

JEE-Main-26-07-2022-Shift-1 (Memory Based)

Physics

Question: A charged particle is having acceleration $\vec{a} = 2\hat{i} + 4\hat{j}$ while moving in an uniform magnetic field $\vec{B} = \alpha\hat{i} - 3\hat{j}$. Find α

Options:

- (a) 10
- (b) 2.5
- (c) 3
- (d) 4.5

Answer: (d)

Solution:

$$\because \vec{F} = q(\vec{v} \times \vec{B})$$

By property of cross product,

$$\vec{F} \perp \vec{B}$$

$$\vec{a} \perp \vec{B}$$

$$\therefore \vec{a} \cdot \vec{B} = 0$$

$$(2\hat{i} + 4\hat{j}) \cdot (\alpha\hat{i} - 3\hat{j}) = 0$$

Question: If in adiabatic process gas is compressed to $\frac{1}{8}$ th of its volume, what will be the final pressure if Initial pressure is P_0 (gas is monatomic)

Options:

- (a) $12 P_0$
- (b) $32 P_0$
- (c) $42 P_0$
- (d) $22 P_0$

Answer: (b)

Solution:

For monoatomic gas $r = \frac{5}{3}$

In adiabatic process

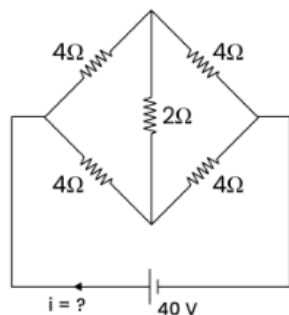
$$P_1 V_1^r = P_2 V_2^r$$

$$\therefore \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^r = \left(\frac{V_0}{V_0/8}\right)^{5/3}$$

$$= (8)^{5/3} = 32$$

$$\therefore P_2 = 32P_1 = 32P_0$$

Question: What is the current through battery?



Options:

- (a) 15 A
- (b) 20 A
- (c) 10 A
- (d) 15 A

Answer: (c)

Solution:

Since it's a balanced Wheatstone bridge equivalent resistance becomes 4Ω . Since 2Ω resistance will not be considered.

$$\therefore i = \frac{40}{4} = 10A$$

Question: A monkey of 50 kg climbs a rope having maximum tension 350 N

Case A: Monkey climbs up with 5 ms^{-2}

Case B: Monkey climbs down with 4 ms^{-2}

In which case Rope will not break

Options:

- (a) Case B: Break, Case A : break
- (b) Case A: Not Break, Case B : break
- (c) Case A: Break, Case B : Not break
- (d) Case B: Not Break, Case A : Not break

Answer: (c)

Solution:

Breaking tension = 350 N

$$\begin{aligned} \text{Case A } T_A &= m(g+a) = 50(10+5) \\ &= 750 \text{ N} \end{aligned}$$

$$\begin{aligned} \text{Case B } T_B &= m(g-a) = 50(10-4) \\ &= 300 \text{ N} \end{aligned}$$

\therefore Rope will break in case A

Question: A neutron and an electron of rest masses m_n and m_e are moving with speeds v and xv resp. If their de Broglie wavelength are equal then approximate value of x is consider $m_e = 9.1 \times 10^{-31} \text{ kg}$ and $m_n = 1.6 \times 10^{-27} \text{ kg}$. Do not consider relativistic effect

Options:

- (a) 1600
- (b) 1758
- (c) 1880
- (d) 1990

Answer: (b)

Solution:

Using de Broglie eq.

$$\lambda = \frac{h}{mv}$$

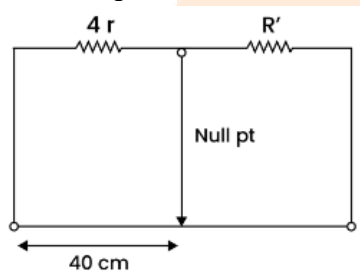
$$\therefore \lambda_e = \lambda_n$$

$$\frac{h}{m_e \times v} = \frac{h}{m_n v}$$

or

$$x = \frac{m_n}{m_e} = 1758$$

Question: A meter Bridge is as shown if a resistance of xr is connected in series with $4r$, new null point comes at 80 cm. Find x ?



Options:

- (a) $x = 30$
- (b) $x = 20$
- (c) $x = 10$
- (d) $x = 30$

Answer: (b)

Solution:

$$\frac{4r}{40} = \frac{R'}{60} \dots(1)$$

$$\& \frac{4r + xr}{80} = \frac{R'}{20} \dots(2)$$

Solving 1 and 2

$$\frac{4r + xr}{8r} = 3$$

$$\boxed{x = 20}$$

Question: If intensity of a wave is 10 W m^{-2} & it is passing through area of 1 cm^2 & wavelength of wave is 900 nm. Find No. of photons passing per second.

