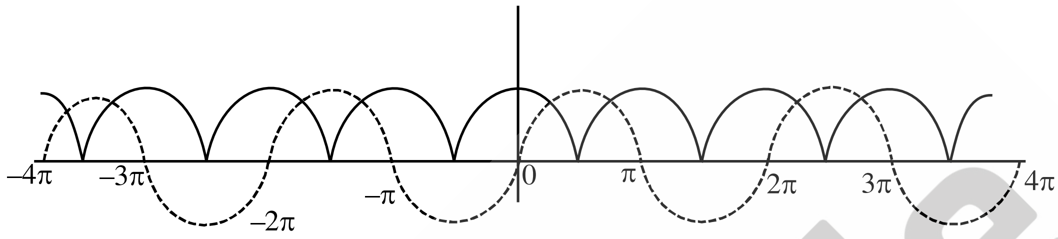
MATHEMATICS

1. If $|\cos x| = \sin x$, then number of solution for $x \in [-4\pi, 4\pi]$

(1) 4 (2) 6 (3) 8 (3) 10

Ans. **(3)**

Sol.



8 solutions

2. If all the words are formed by using all the letters of the word "MANKIND", find the rank of MANKIND

Ans. **(1492)**

Sol. A,D,I,K,M,N,N

$A \to \frac{|\underline{6}}{|\underline{2}} = 360$

$D \to \frac{|\underline{6}}{|\underline{2}} = 360$

$I \to \frac{|\underline{6}}{|\underline{2}} = 360$

$K \to \frac{|\underline{6}}{|\underline{2}} = 360$

$MAD \to \frac{|\underline{4}}{|\underline{2}} = 12$

$MAI \to \frac{|\underline{4}}{|\underline{2}} = 12$

$MAK \to \frac{|\underline{4}}{|\underline{2}} = 12$

$MAND \to |\underline{3} = 6$

$MANI \to |\underline{3} = 6$

$MANKD \to |\underline{2} = 2$

MANKIDN

MANKIND

Rank = $360 \times 4 + 12 \times 3 + 6 \times 2 + 2 \times 1 + 2$

$= 1440 + 36 + 12 + 4$

$= 1492$

3. If f(x) is defind from {1, 2, 3, 4} $\rightarrow$ {1, 2, 3, 4, 5, 6} then find the number of functions such that :

f(1) + f(2) = f(3)

Ans. **(90)**

Sol. x + y = z   x, y, z $\in$ {1, 2, 3, 4, 5, 6}

(x, y) $\equiv$ (1, 1) (1, 2) [1, 3) (1, 4) (1, 5) $\rightarrow$ 5 possibilities

(2, 1) (2, 2) (2, 3) (2, 4) $\rightarrow$ 4 possibilities

(3, 1) (3, 2) (3, 3) $\rightarrow$ 3 possibilities

(4, 1) (4, 2) $\rightarrow$ 2 possibilities

(5, 1) $\rightarrow$ 1 possibility

Total ways = $\frac{5\times 6}{2} \Rightarrow 15\text{ ways}$

Hence total mappings

No. of ways $\underbrace{15}_{\substack{\text{No.of}\\ \text{ways}\\ \text{ofslectic}\\ f(1),f(2)f(3)}} \times \underbrace{6}_{\substack{\text{No.of}\\ \text{ways}\\ \text{ofslectic}\\ f(4)}}$

= 90 ways

4. $\lim\limits_{N\to\infty} (\sqrt{N^2 - N - 1} + N\alpha + \beta) = 0$, then find $8|\alpha + \beta|$

Ans. **(4)**

Sol. Rationalize

$$\frac{N^2(1-\alpha^2) - N(1+2\alpha\beta) - 1 - \beta^2}{\sqrt{N^2 - N - 1} - (N\alpha + \beta)}$$

$1 - \alpha^2 = 0 \Rightarrow \alpha = \pm 1$

$1 + 2\alpha\beta = 0 \Rightarrow \beta = -\frac{1}{2\alpha} = \pm\frac{1}{2}$

$(\alpha, \beta) \equiv \left(1, -\frac{1}{2}\right)$ or $\left(-1, \frac{1}{2}\right) \Rightarrow 8|\alpha + \beta| = 4$

