JEE-Main-25-07-2022-Shift-1 (Memory Based)

Physics

Question: A monoatomic ideal gas expanded isothermally to double its initial volume. It then expanded adiabatically to double the volume again. Find the final pressure if the initial pressure of the gas was $2\times10^7\,Pa$.

Options:

(a) 3.53×10^3

(b) 2.5×10^7

(c) 3.5×10^7

(d) 1.5×10^3

Answer: (a)

Solution:

$V \rightarrow 2V$ (T constant)

$(2V) \rightarrow (4V)$ (Adiabatic)

$p_i \rightarrow \frac{p}{2}(p_f)$ (T constant)

$$\left(\frac{p}{2}\right)(2V)^{1.5} = (p_f)(4V)^{1.5}$$

$$\left(\frac{p}{2p_f}\right) = (2)^{1.5}$$

$$\Rightarrow p_f = \frac{p}{2^{2.5}} = \frac{2\times10^7}{2^{2.5}} = 3.53\times10^3$$

Question: Two square loops of length L and ℓ are concentric and co-planar about centre D. Where $L \gg \ell$. Find the mutual inductance.

Options:

(a) $\frac{\sqrt{2}\mu_0 l^2}{\pi L}$

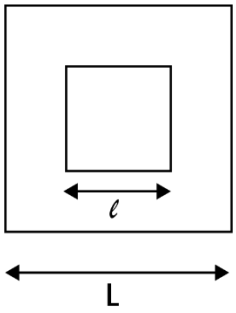
(b) $\frac{2\sqrt{2}\mu_0 l^2}{\pi L}$

(c) $\frac{2\mu_0 l^2 L}{L}$

(d) $\frac{\mu_0 l^2}{\pi L}$

Answer: (a)

Solution:



Magnetic field due to bigger loop $B = \frac{\mu_0 i}{4\pi L}(\sin\theta_1 + \sin\theta_2) \times 4$

$$B = \frac{\mu_0 i}{4\pi L}(\sin 45° + \sin 45°) \times 4$$

$$B = \frac{\mu_0 i}{4\pi L} 4\sqrt{2}$$

Flux linked with the smaller loop $\phi = Bl^2$

$$\phi = \frac{\sqrt{2}\mu_0 i l^2}{\pi L}$$

$$\phi = Mi$$

$$M = \frac{\sqrt{2}\mu_0 i l^2}{\pi L i}$$

$$\boxed{M = \frac{\sqrt{2}\mu_0 l^2}{\pi L}}$$

Question: Find work done in moving a particle of mass 0.5 kg from x = 0 to x = 2. $V = 3x^2 + 4$ m/s

Options:

(a) 20 J

(b) 80 J

(c) 100 J

(d) 60 J

Answer: (d)

Solution:

Work done $= K_f - K_i$

$$= K(2) - K(0)$$
$$= \frac{1}{2}M\left[V(2)^2\right] - \frac{1}{2}M\left[V(0)\right]^2$$
$$= \frac{1}{2}M\left[\left(3(2)^2+4\right)^2 - (4)^2\right]$$
$$= \frac{1}{2}\left(\frac{1}{2}\right)[256-16]$$
$$= \frac{1}{4}\times 240 = 60J$$

Question: A car travelling at speed 150 km/h is stopped after covering a distance of 27 m. If the initial velocity is one-third then after how much distance the car would stop?

Options:

(a) 5 m

(b) 3 m

(c) 9 m

(d) 27 m

Answer: (b)

Solution:

$u, s = 27m, v = 0$

$v^2 = u^2 + 2as$

$0 = u^2 + 2(a)(27)$

$\Rightarrow a = \frac{-u^2}{54}$

If velocity is one-third

$0^2 = \left(\frac{u}{3}\right)^2 + 2as$

$\Rightarrow \frac{u^2}{g} = 2\left(\frac{u^2}{54}\right)(s)$

$s = 3m$

Question: Dimensions of coefficient of viscosity with respect to momentum, area or time is

Options:

(a) $\left[MLT^{-1}\right]$

(b) $\left[PA^{-1}T^0\right]$

(c) $P^1A^{-1}T^0$

(d) $\left[P^1AT\right]$

Answer: (c)

Solution:

Dimension of momentum $(P)=\left[MLT^{-1}\right]$

Dimension of area $(A)=\left[L^2\right]$

So, dimension of viscosity $(\eta)=\dfrac{\left[MLT^{-1}\right]}{\left[L\right]^2}=\dfrac{P}{A}$

Hence $\boxed{[\eta]=\left[P^1A^{-1}T^0\right]}$

Question: If volume of a system is increased at constant pressure, internal energy of system

Options:

(a) Increases

(b) Decreases

(c) Remains same

(d) None of these

Answer: (a)

Solution:

We know $U=f(T)$

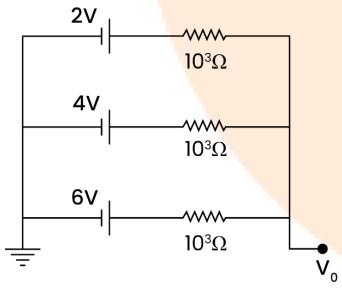
By Ideal gas equation at $(P=C)$

$V \propto T$

Hence, if $V\uparrow$ then $T\uparrow$

So, we can say that if $V\uparrow$ then $U\uparrow$

Question: In the shown circuit, find the value of V_0



Options:

(a) 4 volt

(b) 2 volt

(c) 6 volt

(d) 12 volt

Answer: (a)

Solution:

We can method of parallel combinations of cells heel.

