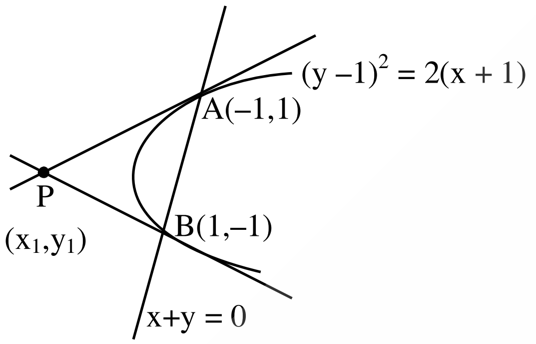
MATHEMATICS

1. A (–1,1) and B(1,–1) are two points on the curve $y^2 = 2x + 2y + 1$. Tangents drawn at points A and B meets at point P, then area of ΔPAB is -

(1) 2 (2) 1 (3) $\frac{1}{2}$ (4) 3

Ans. **(2)**

Sol.



Equation of COC : T = O

$yy_1 = x + x_1 + y + y_1 + 1$

$\Rightarrow x + (1 - y_1)\, y + (x_1 + y_1 + 1) = 0$

It is identical to the line $x + y = 0$

$\therefore\ 1 - y_1 = 1\ \&\ x_1 + y_1 + 1 = 0$

$\Rightarrow P\ (x_1\ y_1) \equiv (-1,0)$

$$\therefore \text{ Area of } \Delta PAB = \frac{1}{2}\left|\begin{vmatrix} 1 & 1 & 1 \\ -1 & -1 & 1 \\ 0 & 1 & -1 \end{vmatrix}\right| = 1 \text{ sq. units}$$

2. $2\sin\frac{\pi}{22}\ \sin\frac{3\pi}{22}\ \sin\frac{5\pi}{22}\ \sin\frac{7\pi}{22}\ \sin\frac{9\pi}{22}$ is equal to -

(1) $\frac{1}{8}$ (2) $\frac{1}{16}$ (3) $\frac{1}{32}$ (4) $\frac{1}{12}$

Ans. **(2)**

Sol. $\frac{\pi}{22} = \theta \Rightarrow \frac{\pi}{2} = 11\theta,\ 7\theta = \frac{\pi}{2} - 4\theta,\ 9\theta = \frac{\pi}{2} - 2\theta$

$3\theta = \frac{\pi}{2} - 8\theta$

$$y = 2\sin\theta\sin3\theta\sin5\theta\sin7\theta\sin9\theta = 2\sin\theta\sin\left(\frac{\pi}{2} - 8\theta\right).\sin5\theta.\sin\left(\frac{\pi}{2} - 4\theta\right)\sin\left(\frac{\pi}{2} - 2\theta\right)$$

$$y = 2\sin\theta\sin5\theta(\cos2\theta\cos4\theta\cos8\theta) = 2\sin\theta\sin5\theta\frac{\sin(16\theta)}{8.\sin(2\theta)} \text{ where } 16\theta = \frac{\pi}{2} + 5\theta$$

$$= \frac{2\sin\theta\sin5\theta\sin\left(\frac{\pi}{2} + 5\theta\right)}{8\sin2\theta} = \frac{2\sin\theta\sin5\theta\cos5\theta}{8\sin2\theta} = \frac{\sin10\theta.\sin\theta}{8\sin2\theta},\ 10\theta = \left(\frac{\pi}{2} - \theta\right)$$

$$= \frac{\sin\left(\frac{\pi}{2} - \theta\right)\sin\theta}{16\sin\theta\cos\theta} = \frac{1}{16}$$

3. Let A = {1,3,5,7,9,…..,99} & B = {2,4,6,…..,100} are two sets. A function y = f(x) defined from set A to set B such that $f(3) \geq f(9) \geq f(15) \geq f(21) \geq \ldots\ldots \geq f(99)$, then number of such bijective functions is -

(1) ${}^{50}C_{17} \times 33!$ (2) ${}^{50}C_{18} \times 32!$ (3) ${}^{50}C_{14} \times 36!$ (4) ${}^{50}C_{12} \times 38!$

Ans. **(1)**

Sol. 3,9,15,…..99 → total 17 elements

Select any 17 elements out of 50 elements from set B and they can be arranged in only one way. Rest 33 elements can be arranged in 33! ways.

