) = \frac{32}{3}$$

6. Which of the following is tautology?

(1) $(\sim p \vee q) \Rightarrow q$ (2) $q \Rightarrow (\sim p \vee q)$

(3) $(\sim p \vee q) \Rightarrow p$ (4) $p \Rightarrow (\sim p \vee q)$

Answer (2)

Sol. (1) $(\sim p \vee q) \Rightarrow q \equiv \sim(\sim p \vee q) \vee q \equiv (p \wedge \sim q) \vee q$

$\equiv p \vee q$

(2) $q \Rightarrow (\sim p \vee q) \equiv \sim q \vee (\sim p \vee q) \equiv (\sim q \vee q) \vee \sim p$

which is a tautology

(3) $(\sim p \vee q) \Rightarrow p \equiv (p \wedge \sim q) \vee p \Rightarrow p \wedge (p \vee \sim q)$

(4) $p \Rightarrow (\sim p \vee q) \equiv \sim p \vee (\sim p \vee q) \Rightarrow \sim p \vee q$

7. If a fair die is thrown two times and α, β are numbers on it, then probability of $x^2 + \alpha x + \beta > 0$

(1) $\frac{1}{2}$ (2) $\frac{19}{36}$

(3) $\frac{4}{9}$ (4) $\frac{17}{36}$

Answer (4)

Sol. Total cases = 36

$\because$ $\alpha^2 < 4\beta$

Value of β	1	2	3	4	5	6
No. of values of α	1	2	3	3	4	4

Favourable ways = 17

Required probability $= \frac{17}{36}$

8. Let $f : \{1, 2, 3, 4\} \longrightarrow \{1, 2, 3, 4, 5, 6\}$, then no. of functions for which $f(1) + f(2) = f(3)$

(1) 108 (2) 90

(3) 15 (4) 18

Answer (2)

Sol. Possible values of $f(3)$ are 2, 3, 4, 5 and 6.

for each k, if $f(3) = k$ then there will be $(k - 1)$ set of values for $f(1)$ and $f(2)$.

So no. of possible combinations of $f(1)$, $f(2)$ and $f(3)$ are $1 + 2 + 3 + 4 + 5 = 15$

Also there are six possibilities for $f(4)$.

Here total number of functions = $15 \times 6 = 90$

9. Argument of $z = 1 + e^{\frac{i6\pi}{5}}$ is equal to

(1) $\frac{2\pi}{5}$ (2) $\frac{3\pi}{5}$

(3) $\frac{7\pi}{5}$ (4) $-\frac{2\pi}{5}$

Answer (4)

Sol. $z = 1 + e^{\frac{i6\pi}{2}}$

$= 1 + \cos\frac{6\pi}{5} + i\sin\frac{6\pi}{5}$

$= 2\cos^2\frac{3\pi}{5} + i2\sin\frac{3\pi}{5}\cos\frac{3\pi}{5}$

$= 2\cos\left(\frac{3\pi}{5}\right)\left(\cos\left(\frac{3\pi}{5}\right) + i\sin\left(\frac{3\pi}{5}\right)\right)$

$= 2\cos\left(\frac{2\pi}{5}\right)\left(\cos\left(-\frac{2\pi}{5}\right) + i\sin\left(-\frac{2\pi}{5}\right)\right)$

$\therefore \quad \text{Arg}(z) = -\frac{2\pi}{5}$

10. $\int_{-10}^{10} f(x)\cos\pi x\, dx$ is equal to, where

$$f(x) = \begin{cases} 1 + [x], & \text{if } [x] \text{ is odd} \\ 1 + [x] + x, & \text{if } [x] \text{ is even,} \end{cases}$$

Where [·] denotes G.I.F.

(1) $\frac{20}{\pi^2}$ (2) $-\frac{20}{\pi^2}$

(3) $\frac{10}{\pi^2}$ (4) $-\frac{10}{\pi^2}$

Answer (2)

Sol. $\int_{-10}^{10} f(x)\cos\pi x\, dx$

$= \int_{-10}^{10} (1 + [x])\cos\pi x\, dx + \int_{-10}^{-9} x\cos\pi x\, dx$

$+ \int_{-8}^{-7} x\cos\pi x\, dx + ... + \int_{8}^{9} x\cos\pi x\, dx$

let $I = \int_{-10}^{10} (1 + [x])\cos\pi x\, dx$

$\Rightarrow I = \int_{-10}^{10} (1 + [-x])\cos\pi x\, dx$

On adding $2I = \int_{-10}^{10} \cos\pi x\, dx = 0$

So, $\int_{-10}^{10} f(x)\cos\pi x\, dx$

$= 0 + \int_{0}^{1} ((x+8) + (x+6) + ... + (x-10))\cos\pi x\, dx$

$= 10\int_{0}^{1} (x-1)\cos\pi x\, dx = 10\left[\frac{(x-1)\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_0^1$

$= -\frac{20}{\pi^2}$

11. Equation of plane containing the straight line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$ and perpendicular to the plane containing the $\text{S}\cdot\text{L}\cdot \frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$ is

(1) $x + 2y - 2z = 0$

(2) $3x + 2y - 2z = 0$

(3) $x - 2y + z = 0$

(4) $5x + 2y - 4z = 0$

Answer (3)

Sol. $\because$ Plane containing the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{4}$

$\therefore$ Plane passes through the point (0, 0, 0)

And plane is perpendicular the plane containing the line $\frac{x}{3} = \frac{y}{4} = \frac{z}{2}$ and $\frac{x}{4} = \frac{y}{2} = \frac{z}{3}$

So, vector parallel to the plane

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 4 & 2 & 3 \end{vmatrix} = 8\hat{i} - \hat{j} - 10\hat{k}$$

Now let equation of plane

$ax + by + cz = 0$

And $2a + 3b + 4c = 0$

$8a - b - 10c = 0$

$\frac{a}{-26} = \frac{b}{52} = \frac{c}{-26}$

$\therefore \quad x - 2y + z = 0$

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. All words formed by arranging the letters of word 'MANKIND' are placed in dictionary. The rank of word 'MANKIND' is

Answer (1492)

Sol. Alphabetical order of words is ADIKMNN

finding rank.