So, $\varepsilon_{eq}=\dfrac{\dfrac{2}{10^3}+\dfrac{4}{10^3}+\dfrac{6}{10^3}}{\dfrac{1}{10^3}+\dfrac{1}{10^3}+\dfrac{1}{10^3}}=\dfrac{2+4+6}{1+1+1}=\dfrac{12}{3}=4V$

Hence $V_0 = 4volt$

Question: Two positively charged particles A and B having same kinetic energy move inside a transverse uniform magnetic field. The ratio of radius of path of charge A to that of charge B is $\frac{3}{5}$, while mass of particle A is $\frac{4}{9}$ times that of particle B. The ratio of charge on particle A to that of B is

Options:

(a) $\frac{9}{10}$

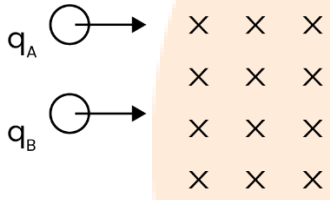
(b) $\frac{10}{9}$

(c) $\frac{2}{5}$

(d) $\frac{5}{3}$

Answer: (b)

Solution:



Given

$KE_A = KE_B$

$\frac{1}{2}M_A V_A^2 = \frac{1}{2}M_B V_B^2$

$\left(\frac{V_A}{V_B}\right)^2 = \frac{M_B}{M_A}$

$\left(\frac{V_A}{V_B}\right) = \sqrt{\frac{9}{4}}$

$\frac{V_A}{V_B} = \frac{3}{2}$

We know $r = \frac{MV}{qB}$

So,

$$r_A = \frac{M_A V_A}{q_A B}$$

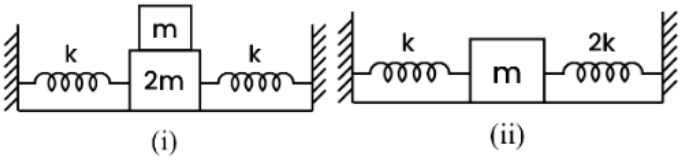
$$r_B = \frac{M_B V_B}{q_B B}$$

$$\left(\frac{r_A}{r_B}\right) = \left(\frac{M_A}{M_B}\right)\left(\frac{V_A}{V_B}\right)\left(\frac{q_B}{q_A}\right)$$

$$\frac{3}{5} = \frac{4}{9} \times \frac{3}{2} \times \left(\frac{q_B}{q_A}\right)$$

$$\left(\frac{q_A}{q_B}\right) = \frac{10}{9}$$

Question: Find the ratio of time period of oscillations in two cases as shown. Assume that there is no slipping between m and 2m blocks in case (1)



Options:

(a) $\frac{3}{2}$

(b) $\frac{3}{\sqrt{2}}$

(c) $\frac{1}{2}$

(d) $\frac{2}{3}$

Answer: (b)

Solution:

For Case Ist: $T = 2\pi\sqrt{\frac{m_{eq}}{k_{eq}}}$

$$T_1 = 2\pi\sqrt{\frac{3m}{2k}} \quad ...(1)$$

For case 2nd :

$$T_2 = 2\pi\sqrt{\frac{m_{eq}}{k_{eq}}}$$

$$T_2 = 2\pi\sqrt{\frac{m}{2k}}$$

$$\frac{T_1}{T_2}=\frac{\sqrt{\frac{3m}{2k}}}{\sqrt{\frac{m}{3k}}}$$

$$\boxed{\frac{T_1}{T_2}=\frac{3}{\sqrt{2}}}$$

Question: Rod is clamped to wall at one end on other end force is applied radius r, length l, force F, increase in length is 5 cm. Then radius 4r, length 4l, force 4F, increase in length is

Options:

(a) 15 cm

(b) 2 cm

(c) 5 cm

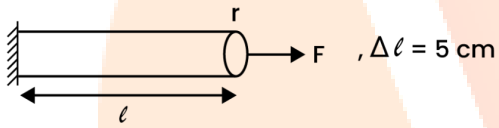
(d) 10 cm

Answer: (c)

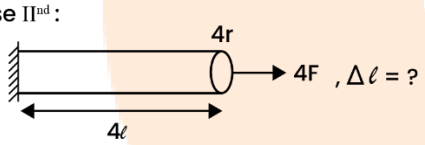
Solution:

Case Ist:

Case Ist :