Number of bijection function = ${}^{50}C_{17} \times 33!$

4. $\lim\limits_{x\to\frac{\pi}{4}} \dfrac{8\sqrt{2}-(\cos x+\sin x)^7}{\left(\sqrt{2}-\sqrt{2}\sin 2x\right)}$ equal to -

(1) 12 (2) –14 (3) 14 (4) –12

Ans. **(3)**

Sol. $\Rightarrow \lim\limits_{x\to\frac{\pi}{4}} \dfrac{-7(\cos x+\sin x)^6(-\sin x+\cos x)}{-2\sqrt{2}\cos 2x}$

$\Rightarrow \lim\limits_{x\to\frac{\pi}{4}} \dfrac{-7(\sqrt{2})^6(-\sin x+\cos x)}{-2\sqrt{2}\cos 2x}$

$\Rightarrow \lim\limits_{x\to\frac{\pi}{4}} \dfrac{14\sqrt{2}(-\cos x-\sin x)}{-2\sin 2x} = 14$

5. $11^{1011} + (1011)^{11}$ is divided by 9, then remainder is -

(1) 6 (2) 7 (3) 8 (4) 5

Ans. **(3)**

Sol. $(9 + 2)^{1011} + (1008 + 3)^{11}$

$= 9k + 2^{1011} + 9P + 3^{11}$

$= 9k_1 + 8^{337} + 3 \times 9^5$

$= 9k_1 + (9 - 1)^{337} + 3 \times 9^5$

$\therefore$ Remainder = – 1 + 9

= 8

6. If $P(A) = \dfrac{1}{3}$, $P(B) = \dfrac{1}{5}$ and $P\left(A \cup B\right) = \dfrac{1}{2}$ then the value of $P\left(\dfrac{A}{\overline{B}}\right) + P\left(\dfrac{B}{\overline{A}}\right)$ is -

(1) $\dfrac{3}{8}$ (2) $\dfrac{5}{8}$ (3) $\dfrac{3}{4}$ (4) $\dfrac{1}{4}$

Ans. **(2)**

Sol. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow \frac{1}{2} = \frac{1}{3} + \frac{1}{5} - (A \cap B)$

$\Rightarrow P(A \cap B) = \frac{8}{15} - \frac{1}{2} = \frac{1}{30}$

We need $\frac{P(A \cap \bar{B})}{P(\bar{B})} + \frac{P(B \cap \bar{A})}{P(\bar{A})}$

$= \frac{P(A) - P(A \cap B)}{1 - P(B)} + \frac{P(B) - P(A \cap B)}{1 - P(\bar{A})}$

$= \frac{\frac{1}{3} - \frac{1}{30}}{\frac{4}{5}} + \frac{\frac{1}{5} - \frac{1}{30}}{\frac{2}{3}}$

$= \frac{9}{24} + \frac{5}{20} = \frac{15}{24} = \frac{5}{8}$

7. $\int_{-3}^{101} ([\sin \pi x] + e^{[\cos 2\pi x]})\, dx$, is- (where [x] denotes greatest integer function of x)

(1) $\frac{52}{e}$ (2) $\frac{26}{e^2}$ (3) $\frac{51}{e}$ (4) $\frac{25}{e^2}$

Ans. (1)

Sol. $\Rightarrow 52 \left(\int_0^2 [\sin \pi x] + e^{[\cos 2\pi x]} \right) dx$

$\Rightarrow 52 \left(\int_0^2 [\sin \pi x]\, dx + \int_0^2 e^{[\cos 2\pi x]}\, dx \right)$

$\Rightarrow 52 \left(\int_1^2 -1 \, . \, dx + 2\int_0^1 e^{[\cos 2\pi x]}\, dx \right)$

$\Rightarrow 52 \left(-1 + 2\left(\int_0^{1/4} 1 \, . \, dx + \int_{1/4}^{3/4} e^{-1} . \, dx + \int_{3/4}^{1} 1 \, . \, dx \right) \right) = \frac{52}{e}$

8. Consider the curve y = f(x) passing through the points (1,2) & (8,1). It the slope of tangent at any point P on the curve is proportional to $-\left(\frac{\text{ordinate of P}}{\text{abscissa of P}} \right)$, then value of $\left| y\left(\frac{1}{8} \right) \right|$ is -

