JEE-Main-25-07-2022-Shift-2 (Memory Based)

Physics

Question: A potentiometer shows reading of 36cm when connected with 1.2V battery, then the same potentiometer is connected to a 1.8V battery, find the difference in the lengths

Options:

(a) 10 cm

(b) 18 cm

(c) 20 cm

(d) 54 cm

Answer: (b)

Solution:

Balancing point = 36 cm

Voltage of cell = 1.2 V

$\therefore$ Potential gradient $= \frac{1.2}{36} = \frac{1}{30} V/cm$

Now, new voltage = 1.8 V

$\therefore$ Balancing length $= \frac{1.8}{\frac{1}{30}}$ or $\frac{1.8}{1.2} \times 36 = 54cm$

$\therefore$ Difference $= 54 - 36 = 18cm$

Question: Two billiard balls of mass 0.05kg moving in opposite directions with velocity of 10m/s and the time of contact between is 0.005sec, find the force of contact.

Options:

(a) 50 N

(b) 100 N

(c) 180 N

(d) 200 N

Answer: (d)

Solution:

$$|F| = \left|\frac{\Delta \vec{P}}{\Delta t}\right|$$

$$= \frac{0.05(\Delta V)}{0.005}$$

$$= \frac{0.05(20)}{0.005} = 200N$$

Question: Max amplitude of AM modulated wave is 6 & min amplitude of AM modulated wave is 2, modulation index in percentage is x% find x.

Options:

(a) 50 N
(b) 80 N
(c) 120 N
(d) 140 N
Answer: (a)
Solution:

Modulation index $= \dfrac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$

$$= \frac{6-2}{6+2} = \frac{4}{8} = 0.5 = 50\%$$

Question: A coil is having 2 turns then the magnetic field at the centre is B_1 when the coil is unwound and recoiled to 5 turns then the magnetic field would become B_2, find $\dfrac{B_2}{B_1}$

Options:
(a) $\dfrac{20}{3}$
(b) $\dfrac{15}{6}$
(c) $\dfrac{25}{4}$
(d) $\dfrac{4}{25}$
Answer: (c)
Solution:

$$B = \frac{\mu_0 i}{2\pi}\left(\frac{N}{r}\right)$$

Now initial turns $= n_1$ (say) and radius $= r_1$ (say)

$$n_1(2\pi r_1) = n_2(2\pi r_2)$$

$$\therefore \frac{n_1}{n_2} = \frac{r_2}{r_1}$$

$$\frac{B_1}{B_2} = \frac{n_1}{n_2} \times \frac{r_2}{r_1} = \left(\frac{n_1}{n_2}\right)^2$$

$$= \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

$$\therefore \frac{B_2}{B_1} = \frac{25}{4}$$

Question: An isolated sphere with radius R_1, when it is surrounded by concentric sphere of R_2 grounded to earth, capacity becomes n times ratio of $\frac{R_2}{R_1}$ is

Options:

(a) $\frac{n}{(n-1)}$

(b) $\frac{(n-1)}{n}$

(c) $\frac{1-n}{n}$

(d) n

Answer: (a)

Solution:

Capacity of isolated sphere $=4\pi\varepsilon_0 R_1$

Capacity of isolated sphere enclosed $=\frac{4\pi\varepsilon_0 R_1 R_2}{R_2-R_1}$

$\therefore n4\pi\varepsilon_0 R_1=\frac{4\pi\varepsilon_0 R_1 R_2}{R_2-R_1}$

$\Rightarrow nR_2-nR_1=R_2$

$\Rightarrow (n-1)R_2=nR_1$

$\frac{R_2}{R_1}=\frac{n}{n-1}$

Question: Heat produced in a resistance R, carrying current l in time t is given as $H=l^2Rt$. If percentage error in measurement of current, resistance and time are 2%, 1% and 1% respectively, then error in measurement of heat would be

Options:

(a) 4%

(b) 3%

(c) 6%

(d) 5%

Answer: (c)

Solution: As $H=I^2Rt$

$\therefore \left[\frac{\Delta H}{H}=\frac{2\Delta I}{I}+\frac{\Delta R}{R}+\frac{\Delta t}{t}\right]\times 100\%$

$\Rightarrow \Delta H\%=2\times 2+1+1\%=6\%$

Question: A ball is projected with 15 m/s at an angle θ such that the range and maximum height is same, then find $\tan\theta$

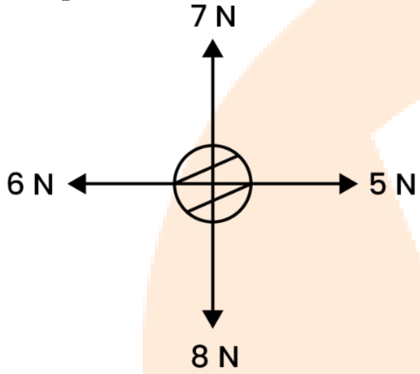
Options:

(a) 0

(b) 2

(c) 4
(d) 6
Answer: (c)
Solution:
$R = H$
$$\frac{v^2 \sin 2\theta}{g} = \frac{u^2 \sin^2 \theta}{2g}$$
$$\Rightarrow 2\sin\theta\cos\theta = \frac{\sin^2\theta}{2}$$
$$\Rightarrow \boxed{\tan\theta = 4}$$

Question: It’s the FBD given for an object, then find the value of force required and its angle with positive x-axis such that its net acceleration becomes zero



Options:
(a) $\sqrt{2}$ and angle 45°
(b) $\sqrt{6}$ and angle 55°
(c) $\sqrt{10}$ and angle 60°
(d) $\sqrt{15}$ and angle 30°
Answer: (a)
Solution:
1 N with angle 45 degrees with x-axis
Net force in x- direction $= -1N$
Net force in y-direction $= -1N$
$\therefore$ Force required to balance $= 1\hat{i} + 1\hat{j}$
$\therefore$ Angle it makes is 45° with x-axis

Question: The phenomenon which makes metal detector alarm is?
Options:
(a) Hall effect
(b) EMI
(c) Interference of EM waves
(d) Gauss law
Answer: (b)

Solution:
The operation of metal detectors is based upon the principles of electromagnetic induction. Metal detectors contain one or more inductor coils that are used to interact with metallic elements on the ground.

Question: A 9.8kg bag is hanging with a rope then a bullet of 200g moving with 10m/s get embedded in it, find the loss in kinetic energy
Options:
(a) 9.8 J
(b) 5.8 J
(c) 7.8 J
(d) 4.8 J
Answer: (a)
Solution:
$m_1u_1 + m_2u_2 = (m_1 + m_2)v$
$0.2(10) + 9.8(0) = (9.8 + 2)v$
$2 = 0.2\, m/s$
Initial K.E $= \frac{1}{2}(0.2)(10)^2$
=10 J
Final K.E $= \frac{1}{2}(0.2)(0.2)^2$
$+\frac{1}{2}(9.8)(0.2)^2$
$= \frac{1}{2}(10)(0.2)^2$
$= 0.2J$
$\therefore$ Loss in K.E = 9.8 J

Question: A body is taken from surface of earth to a height of 5R/4 from length of earth, where R is radius of earth. Percentage decrease in weight of body at height is
Options:
(a) 33.33%
(b) 64%
(c) 25%
(d) 36%
Answer: (d)
Solution:

$$g_h = \frac{GM}{\left(5\frac{R}{4}\right)^2}$$

$$= \frac{16}{25} g_e$$

$$\% \text{change} = \frac{g_h - g_e}{g_e} \times 100\%$$

$$= \left[\left(\frac{16}{25}\right) - 1\right] \times 100\%$$

$$= -36\%$$

Question: A second pendulum is taken to a height of h=2R, find the length of the pendulum at that place.

Options:

(a) $\frac{1}{7}m$

(b) $\frac{1}{9}m$

(c) $\frac{1}{5}m$

(d) $\frac{1}{6}m$

Answer: (b)

Solution:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

At $h = 2R$

$$g = \frac{GM}{(R+2R)^2} = \frac{GM}{9R^2} = \frac{g}{9}$$

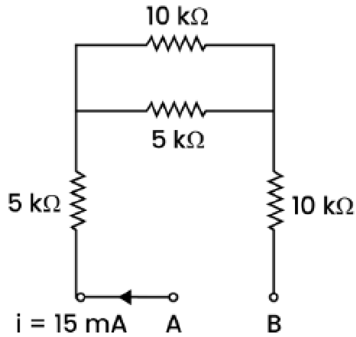
$$\therefore T = 2\pi\sqrt{\frac{l}{g/9}} = 6\pi\sqrt{\frac{l}{g}}$$

But it's a second's pendulum

So time period is 2 sec

$$\therefore 2 = 6\pi\sqrt{\frac{l}{g}} \Rightarrow l \approx \frac{1}{9}m$$

Question: Find potential difference at AB



Options:

(a) 475 V

(b) 385 V

(c) 275 V

(d) 365

Answer: (c)

Solution:

Equivalent resistance

= 5 + (5& 10 in parallel)

+ 10

$$= 5 + \frac{50}{15} + 10$$

$$= 15 + \frac{10}{3} = \frac{55}{3}$$

$$\therefore V = iR$$

$$= 15 \times 10^{-3} \times \frac{55}{3} \times 10^{3}$$

= 275 V

Question: The heat developed in a resistor H has a % error of x % if % error in i is 2%, R is 2%, t = 3%, t = 3% find x.