Options:

- (a) 6.51×10^{16} photos / sec
- (b) 8.51×10^{16} photos / sec
- (c) 3.51×10^{16} photos / sec
- (d) 4.51×10^{16} photos / sec

Answer: (d)

Solution:

G. that

$$I = 100 \text{ w/m}^2$$

$$A = 1 \text{ cm}^2 = 1 \times 10^{-4} \text{ m}^2$$

$$\lambda = 900 \text{ nm} = 900 \times 10^{-9} \text{ m}$$

$$\text{No. of photos} = \frac{IA d}{GC}$$

$$= \frac{100 \times 1 \times 10^{-4} \times 900 \times 10^{-9}}{6.64 \times 10^{-34} \times 3 \times 10^2}$$

$$= 4.51 \times 10^{16} \text{ photos / sec}$$

Question: If in YDSE fringe width is 12 mm. What is new fringe width if whole setup is immersed in water $\mu = \frac{4}{3}$

Options:

- (a) 3 mm
- (b) 9 mm
- (c) 4 mm
- (d) 12 mm

Answer: (b)

Solution:

Given that $\beta = 12 \text{ mm}$

Now set up is immersed in water $\left(\mu = \frac{4}{3} \right)$

$$\beta' = \frac{\beta}{\mu}$$

$$\beta' = \frac{12}{4} \times 3$$

$$\boxed{\beta' = 9 \text{ mm}}$$

Question: Ratio of magnetic field at centre of two circular coils carrying same current and same radius with number of turns 200 and 400. Radius given was 20 cm for both

Options:

- (a) $\frac{1}{2}$
- (b) $\frac{2}{3}$

(c) $\frac{3}{2}$

(d) $\frac{5}{2}$

Answer: (a)

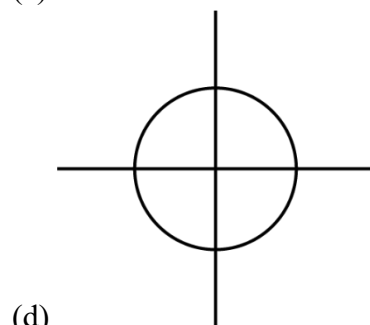
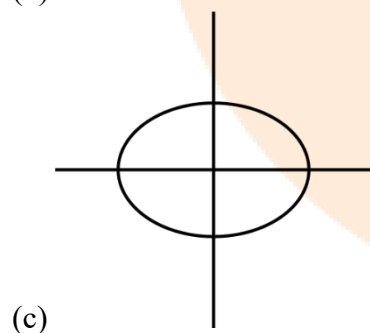
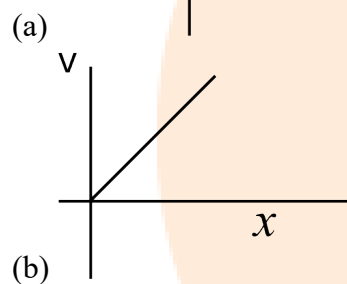
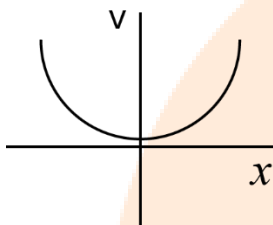
Solution:

$$B = N \frac{\mu_0 i}{2R}$$

$$\frac{B_1}{B_2} = \frac{N_1}{N_2} = \frac{200}{400} = \frac{1}{2}$$

Question: In linear SHM, variation of velocity of body against its displacement is best represented by,

Options:



(d)

Answer: (c)

Solution:

$$v = \pm \omega \sqrt{A^2 - x^2}$$

$$v^2 = \omega^2 (A^2 - x^2)$$

$$v^2 = \omega^2 A^2 - \omega^2 x^2$$

$$v^2 + \omega^2 x^2 = \omega^2 A^2$$

$$\frac{v^2}{\omega^2 A^2} + \frac{\omega^2 x^2}{\omega^2 A^2} = 1$$

$$\frac{v^2}{(\omega A)^2} + \frac{x^2}{A^2} = 1 \quad (\text{ellipse})$$

Question: In LR circuit $X_L = R$ and in LCR circuit $X_L = X_C$. Ratio of power factor in two situations is.