5. If $x^4 + x^3 + x^2 + x + 1 = 0$ has the roots α, β, γ and δ. Find the value of $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021}$

Ans. **(–1)**

Sol. $1 + x + x^2 + x^3 + x^4 = 0$

$\frac{x^5 - 1}{x - 1} = 0 \Rightarrow \mathbf{x^5} = 1$ (fifth root of unity)

So $1^p + \alpha_1^p + \alpha_2^p + \alpha_3^p + \alpha_4^p = 0$ if $p \neq 5\lambda$, $\lambda \in N$

Hence $\alpha^{2021} + \beta^{2021} + \gamma^{2021} + \delta^{2021} = -1$

6. $a_1 = b_1 = 1$

$a_N = a_{N-1} + 2 \quad \forall\ n \geq 2$

$b_N = a_N + b_{N-1}$

Find $S = \sum_{N=1}^{15} a_N b_N$

Ans. **(27560)**

Sol. a_N is in A P $\Rightarrow \alpha_N = 1 + (N - 1)\,2 = 2N - 1$

$B_N = 2N-1 + b_{N-1} = 2N-1 + (2N-3) + b_{N-2} = \ldots.$

$= (2N-1) + (2N - 3) + \ldots.. + 3 + b_1$

$b_N = N^2 - 1 + b_1 \Rightarrow b_N = N^2$

So $a_N = 2N-1$ & $b_N = N^2$

$S = \sum_{N=1}^{15} (2N-1) N^2$

$= 2\sum N^3 - \sum N^2$

$= 28800 - 1240$

$= 27560$

7. Which of the following is tautology.

(1) $\sim p \vee q \rightarrow p$ (2) $p \rightarrow \sim p \vee q$

(3) $q \rightarrow \sim p \vee q$ (4) $\sim p \vee q \rightarrow q$

Ans. **(3)**

Sol.

p	q	~p	$\sim p \vee q$	$\sim p \vee q \rightarrow p$	$p \rightarrow \sim p \vee q$	$q \rightarrow \sim p \vee q$	$\sim p \vee q \rightarrow q$
T	T	F	T	T	T	T	T
T	F	F	F	T	F	T	F
T	T	T	T	F	T	T	T
F	F	T	T	F	T	T	F

8. Remainder, when $(2024)^{2024}$ is divided by 7, is-

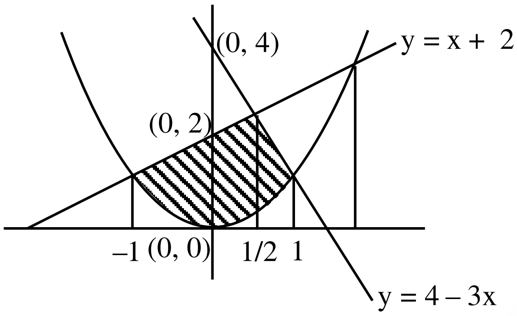
Ans. **1**

Sol. $(2023 + 1)^{2024} = 7\lambda + 1$

9. Find the area bounded by the region $A = \{(x, y) : x^2 \leq y \leq \min (x + 2, 4 - 3x)\}$

Ans. $\frac{17}{6}$

Sol.



$y = x^2 = x + 2 \Rightarrow x^2 - x - 2 = 0 \Rightarrow x = -1$

$y = x^2 = 4 - 3x \Rightarrow x^2 + 3x - 4 = 0 \Rightarrow x = 1$

$$\text{Area} = \int_{-1}^{\frac{1}{2}} ((x+2) - x^2)\,dx + \int_{\frac{1}{2}}^{1} (4 - 3x - x^2)\,dx$$

$$= \left(\frac{x^2}{2}\right)_{-1}^{\frac{1}{2}} + 2\left(\frac{3}{2}\right) - \frac{1}{3}\left(x^3\right)_{-1}^{\frac{1}{2}} + 4\left(\frac{1}{2}\right) - \frac{3}{2}\left(x^2\right)_{\frac{1}{2}}^{1} - \frac{1}{3}\left(x^3\right)_{\frac{1}{2}}^{1}$$