Options:

(a) 9%

(b) 11%

(c) 6%

(d) 10%

Answer: (a)

Solution:

$$H = i^2 Rt$$

$$\frac{DH}{H} = \frac{2\Delta i}{i} + \frac{\Delta R}{R} + \frac{\Delta t}{t}$$

$\therefore \%$ error in H = 2 (2) + 2 + 3

= 9%

Question: Magnetic flux as a function of time is given as $8t^2$-9t+5, and a resistor of 20Ω is connected with t, find the value of induced current at t = 0.25 sec.

Options:

(a) 0.35 mA

(b) 0.15 mA

(c) 0.45 mA

(d) 0.25 mA

Answer: (d)

Solution:

$$\phi = 8t^2 - 9t + 5$$

$$R = 20\Omega$$

$$\varepsilon = -\frac{d\phi}{dt} = -(16t - 9)$$

At t = 0.25

$$\varepsilon = 5V$$

$$\therefore i = \frac{\varepsilon}{R} = \frac{5}{20}$$

$$= 0.25$$

Question: De Broglie wavelength of proton and neutron have ratio $1:\sqrt{2}$ find the ratio of their potential difference through which they were accelerated

Options:

(a) 6:1

(b) 3:1

(c) 1:1

(d) 4:1

Answer: (d)

Solution:

$$\lambda = \frac{h}{\sqrt{2mqv}}$$

$$\frac{\lambda_p}{\lambda_d} = \sqrt{\frac{m_d v_d q_d}{m_p v_p q_p}}$$

$$\frac{1}{\sqrt{2}} = \sqrt{\frac{m_d v_d q_d}{m_p v_p q_p}}$$

$$\frac{1}{2} = \frac{m_d v_d q_d}{m_p v_p q_p} = \left(\frac{2}{1}\right)\left(\frac{v_d}{v_p}\right)\left(\frac{1}{1}\right)$$

$$\Rightarrow \frac{v_d}{v_p} = \frac{1}{4} \text{ or } \frac{v_p}{v_d} = 4:1$$

Question: If electric field of EM wave is $540 \sin \pi \times 10^4 (x - ct)$ and speed of light is 3 x 10^8 m/s then find the amplitude of magnetic field

Options:

(a) 15 x 10^7 T

(b) 18 x 10^{-7} T

(c) 14 x 10^{-6} T

(d) 11 x 10^{-7} T

Answer: (b)

Solution:

$$B_0 = \frac{E_0}{C} = \frac{540}{3 \times 10^8}$$

$$= 180 \times 10^{-8}$$

$$= 18 \times 10^{-7} T$$

JEE-Main-25-07-2022-Shift-2 (Memory Based)

Chemistry

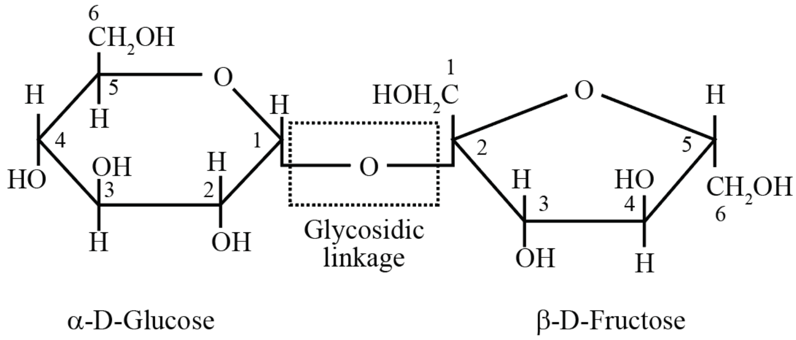
Question: Which of the following is herbicides?
Options:
(a) DDT
(b) Aldrin
(c) Sodium arsenite
(d) Dieldrin
Answer: (c)
Solution: Sodium arsenite is a herbicide

Question: Micelle formation is
Options:
(a) Exothermic, $\Delta S > 0$
(b) Endothermic, $\Delta S < 0$
(c) Exothermic, $\Delta S < 0$
(d) Endothermic, $\Delta S > 0$
Answer: (d)
Solution: $\Delta S > 0$ for micelle formation and the process is endothermic at low temperature.