Options:

(a) $\frac{1}{2}$

(b) $\frac{1}{\sqrt{2}}$

(c) $\sqrt{2}$

(d) $\frac{2}{3}$

Answer: (b)

Solution:

Given that

For LR circuit

$$X_L = R$$

$$\cos \phi = \frac{R}{Z}$$

$$\cos \phi = \frac{R}{\sqrt{R^2 + R^2}}$$

$$(\cos \phi) = \frac{1}{\sqrt{2}} \dots (1)$$

$$X_L = X_C$$

$$\cos \phi = \frac{R}{Z}$$

$$(\cos \phi)_Z = \frac{R}{R} = 1 \dots (2)$$

From equation (1) and (2)

$$\frac{(\cos \phi)}{(\cos \phi)_Z} = \frac{1}{\sqrt{2}}$$

Question: Two projectiles at angles 30° and 45° reach their maximum heights in same time. Find the ratio of their initial velocities.

Options:

- (a) 1 : 1
- (b) 1 : 2
- (c) 2 : 1
- (d) $\sqrt{2} : 1$

Answer: (d)

Solution:

$$\frac{2u_1 \sin 30^\circ}{g} = \frac{2u_2 \sin 45^\circ}{g}$$

$$\frac{u_1}{u_2} = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{\left(\frac{1}{\sqrt{2}}\right)}{\frac{1}{2}} = \sqrt{2} : 1$$

Question: If in EM wave $B_0 = 2 \times 10^{-8} T$ find the amplitude of electric field

Options:

- (a) $2NC^{-1}$
- (b) $3NC^{-1}$
- (c) $6NC^{-1}$
- (d) $8NC^{-1}$

Answer: (c)

Solution:

Given that $B_0 = 2 \times 10^{-8} T$

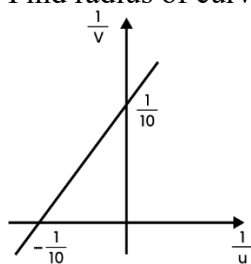
$$E_0 = ?$$

$$E_0 = B_0 C$$

$$E_0 = 2 \times 10^{-8} \times 3 \times 10^8$$

$$E_0 = 6N/C$$

Question: For an equiconvex lens made of refractive index 1.5, following graph is given. Find radius of curvature of lens.



Options:

- (a) 10 cm
- (b) 20 cm

(c) 15 cm

(d) 20 cm

Answer: (a)

Solution:

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

$$\frac{1}{f} = y - x$$

From graph

$$\frac{1}{f} = 0 - \left(-\frac{1}{10}\right)$$

$$f = 10\text{cm}$$

$$\frac{1}{f} = (\mu_R - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

$$\frac{1}{10} = (1.5 - 1) \left(\frac{1}{R} + \frac{1}{R} \right)$$

$$\frac{1}{10} = \frac{1}{2} \left(\frac{2}{R} \right)$$

$$\boxed{R = 10\text{cm}}$$

Question: Radio can tune into 6MHz. Find the corresponding wavelength band

Options:

(a) 20m

(b) 30m

(c) 50m

(d) 70m

Answer: (a)

Solution:

We know $\lambda = \frac{c}{f}$

So, $\lambda = \frac{C}{f_1}$

$$\lambda_1 = \frac{3 \times 10^8}{6 \times 10^6} = 50\text{m}$$

And $\lambda_2 = \frac{c}{f_2}$

$$= \frac{3 \times 10^8}{10 \times 10^6} = 30\text{m}$$

So wavelength bond = $\lambda_1 - \lambda_2$

$$= 50 - 30 = 20\text{m}$$

Question: Find the work done in splitting water droplet of radius $R = 1 \text{ cm}$ into 729 droplets. Surface tension $T = 75 \text{ dyne/cm}^2$.

Options:

- (a) $7.536 \times 10^{-3} \text{ J}$
- (b) $7.536 \times 10^2 \text{ J}$
- (c) $7.536 \times 10^3 \text{ J}$
- (d) $75.36 \times 10^{-3} \text{ J}$

Answer: (a)

Solution:

According to question $\frac{4}{3}\pi R^3 = n \times \frac{4}{3}\pi r^3$

$$(1)^3 = 729(r)^3$$

$$(r)^3 = \frac{1}{729}$$

$$\boxed{r = \frac{1}{9} \text{ m}}$$

$W_D = \text{Surface tension} \times \text{change in area}$

$$W_D = 75 \times 10^{-5+4} \times 4\pi$$

$$\left(729 \times \left(\frac{1}{9}\right)^2 - (1)^2 \right)$$

$$W_D = 7.536 \times 10^{-3} \text{ J}$$

Question: Four capacitors having capacity 1,2,3,4 μF connected in parallel. If 20V battery is connected across the system then find the charge flown through the battery