(1) 2 (2) 4 (3) 6 (4) 5

Ans. **(2)**

Sol. Let $P \equiv (x_1, y_1)$

$$\left.\frac{dy}{dx}\right|_P = k\left(\frac{-y_1}{x_1}\right) \Rightarrow \int \frac{dy}{y} = \int -\frac{k}{x_1}dx$$

$\ell n|y| = -k\ \ell n|x| + c$

$\Rightarrow x^k y = \lambda$

Since it is satisfied by points (1,2) and (8,1)

$\Rightarrow \lambda = 2$ and $8^k = 2$ $\Rightarrow k = \frac{1}{3} \Rightarrow x^{\frac{1}{3}}y = 2$

$$y\left(\frac{1}{8}\right) = \frac{2}{\left(\frac{1}{2}\right)} = 4$$

9. $\lim\limits_{n\to\infty}\frac{1}{2^n}\left(\frac{1}{\sqrt{1-\frac{1}{2^n}}}+\frac{1}{\sqrt{1-\frac{3}{2^n}}}+\frac{1}{\sqrt{1-\frac{7}{2^n}}}+.....+\frac{1}{\sqrt{1-\frac{(2^n-1)}{2^n}}}\right)$ is -

(1) 1 (2) 2 (3) 0 (4) 3

Ans. **(3)**

Sol. $\because \lim\limits_{n\to\infty}\frac{1}{2^n}\left(\frac{1}{\sqrt{1-\frac{1}{2^n}}}+\frac{1}{\sqrt{1-\frac{1}{2^n}}}+......+\frac{1}{\sqrt{1-\frac{1}{2^n}}}\right)$

$$\lim_{n\to\infty}\frac{n}{2^n\sqrt{1-\frac{1}{2^n}}} = 0$$

also, $\lim\limits_{n\to\infty}\frac{1}{2^n}\left(\frac{1}{\sqrt{1-\frac{(2^n-1)}{2^n}}}+....+\frac{1}{\sqrt{1-\frac{(2^n-1)}{2^n}}}\right)$

$$= \lim_{n\to\infty}\frac{1}{2^n}\left(\frac{n}{\sqrt{1-\frac{(2^n-1)}{2^n}}}\right)$$

$$= \lim_{n\to\infty}\frac{n}{\sqrt{2^n}} = 0$$

Hence by sandwitch theorem

Required limit = 0

10. If P : Ram is intelligent

Q : Ram is Honest

R : Ram is Rich

Then the negation of the statement :

"Ram is intelligent & honest if and only if he is not rich", is -

(1) $(\sim P \vee \sim Q) \leftrightarrow \sim R$
(2) $(\sim P \wedge \sim Q) \leftrightarrow \sim R$

(3) $(P \vee \sim Q) \leftrightarrow R$
(4) $(P \vee Q) \leftrightarrow \sim R$

Ans. **(1)**

Sol. $S \equiv (P \wedge Q) \leftrightarrow \sim R$

$\sim S \equiv \sim((P \wedge Q) \leftrightarrow \sim R)$

$\equiv (\sim P \vee \sim Q) \leftrightarrow \sim R$

11. Let f(x) be a monic quadratic polynomial such that $f(1) = \frac{1}{3}$, $f(0) = p$. If $f(f(f(f(x)))) = 0$ and $f(x) = 0$ have a common root, then value of $f(-3)$ is-

(1) 24
(2) 23
(3) 25
(4) 22

Ans. **(3)**

Sol. Let $f(x) = x^2 + ax + p$

$f(1) = 1 + a + p = \frac{1}{3} \Rightarrow a + p = \frac{-2}{3}$

Let 'α' is the common root

$\therefore\ f(f(f(f(\alpha)))) = 0$

$\Rightarrow f(f(f(0))) = 0$

$\Rightarrow f(f(p)) = 0$

$\Rightarrow f\left(\frac{p}{3}\right) = 0$

$\Rightarrow \frac{p^2}{9} + \frac{p}{3}.\,a + p = 0$

$\Rightarrow p + 3a + 9 = 0$

$\Rightarrow p = \frac{7}{2}$ and $a = -\frac{25}{6}$

Now $f(-3) = 9 - 3a + p = 9 + \frac{25}{2} + \frac{7}{2} = 25$