Case IInd :



We know $\Delta l=\frac{Fl}{AY}$

So $\frac{5}{\Delta l}=\frac{\frac{Fl}{\pi r^2\gamma}}{\frac{4F4l}{16\pi r^2\gamma}}$

$\Rightarrow \Delta l=5cm$

Question: Particles a, b, c of mass 100 kg are in a straight line where distance between a,b and b,c is 13 m. A fourth particle p is placed on the perpendicular bisector of AC when BP = 13m. If net force on P is F. Find F in terms of G (Gravitational constant)

Options:

(a) $\frac{G\times10^4}{13^2}\left(\frac{1}{\sqrt{4}}+2\right)$

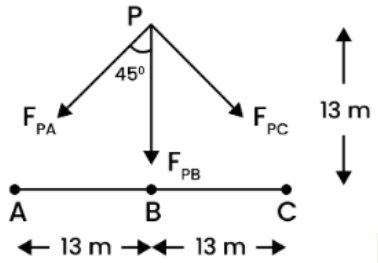
(b) $\frac{G\times10^4}{13^2}\left(\frac{1}{\sqrt{2}}+1\right)$

(c) $\frac{G\times10^4}{13^2}\left(\frac{1}{\sqrt{5}}+3\right)$

(d) $\frac{G\times10^4}{13^2}\left(\frac{1}{\sqrt{6}}+1\right)$

Answer: (b)

Solution:



$$|F_{net}| = |\vec{F}_{PA} + \vec{F}_{PB} + \vec{F}_{PC}|$$

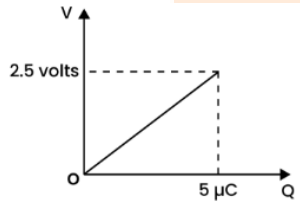
$$= 2\left(\frac{G(100)(100)}{\left(13\sqrt{2}\right)^2}\cos 45°\right) + \left(\frac{G(100)(100)}{13^2}\right)$$

$$= \frac{G\times10^4}{13^2}\left(\frac{1}{\sqrt{2}}+1\right)$$

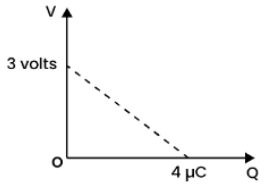
Question: A capacitor having capacity 2μF is charged steadily and charge of 5μc is given to it. Then the graph of potential with charge given is

Options:

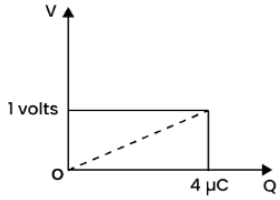
(a)



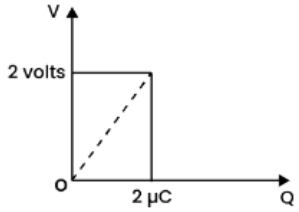
(b)



(c)



(d)



Answer: (a)

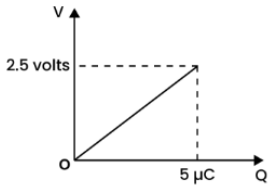
Solution:

We know Q = CV

$$\therefore V = \frac{Q}{C} = \frac{1}{C}(Q) = \frac{5\mu C}{2\mu F}$$

= 2.5 V

Graph will be a straight line as C is constant.



Question: A solid cylinder and solid sphere of same mass are rolled down from the same point on an inclined plane of inclination Q. Find the ratio of their velocity of centre of mass when they reach the bottom of incline.

Options:

(a) $\sqrt{\frac{10}{15}}$

(b) $\sqrt{\frac{18}{15}}$

(c) $\sqrt{\frac{12}{15}}$

(d) $\sqrt{\frac{14}{15}}$

Answer: (d)

Solution:

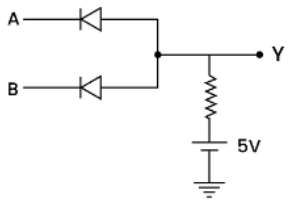
$$\text{Acceleration} \Rightarrow a = \frac{g\sin\theta}{1+\frac{I}{mR^2}}; \text{also } v = \sqrt{2as}$$

$$a_{\text{cylinder}} = \frac{g\sin\theta}{1+\frac{1/2mR^2}{mR^2}} = \frac{2}{3}g\sin\theta$$

$$a_{\text{sphere}} = \frac{g\sin\theta}{1+\frac{2/5mR^2}{mR^2}} = \frac{5}{7}g\sin\theta$$

So ratio of velocity $\frac{v_{\text{cylinder}}}{v_{\text{sphare}}} = \sqrt{\frac{a_{\text{cylinder}}}{a_{\text{sphere}}}} = \sqrt{\frac{2/3}{5/4}} = \sqrt{\frac{14}{15}}$

Question: Which logic gate is this?