Options:

- (a) $50 \mu\text{C}$
- (b) $100 \mu\text{C}$
- (c) $150 \mu\text{C}$
- (d) $200 \mu\text{C}$

Answer: (d)

Solution:

If 1,2,3,4 μF capacitors are connected in parallel then

$$C_{eq} = 1 + 2 + 3 + 4 = 10 \mu\text{F}$$

Given: $V = 20 \text{ volt}$

So charge flown from the battery

$$q = C_{eq}V$$

$$q = 10 \times 20 \mu\text{C}$$

$$q = 200\mu C$$

Question: A mass of M is attached at Top of find disc of same mass M and radius R . If point mass is given gentle push. Find ω of disc when mass reaches bottom.

Options:

(a) $\sqrt{\frac{9g}{3R}}$

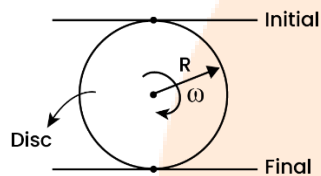
(b) $\sqrt{\frac{8g}{3R}}$

(c) $\sqrt{\frac{R}{3g}}$

(d) $\sqrt{\frac{g}{R}}$

Answer: (b)

Solution:



ω = angular velocity at the instant.

When ball is at bottom.

As here is no friction as well as external force,

So, from conservation of mechanical energy

We have, $\Delta k = -\Delta v$

Let as assume K_i (Initial kinetic energy = 0)

$$\therefore K_f - K_i = -(mg \times 2R)$$

$$\Rightarrow \frac{1}{2} I \omega^2 = mg \times 2R$$

$$\Rightarrow \frac{1}{2} \times \left(\frac{mR^2}{2} + mR^2 \right) \omega^2 = mg \times 2R$$

$$\Rightarrow \frac{3}{4} mR^2 \omega^2 = mg \times 2R$$

$$\Rightarrow \omega^2 = \frac{8g}{3R}$$

$$\Rightarrow \omega = \sqrt{\frac{8g}{3R}}$$

JEE-Main-26-07-2022-Shift-1 (Memory Based)

Chemistry

Question: The product formed in the given reaction



Options:

- (a) H_2
- (b) BeH_2
- (c) Both (a) and (b)
- (d) None of the above

Answer: (b)



Question: Which of the following can help in preventing decomposition of H_2O_2 ?

Options:

- (a) Formaldehyde
- (b) Formic acid
- (c) Ethanol
- (d) Urea

Answer: (d)

Solution: In the presence of metal surfaces or traces of alkali (present in glass containers), the reaction is catalysed. It is, therefore, stored in wax-lined glass or plastic vessels in dark. Urea can be added as a stabiliser.

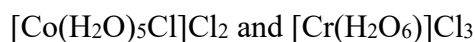
Question: Difference in spin magnetic moment of $[\text{Co}(\text{H}_2\text{O})_5\text{Cl}]\text{Cl}_2$ and $[\text{Cr}(\text{H}_2\text{O})_6]\text{Cl}_3$

Options:

- (a) 1 BM
- (b) 0 BM
- (c) 2 BM
- (d) 3 BM

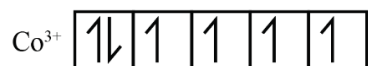
Answer: (a)

Solution:



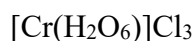
Co is in +3 oxidation state

It means valence shell configuration is $3d^6$ and high spin complex

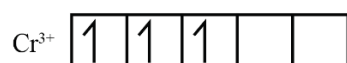


$n = 4$

$$\mu = \sqrt{4(4+2)} = \sqrt{24} \text{ B M} = 4.89 \text{ BM}$$



Cr^{3+} has valence shell configuration $3d^3$



$$n = 3$$

$$\mu = \sqrt{n(n+2)} = \sqrt{3(3+2)} = \sqrt{15} \text{ B M} = 3.87 \text{ BM}$$

$$\text{Difference between spin magnetic moment} = 4.89 - 3.87 \approx 1 \text{ BM}$$

Question: Find the order of the reaction if concentration changes from 0.5 to 1 and half life changes from 100 s to 50 s

Options:

- (a) Zero order
- (b) First order
- (c) Second Order
- (d) None of these

Answer: (c)

Solution:

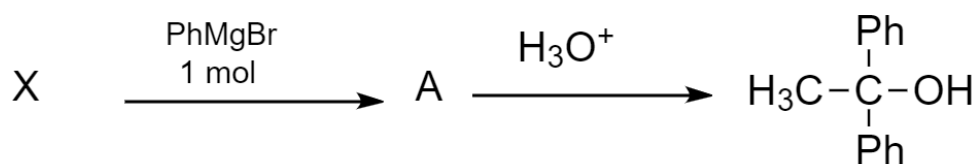
$$t_{1/2} \propto \frac{1}{[A_0]^{n-1}}$$

$$\frac{(t_{1/2})_1}{(t_{1/2})_2} = \frac{[A_0]_2^{n-1}}{[A_0]_1^{n-1}} = \left[\frac{[A_0]_2}{[A_0]_1} \right]^{n-1}$$

$$\frac{100}{50} = \left(\frac{1}{0.5} \right)^{n-1} \Rightarrow 2 = (2)^{n-1}$$

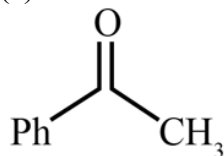
$$\Rightarrow n = 2 \text{ (Second order)}$$

Question: Identify 'X'



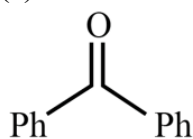
Options:

(a)

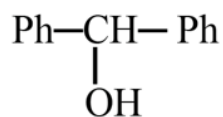


(b) PhCH₂OH

(c)

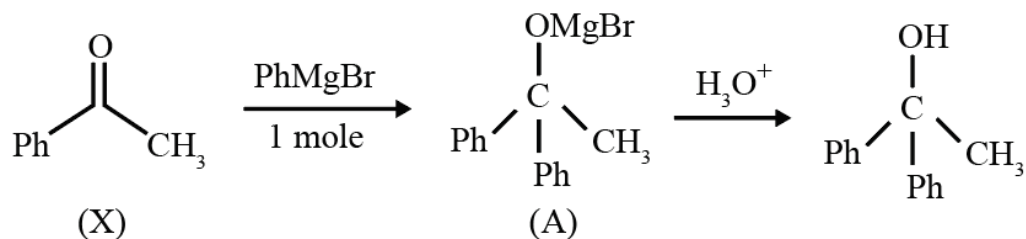


(d)

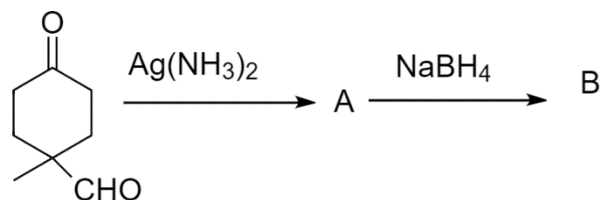


Answer: (a)

Solution:

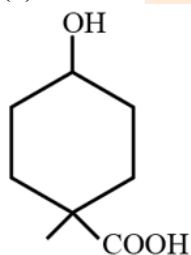


Question: Identify the product B

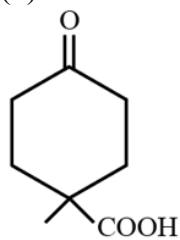


Options:

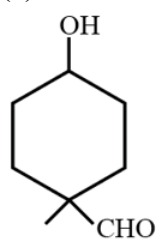
(a)



(b)



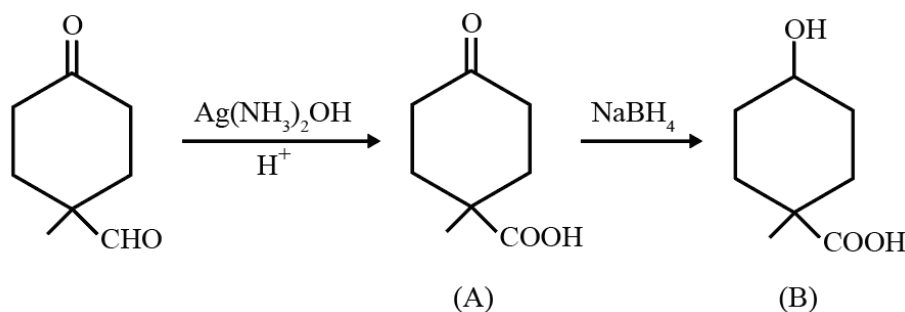
(c)



(d) None of these

Answer: (a)

Solution:



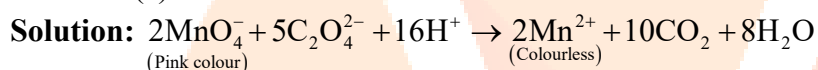
Question: Assertion: Dark purple colour of KMnO_4 in titration with Oxalic acid in acidic medium disappears

Reason: Change in oxidation number of Mn from 7 to 2

Options:

- (a) Both assertion and reason are true, reason is correct explanation of assertion
- (b) Both assertion and reason are true, but reason is not a correct explanation of the assertion.
- (c) Assertion is true, but reason is false.
- (d) Assertion is false, but reason is true

Answer: (a)



Question: A mixture of hydrogen and oxygen contains 40% by mass hydrogen at total pressure 2.2 bar then partial pressure will be

Options:

- (a) 2.01 bar
- (b) 3.81 bar
- (c) 5.2 bar
- (d) 1.2 bar

Answer: (a)

Solution:

% by mass = 40% hydrogen

% by mass = 60% oxygen

Let total weight = 100 g

So, amount of H_2 = 40 g

$$\text{Moles of } \text{H}_2 = \frac{40}{2} = 20 \text{ mol}$$

$$\text{Amount of } \text{O}_2 = \frac{60}{32} = 1.8 \text{ mol}$$

$$X_{\text{H}_2} = \frac{20}{20+1.8} = 0.9$$

$$P_{\text{H}_2} = P_T \times X_{\text{H}_2}$$

$$P_{\text{H}_2} = 2.2 \times 0.9 = 2.01 \text{ bar}$$

Question: Match the column Reaction Catalyst

Reaction	Catalyst
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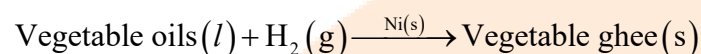
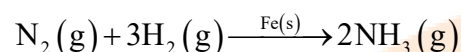
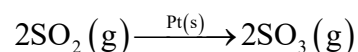
(A) Hydrogenation of oils	(i) Pt
(B) $N_2 + 3H_2 \rightarrow 2NH_3$	(ii) Fe
(C) $SO_2 + O_2 \rightarrow SO_3$	(iii) Ni
(D) Contact process	(iv) V_2O_5

Options:

- (a) A \rightarrow (ii); B \rightarrow (i); C \rightarrow (iii); D \rightarrow (iv)
 (b) A \rightarrow (iii); B \rightarrow (ii); C \rightarrow (i); D \rightarrow (iv)
 (c) A \rightarrow (i); B \rightarrow (iii); C \rightarrow (iv); D \rightarrow (ii)
 (d) A \rightarrow (iv); B \rightarrow (ii); C \rightarrow (i); D \rightarrow (iii)

Answer: (b)

Solution:



Contact process for the manufacture of sulphuric acid require V_2O_5 catalyst

Question: Match the following.

Compounds	Shape
(A) PCl_5	(i) Square Pyramidal
(B) O_3	(ii) Trigonal Bipyramidal
(C) BrF_5	(iii) Bent shape

Options:

- (a) A \rightarrow (i); B \rightarrow (ii); C \rightarrow (iii)
 (b) A \rightarrow (ii); B \rightarrow (iii); C \rightarrow (i)
 (c) A \rightarrow (i); B \rightarrow (iii); C \rightarrow (ii)
 (d) A \rightarrow (iii); B \rightarrow (ii); C \rightarrow (i)

Answer: (b)

Solution:

(A) $PCl_5 \Rightarrow$ Trigonal Bipyramidal

(B) $O_3 \Rightarrow$ Bent shape

(C) $BrF_5 \Rightarrow$ Square Pyramidal

Question: Liquation refining is based on

Options:

- (a) Low melting point
 (b) High melting point
 (c) Less soluble impurities
 (d) More soluble impurities

Answer: (a)

Solution: Liquation is used for the refining of metals having low melting point and are associated with high melting impurities. For example, Pb, Sn, Sb, Bi and Hg.

Question: If stearic acid and polyethylene glycol reacts then which of the following soap/detergent will be formed?

Options:

- (a) Cationic Detergent
- (b) Soap
- (c) Anionic detergent
- (d) Non Ionic Detergent

Answer: (d)

Solution: If sodium stearate and polyethylene glycol reacts, then non-ionic detergents are formed

Question: $\text{Cl}^\bullet + \text{CH}_4$ in atmosphere gives

Options:

- (a) $\dot{\text{C}}\text{H}_3$
- (b) Cl_2
- (c) HOCl
- (d) None of these

Answer: (a)

Solution: $\dot{\text{C}}\text{l}(\text{g}) + \text{CH}_4(\text{g}) \rightarrow \dot{\text{C}}\text{H}_3(\text{g}) + \text{HCl}(\text{g})$