Question: Glycosidic linkage between alpha glucose and beta fructose is present in
Options:
(a) lactose
(b) Sucrose
(c) Maltose
(d) None of these
Answer: (b)
Solution: Sucrose: One of the Common disaccharides is sucrose which on hydrolysis gives equimolar mixture of D-(+)-glucose and D-(-) fructose.

$$\underset{\text{Sucrose}}{C_{12}H_{22}O_{11}} + H_2O \rightarrow \underset{\text{D-(+)-Glucose}}{C_6H_{12}O_6} + \underset{\text{D-(-)-Fructose}}{C_6H_{12}O_6}$$

These two monosaccharides are held together by a glycosidic linkage between C1 of α-D-glucose and C2 of β-D-fructose. Since the reducing groups of glucose and fructose are involved in glycosidic bond formation, sucrose is a non reducing sugar.



Sucrose

Question: What is the correct order of density for group 2 elements?
Options:
(a) Be > Mg > Ca > Sr
(b) Ca > Mg > Be > Sr
(c) Mg < Ca < Sr < Be
(d) Ca < Mg < Be < Sr
Answer: (d)
Solution: The size of alkali metals increases down the group, and volume also shows an increase. Since volume is inversely proportional to the density, there is an increase in the volume, which is lesser than increasing mass in the case of Sr and Ba. So as moving down the group, density decreases first and then increases.

Question: What is the hybridization of Xe in the following compounds XeO_3, XeF_6, XeO_2F_2
Options:
(a) XeO_3–sp^3, XeF_6–sp^3d^3, XeO_2F_2–sp^3d
(b) XeO_3–sp^3d, XeF_6–sp^3, XeO_2F_2–sp^3d^3
(c) XeO_3–sp^3d^2, XeF_6–sp^3, XeO_2F_2–sp^3
(d) XeO_3–sp^3d, XeF_6–sp^3d^2, XeO_2F_2–sp^3d
Answer: (a)
Solution: Hybridization of Xe in
$XeO_3 \Rightarrow sp^3$
$XeF_6 \Rightarrow sp^3d^3$
$XeO_2F_2 \Rightarrow sp^3d$

Question: 99.9 % pure dihydrogen can be prepared by
Options:
(a) Reaction of methane with steam
(b) Mixing natural hydrocarbons of high molecular weight
(c) Electrolysis of water
(d) Reaction of salts like hydride with water
Answer: (c)
Solution: Highly pure hydrogen can be obtained by electrolysis of water.
It is the decomposition of water into O_2 & H_2 gas by passing electric current.
At anode, $6H_2O(l) \rightarrow O_2(g) + 4H_3O^+ + 4e^-$
cathode, $4e^- + 4H_2O(l) \rightarrow 2H_2(g) + 4OH^-(ag)$

Overall → $2H_2O(l) \rightarrow 2H_2(g) + O_2(g)$

Question: The first ionization energy order B, Be, C, O, N among is ______
Options:
(a) B < Be < C < O < N
(b) B < Be < C < N < O
(c) Be < B < C < N < O
(d) Be < B < C < O < N
Answer: (a)
Solution: The ionisation energy increases across a period as atomic size decrease
Therefore, correct order is B < Be < C < O < N

Question: Drugs which do not bind to its active site is called
Options:
(a) Allosteric site
(b) Non active site
(c) Both (a) and (b)
(d) None of the above
Answer: (a)
Solution: Some drugs do not bind to enzyme's active site.
These bind to a different site of enzyme called allosteric site

Question: Match the following.

Column-I (polymer)	Column-II (Uses)
(A) Nylon 6	(i) non sticking Utensils
(B) HDP	(ii) Buckets
(C) LDP	(iii) Brush Bristles
(D) Teflon	(iv) Toys

Options:
(a) A → (i); B → (iii); C → (iv); D → (ii)
(b) A → (iii); B → (ii); C → (iv); D → (i)
(c) A → (ii); B → (i); C → (iv); D → (iii)
(d) A → (iv); B → (iii); C → (ii); D → (i)
Answer: (b)
Solution:
(A) Nylon 6 ⇒ (iii) Brush Bristles
(B) HDP ⇒ (ii) Buckets
(C) LDP ⇒ (iv) Toys
(D) Teflon ⇒ (i) non sticking Utensils

Question: Statement-I: Pig iron can be obtained from cast Iron.
Statement-II: Cast iron has least carbon content
Options:
(a) Both Statement I and Statement II are correct.
(b) Both Statement I and Statement II are incorrect.
(c) Statement I is correct, but Statement II is incorrect.

(d) Statement I is incorrect, but Statement II is correct.

Answer: (b)

Solution: Cast iron is made from pig iron

Wrought iron has least carbon content

Both S-I and S-II are false

Question: $CH_3-CH_2-CN \xrightarrow{CH_3MgBr} A \xrightarrow{H_3O^+} B \xrightarrow{Zn.Hg/HCl} C$

What is C?