$$= \frac{1}{2}\left(\frac{1}{4} - 1\right) + 3 - \frac{1}{3}\left(\frac{1}{8} + 1\right) + 2 - \frac{3}{2}\left(1 - \frac{1}{4}\right) - \frac{1}{3}\left(1 - \frac{1}{8}\right)$$

$$= -\frac{3}{8} + 3 - \frac{3}{8} + 2 - \frac{9}{8} - \frac{7}{24}$$

$$= -\frac{3}{4} + 5 - \frac{9}{8} - \frac{7}{24} = 5 - \frac{(18 + 27 + 7)}{24} = 5 - \frac{52}{24} = \frac{68}{24} = \frac{17}{6}$$

10. Sum and product of mean and variance of binomial distribution are 24 and 128 respectively. Probability of 1 or 2 successes is-

(1) $\dfrac{\left({}^{32}C_1 + {}^{32}C_2\right)}{2^{32}}$

(2) $\dfrac{\left({}^{32}C_0 + {}^{32}C_2\right)}{2^{32}}$

(3) $\dfrac{\left({}^{16}C_1 + {}^{16}C_2\right)}{2^{16}}$

(4) $\dfrac{\left({}^{20}C_1 + {}^{20}C_2\right)}{2^{20}}$

Ans. **(1)**

Sol. Mean = np = a

Var = npq = np(1–p) = b

Here, a + b = 24, ab = 128

$\Rightarrow$ a = 16 and b = 8

$\therefore$ n = 32, p = q = $\frac{1}{2}$

P(1or 2 successes) = $\dfrac{\left({}^{32}C_1 + {}^{32}C_2\right)}{2^{32}}$

11. If $z = 1 + \cos\frac{6\pi}{5} + i\sin\frac{6\pi}{5}$, then argument of z is–

(1) $-\frac{3\pi}{5}$ (2) $-\frac{2\pi}{5}$ (3) $\frac{3\pi}{5}$ (4) $\frac{2\pi}{5}$

Ans. **(2)**

Sol. $z = 2\cos^2\frac{3\pi}{5} + i\left(2\sin\frac{3\pi}{5}\cos\frac{3\pi}{5}\right)$

$z = -2\cos\frac{3\pi}{5}\left(-\cos\frac{3\pi}{5} - i\sin\frac{3\pi}{5}\right)$

$= r\ \text{cis}\left(-\frac{2\pi}{5}\right)$

12. If the locus of center (α, β) of circle which touches the circle $x^2 + (y - 1)^2 = 1$ externally and also touches the x-axis is $f(\alpha, \beta) = 0$ then the area bounded by the locus $f(\alpha, \beta) = 0$ and $x = 1$ is (Assume $\beta > 0$)

(1) $\frac{1}{36}$ (2) $\frac{1}{12}$ (3) $\frac{1}{18}$ (4) $\frac{1}{4}$

Ans. **(B)**

Sol. Let center of circle is $C_1(\alpha, \beta)$

$\because$ It touches the x-axis $\Rightarrow$ radius $(r_1) = |\beta|$

$C_1C_2 = r_1 + r_2$ where $C_2 = (0, 1)$ and $r_2 = 1$

$\Rightarrow \alpha^2 + (\beta - 1)^2 = (|\beta| + 1)^2$

$\Rightarrow \alpha^2 = 4\beta \Rightarrow x^2 = 4y\ (\because \beta > 0)$

Area $= \int_0^1 \frac{x^2}{4}dx = \frac{1}{12}$

13. Find the probability that the expression $x^2 + \alpha x + \beta > 0\ \forall\ x \in R$ where α & β are obtained by throwing a pair of dice