Options:

(a) NAND gate

(b) OR gate

(c) AND gate

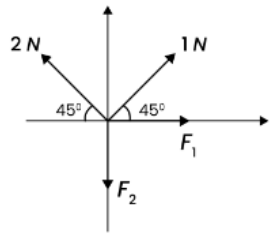
(d) XOR gate

Answer: (c)

Solution:

In 1	In 2	Out
0	0	0
0	1	0
1	0	0
1	1	1

Question: The ratio of $F_1:F_2 = 1 : x$. Then find the value of x, when net force is zero



Options:

(a) 3

(b) 2

(c) 1

(d) 4

Answer: (a)

Solution:

F_1 must balance net horizontal force

i.e $F_1 = 2\cos 45° - 1\cos 45°$

$= \sqrt{2} - \frac{1}{\sqrt{2}} = \frac{2-1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

F_2 must balance net vertical force

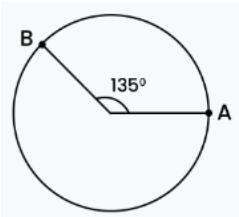
i.e. $F_2 = 2\sin 45° + 1\sin 45°$

$= \sqrt{2} + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}}$

$\therefore \frac{F_1}{F_2} = \frac{1}{3}$

$\therefore x = 3$

Question: When a particle moves on a circular track from point A to B as shown, it covers a distance of 80 m. Find the displacement of the particle. (cos 135° = -0.7)



Options:

(a) 46.2 m

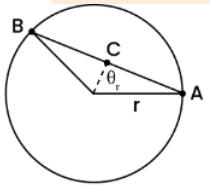
(b) 62.6 m

(c) 45.2 m

(d) 58.1 m

Answer: (b)

Solution:



$\theta = 135° = \frac{3\pi}{4}$ radiaus

We know

$\text{angle} = \frac{\text{arc}}{\text{radius}}$

$\Rightarrow r = \frac{\text{arc}}{\text{angle}} = \frac{80}{\frac{3\pi}{4}}$

So disp $AB = 2AC = 2 \times r \sin\frac{\theta}{2}$

$= 2 \times \frac{80}{\frac{3\pi}{4}} \left(\sqrt{\frac{1-\cos\theta}{2}} \right)$

$= \frac{640}{3\pi} \sqrt{\frac{1.7}{2}} = 62.6\,m$

Question: Conduction (RMS) current 6.6 μA in a circuit with capacitor connected across 220 V source. Angular frequency is 600 rad/sec. Value of capacitance is?

Options:

(a) 50 pF
(b) 20 pF
(c) 40 pF
(d) 10 pF

Answer: (a)

Solution:

$\omega = 600\, rad/s$

$I = \frac{V}{X_c} = \frac{220}{(1/\omega c)}$

$6.6\times10^{-6} = 220\times(600)\times C$

$C = \frac{6.6\times10^{-6}}{220\times600} = 50\, pF$

Question: Speed of light in two media of refractive index μ_A and μ_b is V_A and V_B respectively. The ratio of μ_A and μ_B is 1 : 2. If the time taken by light to travel same distance in both media is t_1 and t_2 respectively such that $t_2 - t_1 = 5 \times 10^{-10}$s. Calculate the distance travelled by the light.

Options:

(a) $V_B\left(10^{-9}\right)$
(b) $V_B\left(15^{-9}\right)$
(c) $V_B\left(20^{-9}\right)$
(d) $V_B\left(30^{-9}\right)$

Answer: (a)

Solution:

Let the distance travelled be 's'

$s = V_A t_1 \qquad s = V_B t_2$

$V_A t_1 = V_B t_2 \Rightarrow \frac{V_B}{V_A} = \frac{t_1}{t_2}$

Now, $V = \frac{c}{\mu} \quad \frac{\mu_A}{\mu_B} = \frac{t_1}{t_2}$

$\frac{1}{2} = \frac{t_1}{t_2} \Rightarrow t_2 = 2t_1$

$t_2 - t_1 = 5\times10^{-10}\ \text{sec}$

$2t_1 - t_1 = 5\times10^{-10} \Rightarrow t_1 = 5\times10^{-10}\ \text{sec}$

$t_2 = 10^{-9}\ \text{sec}$

Now, $s = V_A\left(5\times10^{-10}\right) = V_B\left(10^{-9}\right)$

Question: A cyclist is moving with speed 18 km/h towards a hill. A sound source kept behind (at some distance) cyclist produces sound of frequency 640 Hz. speed of sound = 320 m/s. Find the beats heard by cyclist due to source and echo.