Question: Phenol + Br_2 (in CCl_4) \rightarrow Product

Phenol + Br_2 (in water) \rightarrow Product

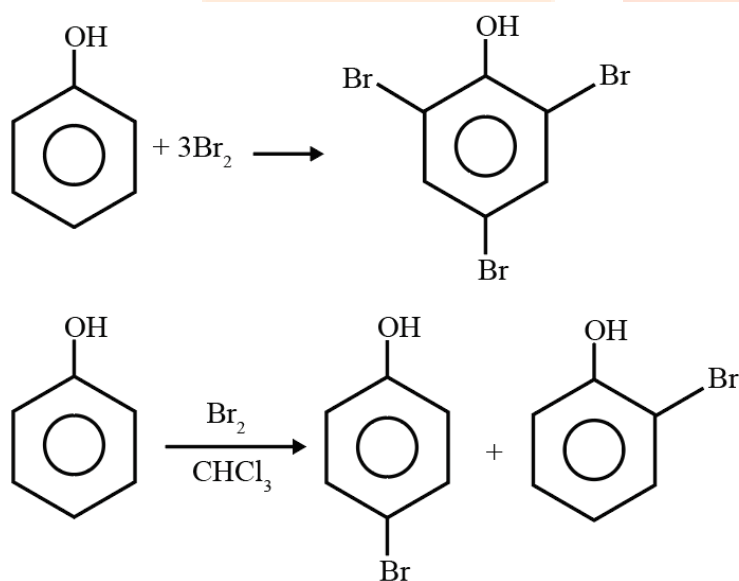
The difference in two products due to

Options:

- (a) Polarity of solvent
- (b) Electronegativity
- (c) High activating effect of OH group
- (d) Both (a) and (c)

Answer: (d)

Solution: In case of phenol, polarization of bromine takes place even in the absence of Lewis acid due to high activating effect of $-\text{OH}$ group attached to benzene



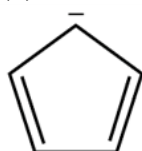
Question: Which of the following is not an aromatic compound?

Options:

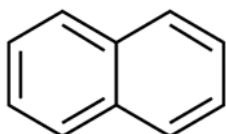
(a)



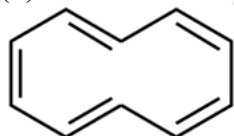
(b)



(c)



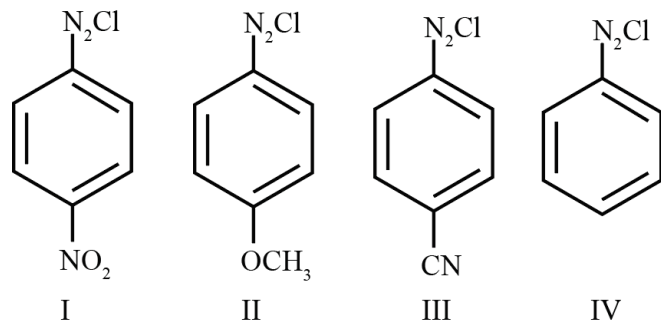
(d)



Answer: (d)

Solution: It doesn't satisfy the condition of an aromatic compound

Question: The correct order of stability



Options:

- (a) II > III > I > IV
- (b) II > IV > III > I
- (c) III > II > I > IV
- (d) I > II > IV > III

Answer: (b)

Solution: More is the electron withdrawing group deactivating, the order of stability of diazonium salt decreases on benzene ring.

Question: Borazine is an inorganic benzene like compound formed by 3 equivalent of element X and 6 equivalent of element Y, Identify X and Y

Options:

- (a) B₂H₆, NH₃

(b) B₂H₆, HN₃

(c) NH₃, B₂O₃

(d) NH₃, B₂H₆

Answer: (a)



Question: If wavelength of first line of Lyman series of H spectrum is λ and wavelength difference between second transition of Balmer and third transition of Paschen series of line spectrum of H atom is $x\lambda$. Find the value of x .

Answer: 5.00

Solution:

$$\frac{1}{\lambda} = R \left[\frac{1}{1} - \frac{1}{4} \right] = \frac{3R}{4} \Rightarrow \lambda = \frac{4}{3R}$$