Ans. $\frac{17}{36}$

Sol. $D < 0 \Rightarrow \alpha^2 < 4\beta$

α	β
1	1 – 6
2	2 – 6
3	3 – 6
4	5 – 6
5	0
6	0

$\therefore$ Probability $= \frac{17}{36}$

14. Let x * y = $x^2 + y^3$ and (x * 1) * 1 = x * (1 * 1). Then a value of $2\sin^{-1}\left(\frac{x^4+x^2-2}{x^4+x^2+2}\right)$ is

Ans. $\frac{\pi}{3}$

Sol. $x * y = x^2 + y^3$

$(x * 1) = x^2 + 1$

$(x * 1) * 1 = (x^2 + 1)^2 + 1$

$x * (1 * 1) = x^2 + 2^3$

$x^4 + 2x^2 + 2 = x^2 + 8$

$\Rightarrow x^2 = 2$

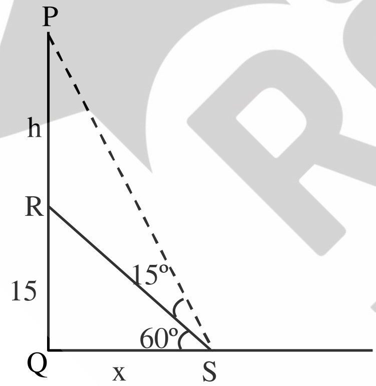
$\therefore \quad 2\sin^{-1}\left(\frac{x^4+x^2-2}{x^4+x^2+2}\right) = 2\sin^{-1}\left(\frac{4+2-2}{4+2+2}\right) = \frac{\pi}{3}$

15. Let Q be the foot of the tower PQ and R cuts the tower such that RQ = 15cm. Let S be any point on gound such that angle of elevation of R from S is 60º and PR makes an angle 15º on S. Then, height of tower is

(1) $10(\sqrt{3}+1)$
(2) $10(\sqrt{3}-1)$
(3) $5\sqrt{3}\,(2+\sqrt{3})$
(4) $5\sqrt{3}\,(2-\sqrt{3})$

Ans. **(3)**

Sol.



$\tan 60° = \frac{15}{x}$

$\Rightarrow x = \frac{15}{\sqrt{3}}$

$\tan 75° = \frac{h+15}{x}$

$\Rightarrow h + 15 = \frac{15}{\sqrt{3}}\left(2+\sqrt{3}\right)$

16. The general solution of $(x - y^2)dx + y(5x + y^2)dy = 0$ is

(1) $(y^2 + x)^4 = C(y + 2x)^3$
(2) $(y^2 + x)^4 = C(y^2 + 2x)^3$
(3) $(y^2 + 2x)^4 = C(y^2 + x)^3$
(4) $(y^2 - x)^3 = C(y^2 + x)^3$

Ans. **(2)**

Sol. Let $y^2 = t \Rightarrow 2ydy = dt$

$$xdx - tdx + \frac{5xdt}{2} + \frac{t.dt}{2} = 0$$

$$2(x - t)dx + (5x + t)dt = 0$$

$$\Rightarrow \frac{dt}{dx} = \frac{-2(x-t)}{5x+t}$$

Let $t = mx \Rightarrow \frac{dt}{dx} = m + x\frac{dm}{dx}$

$$m + x\frac{dm}{dx} = \frac{2(mx - x)}{5x + mx} = \frac{2m-2}{5+m}$$

$$x\frac{dm}{dx} = \frac{2m-2}{m+5} - m = \frac{-m^2 - 3m - 2}{m+5}$$

$$\int\frac{(m+5)dm}{m^2+3m+2} = -\int\frac{dx}{x} \Rightarrow \frac{(m+5)dm}{(m+1)(m+2)} = -\ell nx + \ell nc$$

$$\frac{m+5}{(m+1)(m+2)} = \frac{A}{m+1} + \frac{B}{m+2}$$

$m + 5 = A(m + 2) + B(m + 1) \Rightarrow A + B = 1, 2A + B = 5, A = 4, B = -3$

$$\int\left(\frac{4}{m+1} - \frac{3}{m+2}\right)dm = -\ell nx + \ell nc$$

on simplifying, we get $(y^2 + x)^4 = C(y^2 + 2x)^3$