Options:

(a) $CH_3CH_2CH_3$

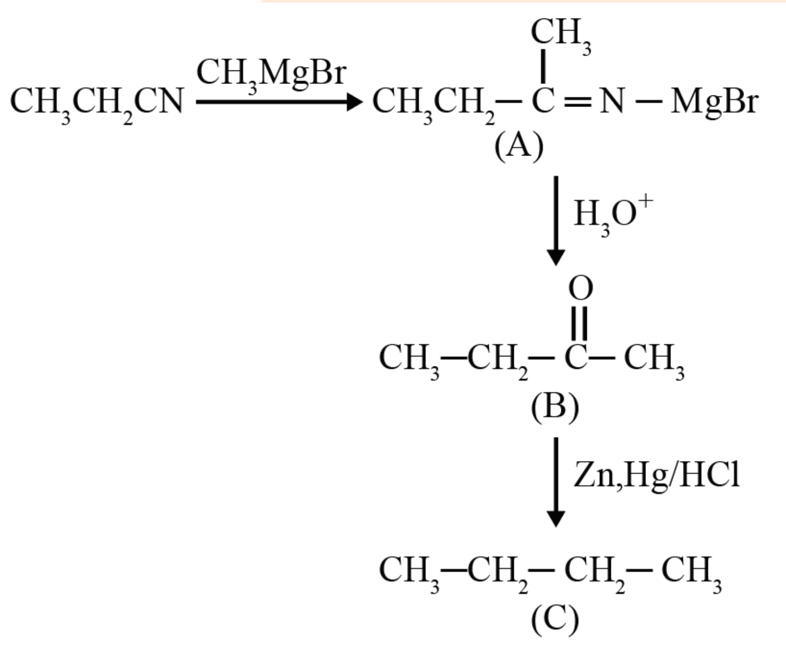
(b) $CH_3CH_2CH_2OH$

(c) $CH_3CH_2CH_2CH_3$

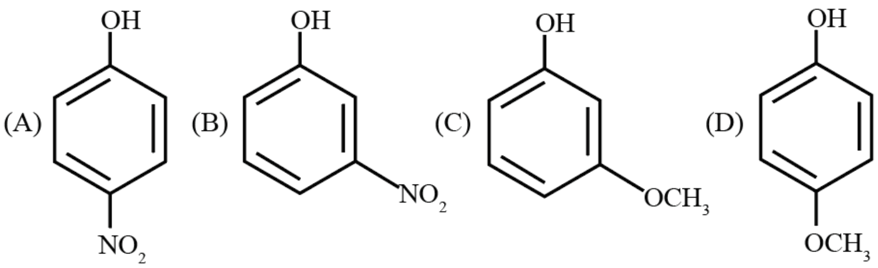
(d) $CH_3CH_2COCH_3$

Answer: (c)

Solution:



Question: Which of the following is correct decreasing order of acidity?



Options:

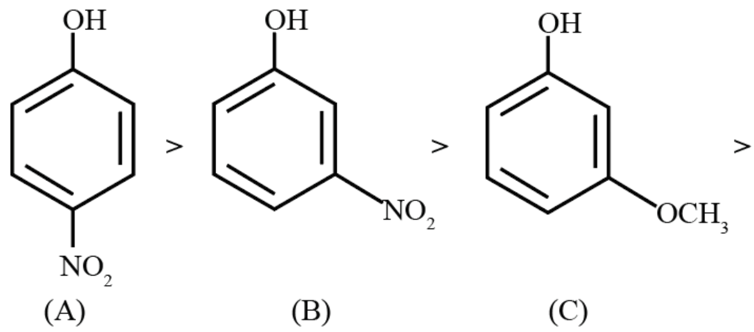
(a) A > B > C > D

(b) B > C > A > D

(c) C > A > B > D

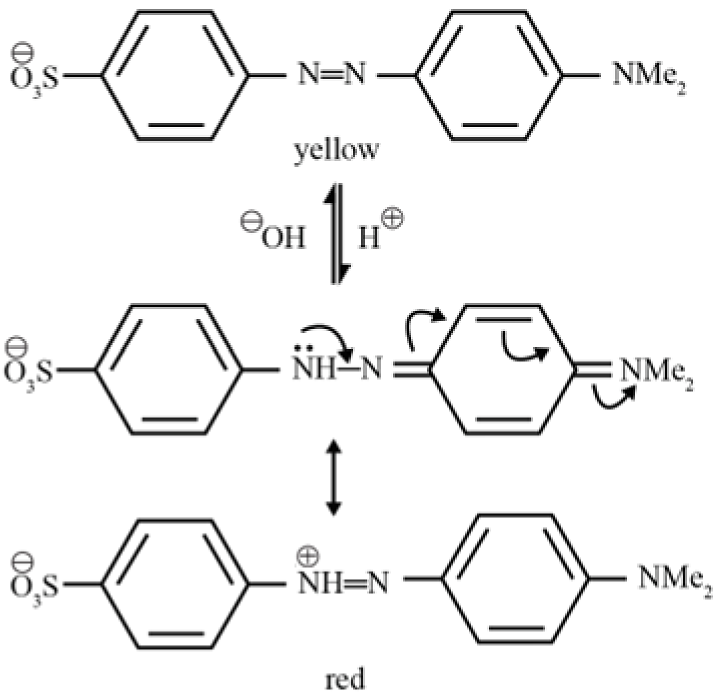
(d) D > A > B > C

Answer: (a)
Solution:



A shows –I effect, –m effect
B shows –I effect due to $-NO_2$ group
C shows –I effect due to $-OCH_3$ group
D shows +m effect and –I but +m effect is dominating here
$\therefore$ Order is A > B > C > D

Question: Methyl orange structure at end point?
Options:
(a) Quinoid form
(b) Benzenoid form
(c) Both (a) and (b)
(d) None of these
Answer: (a)
Solution:



Question: Mn^{3+}/Mn^{2+}, Fe^{3+}/Fe^{2+}, Cr^{3+}/Cr^{2+}, Co^{3+}/Co^{2+}.
Find the magnetic moment in M^{2+} which has negative E_{red}.
Answer: 4.90

Solution:

$E^{o}_{Mn^{3+}/Mn^{2+}} = +1.57$

$E^{o}_{Fe^{3+}/Fe^{2+}} = +0.77$

$E^{o}_{Co^{3+}/Co^{2+}} = +1.97$

$E^{o}_{Cr^{3+}/Cr^{2+}} = -0.41$

Magnetic moment of $Cr^{2+} = \sqrt{4(4+2)} = 4.9$ BM

Question: XeO_3, XeF_6, XeO_2F_2 sum of lone pair of central atom is ___

Answer: 3.00

Solution:

$XeO_3 \Rightarrow$ 1 lone pair

$XeF_6 \Rightarrow$1 lone pair

$XeO_2F_2 \Rightarrow$ 1 lone pair

Sum = 1 + 1 + 1 = 3 lone pair

Question: Total number of spectral line emitted when electrons jumps from n = 5 to ground state?

Answer: 10.00

Solution: If the electron jumps from $n_2 = 5$ to $n_1 = 1$

Then following transition possible

$5 \rightarrow 4$, $5 \rightarrow 3$, $5 \rightarrow 2$, $5 \rightarrow 1$

$4 \rightarrow 3$, $4 \rightarrow 2$, $4 \rightarrow 1$

$3 \rightarrow 3$, $3 \rightarrow 1$

$2 \rightarrow 1$

Hence, 10 transitions are possible

Question: Total number of acidic oxides among is/are ___

N_2O, CO, N_2O_5, CO_2, P_2O_5

Answer: 3.00

Solution:

CO, N_2O are neutral oxide

N_2O_5, CO_2, P_2O_5 are acidic oxides

JEE-Main-25-07-2022-Shift-2 (Memory Based)

MATHEMATICS

Question: The number of bijective function $f(1,3,5,7,...,99) \rightarrow (2,4,6,8,...,100)$ if $f(3) \geq f(5) \geq .... \geq f(99)$ is:

Options:

(a) ${}^{50}C_1$

(b) ${}^{50}C_2$

(c) $\frac{50!}{2}$

(d) ${}^{50}C_3 \times 3!$

Answer: (a)

Solution:

Bijective function means one-one and onto.

That means for every input unique output which is non-repeating so, set $A(1,3,5,7,...,99)$ has 50 elements and set $B(2,4,6,....,100)$ has 50 elements.

Such that $f(3) \geq f(5) \geq .... \geq f(99)$

This can be done in ${}^{50}C_1$ ways

Question: If $P(A) = \frac{1}{3}, P(B) = \frac{1}{5}$ and $P(A \cup B) = \frac{1}{2}$ then $P\left(\frac{A}{B'}\right) + P\left(\frac{A'}{B}\right) =$

Options:

(a) $\frac{5}{8}$

(b) $\frac{4}{9}$

(c) $\frac{29}{24}$

(d) 3

Answer: (c)

Solution:

As $P(A) = \frac{1}{3}, P(B) = \frac{1}{5}$ and $P(A \cup B) = \frac{1}{2}$

So, $P(A \cap B) = \frac{1}{30}$

Now, $P\left(\frac{A}{B'}\right)=\frac{P(A\cap B')}{P(B')}=\frac{\frac{1}{3}-\frac{1}{30}}{\frac{4}{5}}=\frac{3}{8}$