A _ _ _ _ _ _ $\rightarrow \frac{6!}{2!}$ = 360 words

D _ _ _ _ _ _ $\rightarrow$ 360 words

I _ _ _ _ _ _ $\rightarrow$ 360 words

K _ _ _ _ _ _ $\rightarrow$ 360 words

M A D _ _ _ _ $\rightarrow \frac{4!}{2!}$ = 12 words

M A I _ _ _ _ $\rightarrow$ 12

M A K _ _ _ _ $\rightarrow$ 12

M A N D _ _ _ $\rightarrow$ 6

M A N I _ _ _ $\rightarrow$ 6

M A N K D _ _ $\rightarrow$ 2

M A N K I D _ $\rightarrow$ 1

M A N K I N D $\rightarrow$ 1

$\therefore$ Rank is (4 × 360) + (3 × 12) + 2(6) + 2 + 2

= 1492

22. If $\lim\limits_{x\to\infty}\sqrt{x^2-x-1}+\alpha x+\beta=0$, then $|8(\alpha+\beta)|$ is

Answer (4)

Sol. $\lim\limits_{x\to\infty}\sqrt{x^2-x-1}+(\alpha x+\beta)=0$

It will be indeterminate form when $\alpha < 0$

$\Rightarrow \lim\limits_{x\to\infty}\frac{(x^2-x-1)-(\alpha x+\beta)^2}{\sqrt{x^2-x-1}-(\alpha x+\beta)}=0$

$\Rightarrow \lim\limits_{x\to\infty}\frac{x^2(1-\alpha^2)+x(-1-2\alpha\beta)+(-1-\beta^2)}{\sqrt{x^2-x-1}-(\alpha x+\beta)}=0$

$\Rightarrow$ Clearly, $\alpha^2 = 1$ and $-1-2\alpha\beta = 0$

$\Rightarrow \alpha = -1$ and $\beta=\frac{1}{2}$

So, $|8(\alpha+\beta)| = 4$

23. Given $a_1 = b_1 = 1$, $a_n = a_{n-1} + 2$ and $b_n = a_n + b_{n-1}$ find the value of $\sum\limits_{n=1}^{15} a_n \cdot b_n$

Answer (27560)

Sol. $\because$ $a_n - a_{n-1} = 2$

$\Rightarrow \langle a_n\rangle$ is an A.P.

So $a_n = a_1 + (n-1)d = 2n - 1$

Also $a_n = b_n - b_{n-1}$

$\Rightarrow 2n - 1 = n^2 - (n-1)^2$

Clearly $b_n = n^2$

Now $\sum\limits_{n=1}^{15} a_n \cdot b_n = \sum\limits_{n=1}^{15}\left(2n^3 - n^2\right)$

$= 2\left(\frac{15\cdot 16}{2}\right)^2 - \frac{15\cdot 16\cdot 31}{6}$

= 27560

24. Let a, b be two non-zero real numbers. If p, r are roots of $x^2 - 8ax + 2a = 0$ and q, s are the roots of $x^2 + 12bx + 6b = 0$ such that $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P., then the value of $a^{-1} - b^{-1}$ is equal to

Answer (38)

Sol. Clearly the equation $2ax^2 - 8ax + 1 = 0$ has roots $\frac{1}{p}, \frac{1}{r}$ and $6bx^2 + 12bx + 1 = 0$ has roots $\frac{1}{q}, \frac{1}{s}$.

$\because$ $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}, \frac{1}{s}$ are in A.P., so let

$\frac{1}{p} = \alpha - 3\beta$, $\frac{1}{q} = \alpha - \beta$, $\frac{1}{r} = \alpha + \beta$ and

$\frac{1}{s} = \alpha + 3\beta$

Using sum of roots in both equations, we get

$2\alpha - 2\beta = 4$ and $2\alpha + 2\beta = -2$

$\Rightarrow \quad \alpha = \frac{1}{2}$ and $\beta = -\frac{3}{2}$

Using product of roots in both equations we get

$\frac{1}{2a} = -5 \Rightarrow \frac{1}{a} = -10$ and $\frac{1}{6b} = -8 \Rightarrow \frac{1}{b} = -48$

So, $\frac{1}{a} - \frac{1}{b} = 38$

25. The remainder of $(2024)^{2024}$ when divided by 7

Answer (1)

Sol. $\because \quad 2024 \equiv 1 \pmod 7$

$\Rightarrow \quad (2024)^{2024} \equiv 1 \pmod 7$

When $(2024)^{2024}$ is divided by 7, remainder is 1.

26.

27.

28.

29.

30.

❑ ❑ ❑