Options:

(a) 10 Hz

(b) 30 Hz

(c) 20 Hz

(d) 50 Hz

Answer: (c)

Solution:

Let f' = apparent freq. of the source

$= f\left(\frac{v-v_0}{v-vs}\right)$

$= 640\left(\frac{320-5}{320}\right) = 640\left(\frac{315}{320}\right)$

$f'' =$ apparent freq of reflection

$= 640\left(\frac{320+5}{320}\right) = 640\left(\frac{325}{320}\right)$

$\left|f''-f'\right| = f_B = 20\,Hz$

JEE-Main-25-07-2022-Shift-1 (Memory Based)

Chemistry

Question: A interhalogen compound of Bromine reacts with excess of flourine to form?
Options:
(a) BrF_5
(b) BrF
(c) BrF_3
(d) None of these
Answer: (a)
Solution: $Br_2 + \underset{(excess)}{5F_2} \rightarrow 2BrF_5$

Question: Which of the following structure of protein does not change on denaturation?
Options:
(a) Primary
(b) Secondary
(c) Tertiary
(d) Quaternary
Answer: (a)
Solution: Chemically denaturation does not change the primary structure of proteins.

Question: Which one of the following is not present in photochemical smog?
Options:
(a) NO
(b) NO_2
(c) HCHO
(d) SO_2
Answer: (d)
Solution: SO_2 is not present in photochemical smog

Question: Statement-I: $KHSO_4$ dehydrates glycerol to form acrolein.
Statement-II: Acrolein has a fruity smell and can be used to detect glycerol.
Options:
(a) Both Statement I and Statement II are correct.
(b) Both Statement I and Statement II are incorrect.
(c) Statement I is correct but Statement II is incorrect.
(d) Statement I is incorrect but Statement II is correct.
Answer: (c)
Solution: Acrolein is a colourless, yellow liquid with and pungent suffocating odour
Statemen-I is true, statement-II is false

Question: Monomer of melamine?
Options:
(a) Formaldehyde

(b) Caprolactum
(c) Styrene
(d) Acrylonitrile
Answer: (a)
Solution: Monomer of melamine is formaldehyde.

Question: The correct order of density of s-Block elements
Options:
(a) Li < Na < Rb < K < Cs
(b) Cs < Rb < K < Na < Li
(c) Cs < Rb < Na < K < Li
(d) Li < K < Na < Rb < Cs
Answer: (d)
Solution: On moving down in a group, density increase but the density of K is somewhat lesser than Na.
Thus, the order of density is Li < K < Na < Rb < Cs

Question: Drugs that inhibits natural functions
Options:
(a) Agonist
(b) Antagonist
(c) Allosteric
(d) Anti Histamines
Answer: (b)
Solution: Antagonist inhibits natural functions

Question: IUPAC name of element with atomic no 103 is
Options:
(a) Unnilunium
(b) Unnilbium
(c) Unnitirium
(d) Unnilqadium
Answer: (c)
Solution: Unnitirium

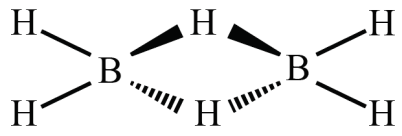
Question: Which absorbs energy of minimum wavelength?
Options:
(a) $[CO(NH_3)_6]^{+3}$
(b) $[Co(CN)_6]^{3-}$
(c) $[Co(NH_3)_5(H_2O)]^{3+}$
(d) $[Co(CN)_6]^{3-}$ and $[Co(NH_3)_5(H_2O)]^{3+}$
Answer: (b)
Solution: $\text{Wavelength} \propto \frac{1}{\text{stability of complex}}$
In all the complexes Co is present in +3 state and among the given ligands CN^- is the strongest ligand.
Hence, $[Co(CN)_6]^{3-}$ is most stable, Thus it absorbs light of minimum wavelength.

Question: What is the name of element having electronic configuration $[Rn]5f^{14}7s^27p^1$
Options:

(a) No
(b) Rf
(c) Lr
(d) Lu
Answer: (c)
Solution: Lawrenium (Lr) has electronic configuration $[Rn]5f^{14}7s^{2}7p^{1}$

Question: Which of the following sets of quantum numbers not possible?
Options:
(a) $n = 1,\ l = 0, m_l = 0, m_s = -\frac{1}{2}$
(b) $n = 1,\ l = 1, m_l = 0, m_s = +\frac{1}{2}$
(c) $n = 2,\ l = 1, m_l = 0, m_s = +\frac{1}{2}$
(d) $n = 3,\ l = 1, m_l = 0, m_s = +\frac{1}{2}$
Answer: (b)
Solution: $n = 1,\ l = 1, m_l = 0, m_s = +\frac{1}{2}$ is the wrong set of quantum numbers as maximum l value should be equal to n–1 and it cannot be equal to n

Question: Geometry around borane in the product
$BF_3 + NaH \rightarrow Product + NaF$
Options:
(a) Trigonal planar
(b) Tetrahedral
(c) Pyramidal
(d) Planar
Answer: (b)
Solution: $BF_3 + NaH \rightarrow B_2H_6 + NaF$



(Tetrahedral)

Question: The pK_a value of weak acid HA is 4.8 and pK_b of weak base BOH is 4.78. The pH of corresponding salt BA will be
Options:
(a) 8.58
(b) 7.01
(c) 4.79
(d) 9.22
Answer: (b)
Solution: $pH = 7 + \left(\frac{pK_a - pK_b}{2}\right) = 7 + \left(\frac{0.02}{2}\right) = 7.01$