And $P\left(\frac{A'}{B}\right)=\frac{P(A'\cap B)}{P(B)}=\frac{\frac{1}{5}-\frac{1}{30}}{\frac{1}{5}}=\frac{5}{6}$

So, $P\left(\frac{A}{B'}\right)+P\left(\frac{A'}{B}\right)=\frac{3}{8}+\frac{5}{6}=\frac{29}{24}$

Question: Let $f(x)=\left[x^2-2x\right]+|5x-7|$, and let m be minimum value of $f(x)$ and M be maximum value of $f(x)$ in $\left[\frac{5}{4},2\right]$, then:

Options:

(a) $m=-1, M=2$

(b) $m=0, M=3$

(c) $m=-1, M=4$

(d) $m=-2, M=2$

Answer: (a)

Solution:

For $x\in\left[\frac{5}{4},2\right],\left[x^2-2x\right]=-1$

So, $f(x)=-1+|5x-7|$ is lost at $x=\frac{7}{5}$ and greatest at $x=2$

$m=f\left(\frac{7}{5}\right)=-1$

And $M=f(2)=2$

Question: The tangent at the point at $A(1,3)$ & $B(1,-1)$ on the parabola $y^2-2x-2y=1$ meet at point P. Find area of ΔPAB.

Options:

(a) 4

(b) 6

(c) 7

(d) 8

Answer: (b)

Solution:

Tangent at $A(1,3)$

$3y-(x+1)-(y+3)=1$

$\Rightarrow x-2y+5=0$

Tangent at $B(1,-1)$

$-y-(x+1)-(y-1)=1$

$x+2y-1=0$

$\therefore$ P is $\left(-2,\frac{3}{2}\right)$

$$\therefore \text{ Area of } \Delta PAB=\frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 1 & -1 & 1 \\ -2 & \frac{3}{2} & 1 \end{vmatrix}$$

$$=\frac{1}{2}\left[1-\left(-1-\frac{3}{2}\right)-3(1+2)+1\left(\frac{3}{2}-2\right)\right]$$

$$=\frac{1}{2}\left[-\frac{5}{2}-9-\frac{1}{2}\right]$$

$=6$

Question: If $\vec{a}=\hat{i}-\hat{j}+2\hat{k}$ & $\vec{a}\times\vec{b}=2\hat{i}-\hat{k}, \vec{a}.\vec{b}=3,$ find projection of $\vec{b}$ on $\vec{a}-\vec{b}$

Options:

(a) $\frac{2}{\sqrt{21}}$

(b) $\frac{\sqrt{3}}{7}$

(c) $\frac{\sqrt{7}}{3}$

(d) $\frac{2}{3}$

Answer: (a)

Solution:

As $|a\times b|^2+(\vec{a}.\vec{b})^2=|a|^2|b|^2$

$(\sqrt{5})^2+(3)^2=(\sqrt{6})^2|b|^2$

Now, projection of $\vec{b}$ on $\vec{a}-\vec{b}=\frac{(\vec{b}).(\vec{a}-\vec{b})}{|\vec{a}-\vec{b}|}$

$$=\frac{\vec{a}.\vec{b}-|b|^2}{|\vec{a}-\vec{b}|}$$

Now, $\left|\vec{a}-\vec{b}\right|^2 = |a|^2 + |b|^2 - 2\vec{a}.\vec{b} = \frac{7}{3}$

$|a-b| = \frac{\sqrt{7}}{3}$

$\therefore$ Projection is $\frac{3-\frac{7}{3}}{\frac{\sqrt{7}}{3}} = \frac{2}{\sqrt{21}}$

Question: Shortest distance between the lines $\frac{x+7}{-6} = \frac{y-6}{7} = 7$ and $\frac{7-x}{2} = y-2 = z-6$ is

Options:

(a) $2\sqrt{29}$

(b) 1

(c) $\frac{\sqrt{37}}{29}$

(d) $\frac{\sqrt{29}}{22}$

Answer: (a)

Solution:

$L_1 : \frac{x+7}{-6} = \frac{y-6}{7} = \frac{z-0}{1}$

$L_2 = \frac{x-7}{-2} = \frac{y-2}{1} = \frac{z-6}{1}$

$d = \left| \frac{(\vec{a}_2 - \vec{a}_1)\cdot(\vec{b}_1 \times \vec{b}_2)}{|b_1 \times b_2|} \right|$

Here, $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 7 & 1 \\ -2 & 1 & 1 \end{vmatrix} = 6\hat{i} + 4\hat{j} + 8\hat{k}$

$\vec{a}_2 - \vec{a}_1 = 14\hat{i} - 4\hat{j} + 6\hat{k}$

$\therefore d = \left| \frac{(14\hat{i} - 4\hat{j} + 6\hat{k})\cdot(6\hat{i} + 4\hat{j} + 8\hat{k})}{\sqrt{36+16+64}} \right| = 2\sqrt{29}$

Question: $\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right) = ?$

Answer: $\frac{1}{32}$

Solution:

$$\sin\left(\frac{\pi}{22}\right)\sin\left(\frac{3\pi}{22}\right)\sin\left(\frac{5\pi}{22}\right)\sin\left(\frac{7\pi}{22}\right)\sin\left(\frac{9\pi}{22}\right)$$

$$\cos\left(\frac{\pi}{2}-\frac{\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{3\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{5\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{7\pi}{22}\right)\cos\left(\frac{\pi}{2}-\frac{9\pi}{22}\right)$$

$$\cos\left(\frac{10\pi}{22}\right)\cos\left(\frac{8\pi}{22}\right)\cos\left(\frac{6\pi}{22}\right)\cos\left(\frac{4\pi}{22}\right)\cos\left(\frac{2\pi}{22}\right)$$

$$\cos\left(\frac{\pi}{11}\right)\cos\left(\frac{2\pi}{11}\right)\cos\left(\frac{3\pi}{11}\right)\cos\left(\frac{4\pi}{11}\right)\cos\left(\frac{5\pi}{11}\right)=\frac{1}{2^5}=\frac{1}{32}$$

Question: $\sum_{n=1}^{21}\frac{3}{(4n-3)(4n+1)}=?$

Answer: $\frac{63}{85}$

Solution:

$$T_n=\frac{3}{(4n-3)(4n+1)}$$

$$T_n=\frac{3}{4}\left(\frac{4}{(4n-3)(4n+1)}\right)$$

$$=\frac{3}{4}\left[\frac{(4n+1)-(4n-3)}{(4n-3)(4n+1)}\right]$$

$$T_n=\frac{3}{4}\left[\frac{1}{4n-3}-\frac{1}{4n+1}\right]$$

$$\Rightarrow T_1=\frac{3}{4}\left(\frac{1}{1}-\frac{1}{5}\right)$$

$$\Rightarrow T_2=\frac{3}{4}\left(\frac{1}{5}-\frac{1}{9}\right)$$

$$\vdots$$

$$T_{21}=\frac{3}{4}\left(\frac{1}{81}-\frac{1}{85}\right)$$

$$S_{21}=\frac{3}{4}\left(1-\frac{1}{85}\right)$$

$$=\frac{3}{4}\left(\frac{84}{85}\right)=\frac{63}{85}$$

Question: Find remainder when $(11)^{1011}+(1011)^{11}$ is divided by 9.

Answer: 8.00

Solution:

Given, $(11)^{1011}+(1011)^{11}$

$\Rightarrow (9+2)^{1011}+(1008+3)^{11}$

$\Rightarrow 9 \text{ Integer}+2^{1011}+9 \text{ Integer}+311$

$\Rightarrow \left(2^3\right)^{337}+3\left(3^2\right)^5$

$\Rightarrow (9-1)^{337}$

$\Rightarrow 9 \text{ Integer}-1$

$\Rightarrow 9 \text{ Integer}-1-8+8$

$\therefore$ Remainder will be 8.

Question: $\lim\limits_{x\to\frac{\pi}{4}} \dfrac{2\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x}$

Answer: 14.00

Solution:

$$\lim_{x\to\frac{\pi}{4}} \frac{2\sqrt{2}-(\cos x+\sin x)^7}{\sqrt{2}-\sqrt{2}\sin 2x}$$

Since its $\frac{0}{0}$ form, lets apply L-Hospital rule

$$\lim_{x\to\frac{\pi}{4}} 0-\frac{7(\cos x+\sin x)^6(-\sin x+\cos x)}{0-\sqrt{2}\cos 2x.(2)}$$

$$\lim_{x\to\frac{\pi}{4}} \frac{7}{2\sqrt{2}}\frac{(\cos x+\sin x)^5\left(\cos^2 x-\sin^2 x\right)}{\cos^2 x}$$

$$\frac{7}{2\sqrt{2}}\left(\sqrt{2}\right)^5=\frac{7\left(2\times2\sqrt{2}\right)}{2\sqrt{2}}=14$$

Question: If $x^2+px^2+qx+1=0\ (p<q)$ has only one root α, then α belongs to:

Answer:

Solution:

$f(0)=1$

& $f(-1)=-1+p-q+1=p-q<0$

$\therefore f(0)>0$ & $f(-1)<0$

$\therefore f(x)$ must have root between $(-1,0)$