Question: Which of the following comes out as slag in metallurgy of copper?
Options:
(a) FeO
(b) Al_2O_3
(c) ZnO
(d) NiO
Answer: (a)
Solution: $FeO + SiO_2 \rightarrow \underset{(Slag)}{FeSiO_3}$

Question: Total number of diamagnetic species is/are
N_2, N_2^+, N_2^-, N_2^{2-}, O_2, O_2^+, O_2^-, O_2^{2-}
Answer: 2.00
Solution: N_2 and O_2^{2-} are diamagnetic species

Question: How many compounds have same degree of unsaturation?
A) $CH_3–CH=CH–C\equiv N$
B) $CH_3–N=CH–C\equiv CH$
C) $CH\equiv C–NH–CH=CH_2$
D) $CH_2=CH–CH_2–C\equiv N$
Answer: 4.00
Solution: Degree of unsaturation = Number of π bonds + Number of rings
$CH_3CH=CH–C\equiv N = 2 + 1 = 3$
$CH_3–N=CH–C\equiv CH = 2 + 1 = 3$
$CH\equiv C–NH–CH=CH_2 = 2 + 1 = 3$
$CH_2=CH–CH_2–C\equiv N = 2 + 1 = 3$
All the compounds have 3 as degree of unsaturation

JEE-Main-25-07-2022-Shift-1 (Memory Based)

MATHEMATICS

Question: $x^4+x^3+x^2+x+1=0$ equation has 4 roots $\alpha\beta\gamma\delta$. Then $\alpha^{2021}+\beta^{2021}+\gamma^{2021}+\delta^{2021}=?$

Options:

(a) 4

(b) 1

(c) -1

(d) 4

Answer: (c)

Solution:

Given, $x^4+x^3+x^2+x+1=0$

$\Rightarrow x^5+x^4+x^3+x^2+x=0$

$\Rightarrow x^5-1=0$

$\Rightarrow x^5=1$

$\alpha^{2021}+\beta^{2021}+\gamma^{2021}+\delta^{2021}$

$=\alpha\left(\alpha^5\right)^{400}+\beta\left(\beta^5\right)^{400}+\gamma\left(\gamma^5\right)^{400}+\delta\left(\delta^5\right)^{400}$

$=\alpha+\beta+\gamma+\delta$

$=-1$

Question: A die is rolled twice and let the outcomes be α,β. Then probability such that $x^2+\alpha x+\beta>0$

Options:

(a) $\frac{17}{36}$

(b) $\frac{9}{36}$

(c)

(d)

Answer: (a)

Solution:

$x^2+\alpha x+\beta>0$

$\Rightarrow \alpha^2-4\beta<0$

$\Rightarrow \alpha=1,\ \beta=1,2,3,4,5,6$

$\Rightarrow \alpha=2,\ \beta=2,3,4,5,6$

$\Rightarrow \alpha=3,\ \beta=3,4,5,6$

$\Rightarrow \alpha=4,\ \beta=5,6$

$\Rightarrow \alpha=5,$ not possible

Favourable outcomes $=6+5+4+2=17$

$\therefore\ p=\dfrac{17}{36}$

Question: $\lim\limits_{n\to\infty}\sqrt{n^2-n+1}+n\alpha+\beta=0$ then $8(\alpha+\beta)$

Options:

(a) -8

(b) -4

(c)

(d)

Answer: (b)

Solution:

$\lim\limits_{n\to\infty}\sqrt{n^2-n+1}+n\alpha+\beta=0$

$\lim\limits_{n\to\infty}\left(\dfrac{\sqrt{n^2-n+1}+n\alpha+\beta}{\sqrt{n^2-n+1}-(n\alpha+\beta)}\right)\left(\sqrt{n^2-n+1}-(n\alpha+\beta)\right)=0$

$\Rightarrow\lim\limits_{n\to\infty}\dfrac{n^2-n+1-(n\alpha+\beta)^2}{\sqrt{n^2-n+1}-(n\alpha+\beta)}=0$

$\Rightarrow\lim\limits_{n\to\infty}\dfrac{n^2(1-\alpha^2)+n(-1-2\alpha\beta)+1-\beta^2}{\sqrt{n^2-n+1}-(n\alpha+\beta)}=0$

$\Rightarrow 1-\alpha^2=0$

$\Rightarrow\alpha=\pm1$

$-1-2\alpha\beta=0$

$\beta=\dfrac{-1}{2\alpha}$

$\alpha=1,\beta=\dfrac{-1}{2}$

$\alpha=-1,\ \beta=\dfrac{1}{2}$

$8(\alpha+\beta)=\pm4$

Question: The number of solution of $|\cos x|=\sin x$ such that $-4\pi\le x\le 4\pi$ is

Options:

(a) 4

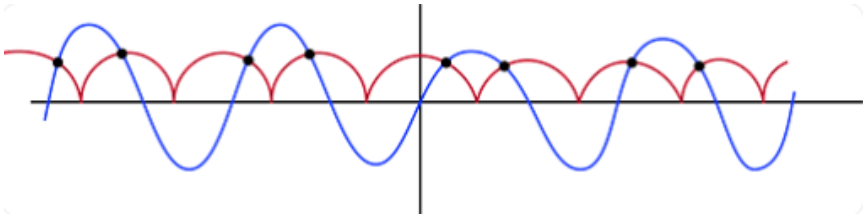
(b) 6

(c) 8

(d) 12

Answer: (c)

Solution:



Number of solutions = 8

Question: Which of the following is a tautology?
Options:
(a) $(\sim p \vee q) \Rightarrow p$
(b) $p \Rightarrow (\sim p \vee q)$
(c) $(\sim p \vee q) \Rightarrow q$
(d) $q \Rightarrow (\sim p \vee q)$
Answer: (d)
Solution:
$q \Rightarrow (\sim p \vee q)$
$\sim q \vee (\sim p \vee q)$
$(\sim q \vee \sim p) \vee (\sim q \vee q)$
$(\sim q \vee \sim p) \vee (T)$
T

Question: Set $A = \{1,2,3,4\}$ and $B = \{1,2,3,4,5,6\}$. Find the number of functions from A to B such that $f(1) + f(2) = f(3)$
Answer: 40.00
Solution:
$A = \{1,2,3,4\}$, $B = \{1,2,3,4,5\}$
$f(1) + f(2) = f(3)$
$1+2=3 \rightarrow 2 \times 5 = 10$
$1+3=4 \rightarrow 2 \times 5 = 10$
$1+4=5 \rightarrow 2 \times 5 = 10$
$2+3=5 \rightarrow 2 \times 5 = 10$
Number of functions $= 10+10+10+10 = 40$

Question: If roots of $x^2 - 8ax + 2a = 0$ are p & r, while q & s are roots of $x^2 + 12bx + 6b = 0$ then find $\frac{1}{a} - \frac{1}{b}$ if $\frac{1}{p}, \frac{1}{q}, \frac{1}{r}$ & $\frac{1}{s}$ are in AP.
Answer:
Solution:
$p + r = 8a,\ pr = 2a$
$q + s = -12b,\ qs = 6b$

$\Rightarrow \frac{1}{p}+\frac{1}{r}=4, \frac{1}{q}+\frac{1}{s}=-2$

$\Rightarrow \frac{1}{p}-\frac{1}{q}+\frac{1}{r}-\frac{1}{s}=6$

$\Rightarrow -2d=6$

$\Rightarrow d=-3$

$\frac{1}{p}+\frac{1}{p}-3=4$

$\Rightarrow \frac{2}{p}=7$

$\Rightarrow \frac{1}{p}=\frac{2}{7}, \frac{1}{q}=\frac{-19}{7}, \frac{1}{r}=\frac{-40}{7}, \frac{1}{s}=\frac{-61}{7}$

$\frac{1}{a}=\frac{2}{pr}=2\cdot\frac{2}{7}\cdot\left(\frac{-40}{7}\right)=\frac{-80}{49}$

$\frac{1}{b}=\frac{6}{qs}=6\cdot\left(\frac{-19}{7}\right)\left(\frac{-61}{7}\right)=\frac{6954}{49}$

$\frac{1}{a}-\frac{1}{b}=\frac{-80}{49}-\frac{6954}{49}$

$=\frac{-7034}{49}$

Question: If $a_1=b_1=1, a_n=a_{n-1}+2$ & $b_n=a_n+b_{n-1}$, then find $\sum_{n=1}^{15} a_n\times b_n$

Answer: 27560.00

Solution:

$a_1=b_1=1,\ a_n=a_{n-1}+2$

$a_r=1+(r-1)2=2r-1$

$b_r=a_r+b_{r-1}=a_r+a_{r-1}+b_{r-2}$

$=a_r+a_{r-1}+a_{r-2}+....+a_2+b_1$

$=\sum(2r-1)$

$=\frac{2(r)(r+1)}{2}-r=r^2$

$\sum a_n b_n=\sum(2n-1)n^2$

$=\sum 2n^3-\sum n^2$

$=2\left(\frac{15\times16}{2}\right)^2-\frac{15\times16\times31}{6}$

$=27560$

Question: A line with slope greater than 1, passes through $A(4,3)$. Line $x-y=2$ intersects former line at B. Find B if $AB=\frac{\sqrt{29}}{3}$.

Answer: $\frac{17}{3},\frac{11}{3}$

Solution:

$$\frac{x-4}{\cos\theta}=\frac{y-3}{\sin\theta}=r$$

$$x=\frac{\sqrt{29}}{3}\cos\theta+4,\ y=\frac{\sqrt{29}}{3}\sin\theta+3$$

$$x-y=2$$

$$\Rightarrow\frac{\sqrt{29}}{3}\cos\theta+4-\frac{\sqrt{29}}{3}\sin\theta-3=2$$

$$\Rightarrow\frac{\sqrt{29}}{3}(\cos\theta-\sin\theta)=1$$

$$\Rightarrow\cos\theta-\sin\theta=\frac{3}{\sqrt{29}}$$

$$\Rightarrow1-\sin2\theta=\frac{9}{29}$$

$$\Rightarrow\sin2\theta=\frac{20}{29}$$

$$\Rightarrow\sin\theta+\cos\theta=\sqrt{1+\left(\frac{20}{29}\right)}$$

$$\Rightarrow\sin\theta+\cos\theta=\frac{7}{\sqrt{29}}$$

$$\sin\theta=\frac{2}{\sqrt{29}},\cos\theta=\frac{5}{\sqrt{29}}$$

$$x=\frac{\sqrt{29}}{3}\times\left(\frac{5}{\sqrt{29}}\right)+4=\frac{5}{3}+4=\frac{17}{3}$$

$$y=\frac{\sqrt{29}}{3}\times\frac{2}{\sqrt{29}}+3=\frac{2}{3}+3=\frac{11}{3}$$

$$B\equiv\left(\frac{17}{3},\frac{11}{3}\right)$$

Question: Find remainder when $(2024)^{2024}$ is divided by 7.

Answer: 1.00

Solution:

$$(2024)^{2024}$$

$$=(289\times+1)^{2024}$$

$= {}^{2024}C_0 + {}^{2024}C_1(289\times 7) + ...$

Remainder = 1

Question: $z = 1+\cos\frac{6\pi}{5}+i\sin\frac{6\pi}{5}$, find arg.

Answer: $\frac{3\pi}{5}$

Solution:

$z = 1+\cos\frac{6\pi}{5}+i\sin\frac{6\pi}{5}$

$= 2\cos^2\frac{6\pi}{10}+2i\sin\frac{6\pi}{10}\cos\frac{6\pi}{10}$

$= 2\cos\frac{3\pi}{5}\left(\cos\frac{3\pi}{5}+i\sin\frac{3\pi}{5}\right)$

$\arg(z) = \frac{3\pi}{5}$

Question: Sum & product of mean & variance of Binomial distribution are 24 & 128 respectively. Find probability of 1 or 2 successes.

Answer:

Solution:

Mean and variance are roots of $x^2 - 24x + 128 = 0$

$\Rightarrow$ Mean $= np = 16$

Variance $= npq = 8$

$\Rightarrow p = q = \frac{1}{2},\ n = 32$

$p(x=1)+p(x=2) = {}^{32}C_1\left(\frac{1}{2}\right)^{32} + {}^{32}C_2\left(\frac{1}{2}\right)^{32}$

$= \left({}^{32}C_1 + {}^{32}C_2\right)\left(\frac{1}{2}\right)^{32}$

$= {}^{33}C_2\left(\frac{1}{2}\right)^{32}$

Question: Find dictionary rank of MANKIND.

Answer: 1492.00

Solution:

M A N K I N D

$\frac{4\cdot 6!}{2!}+0+\frac{3(4!)}{2!}+2(3!)+2!+1+1$

$= 1440+36+12+2+1+1$

$= 1492$

Question: If the locus of centre (α,β) of circle that touches both $x^2+(y-1)^2=1$ & x-axis, then find area enclosed by locus & $y=4$

Answer: $\frac{64}{3}$

Solution:

$(x-\alpha)^2+(y-\beta)^2=\beta^2$

$|\beta+1|=\sqrt{(\alpha)^2+(\beta-1)^2}$

$\Rightarrow \alpha^2=4\beta$

$\Rightarrow x^2=4y$

$\Rightarrow y=\frac{x^2}{4}$

$\text{Area}=2\int_0^4\left(4-\frac{x^2}{4}\right)=\frac{64}{3}$

Question: $p(x)=x^2+ax^2+bx+c$, $y=p(x)$ touches x-axis at $(-2,0)$ & $p'(0)=3$. Find local maxima.

Answer: 0.00

Solution:

$p(x)=x^3+ax^2+bx+c$

$p(-2)=0$

$0=-8+4a-2b+c$

$p'(-2)=0$

$\Rightarrow 12-4a+b=0$

$p'(0)=3$

$\Rightarrow c=3$

$\Rightarrow b=7$

$\Rightarrow a=\frac{19}{4}$

$p(x)=x^3+\frac{19}{4}x^2+7x+3$

$p'(x)=3x^2+\frac{19}{2}x+7$

$x=\frac{\frac{-19}{2}\pm\sqrt{\left(\frac{19}{2}\right)^2-4\cdot7\cdot3}}{6}$

$=\frac{\frac{-19}{2}\pm\frac{5}{2}}{6}=-2,\frac{-7}{6}$

$p''(x) = 6x + \frac{19}{2}$

$p''(-2) = -12 + \frac{19}{2} < 0$

$p''\left(\frac{-7}{6}\right) > 0$

Maxima = $p(-2) = 0$