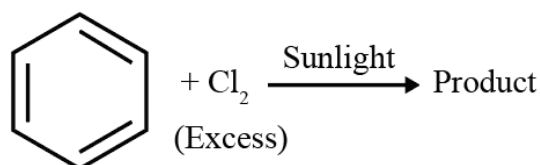
$$\frac{1}{\lambda_1} = R \left[\frac{1}{4} - \frac{1}{16} \right] = \frac{12R}{64} \Rightarrow \lambda_1 = \frac{16}{3R}$$

$$\frac{1}{\lambda_2} = R \left[\frac{1}{9} - \frac{1}{36} \right] = \frac{27R}{9 \times 36} \Rightarrow \lambda_2 = \frac{36}{3R}$$

$$\lambda_2 - \lambda_1 = x\lambda = \frac{36}{3R} - \frac{16}{3R} = \frac{20}{3R}$$

$$\frac{20}{3R} = 5 \times \frac{4}{3R} = x\lambda = x = 5$$

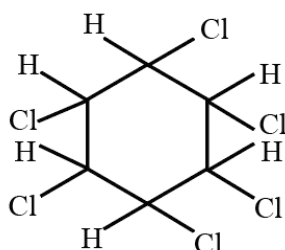
Question:



Number of hydrogen atoms in the product are

Answer: 6.00

Solution:



Question: The velocity of electron is x times the velocity of a neutron. If the wavelength of electron is equal to the wavelength of neutron, find the value of x .

Given: Mass of electron = 9.1×10^{-31} kg.

Mass of neutron = 1.6×10^{-27} kg. (Round off to the nearest integer)

Answer: 1758.00

Solution:

$$V_e = xV_n$$

$$\lambda_e = \lambda_n$$

$$\frac{h}{m_e v_e} = \frac{h}{m_n v_n}$$

$$m_e x v_n = m_n v_n$$

$$x = \frac{m_n}{m_e} = \frac{1.6 \times 10^{-27}}{9.1 \times 10^{-31}} = 0.17582 \times 10^4$$

$$= 1758.2$$

$$x \approx 1758$$



JEE-Main-26-07-2022-Shift-1 (Memory Based)

MATHEMATICS

Question: How many 5 digit number can be formed such that product of digits is 30.

Answer: 80.00

Solution:

$$x_1 \cdot x_2 \cdot x_3 \cdot x_4 \cdot x_5 = 30 = 5 \times 2 \times 3$$

$$1 \ 1 \ 2 \ 3 \ 5 \rightarrow \frac{5!}{2!} = 60$$

$$1 \ 1 \ 1 \ 6 \ 5 \rightarrow \frac{5!}{3!} = 20$$

$$60 + 20 = 80$$

Question: $f(3x) - f(x) = x$, $f(8) = 7$, find $f(14)$.

Answer: 10.00

Solution:

$$f(3x) - f(x) = x$$

$$f(x) - f\left(\frac{x}{3}\right) = \frac{x}{3}$$

$$f\left(\frac{x}{3}\right) - f\left(\frac{x}{3^2}\right) = \frac{x}{3^2}$$

⋮

On adding, we get

$$f(x) - \lim_{n \rightarrow \infty} f\left(\frac{x}{3^n}\right) = x\left(\frac{1}{3} + \frac{1}{3^2} + \dots + \infty\right)$$

$$\Rightarrow f(x) - f(0) = \frac{x}{2}$$

$$\because f(8) = 7, \text{ so } f(0) = 3$$

$$\therefore f(x) = \frac{x}{2} + 3$$

$$\therefore f(14) = 10$$

Question: Coefficient of x & x^2 in $(1+x)^p \times (1-x)^q$ are -3 & -5 respectively. Find coefficient of x^3 .

Answer: 23.00

Solution:

Given $(1+x)^p \times (1-x)^q$

$$\left({}^p C_0 + {}^p C_1 x + {}^p C_2 x^2 + {}^p C_3 x^3 \dots \right) \left({}^q C_0 - {}^q C_1 x + {}^q C_2 x^2 - {}^q C_3 x^3 \dots \right)$$

$$p - q = -3$$

$$-pq + \frac{q(q-1)}{2} + \frac{p(p-1)}{2} = -5$$

$$-2pq + q^2 - q + p^2 - p = -10$$

$$(p-q)^2 - p - q = -10$$

$$9 - p - q = -10$$

$$p + q = 19$$

$$\Rightarrow p = 8, q = 11$$

Coefficient of $x^3 = -{}^q C_3 + {}^p C_3 + p {}^q C_2 - q {}^p C_2$

$$= -{}^{11} C_3 + {}^8 C_3 + 8 {}^{11} C_2 - 11 {}^8 C_2$$

$$= \frac{-11 \cdot 10 \cdot 9}{6} + \frac{8 \cdot 7 \cdot 6}{6} + \frac{8 \cdot 11 \cdot 10}{2} - \frac{11 \cdot 8 \cdot 7}{2}$$

$$= 23$$

Question: $\frac{dy}{dx} + (2 \tan x)y = \sin x, y\left(\frac{\pi}{3}\right) = 0$, find $f(x)|_{\max}$

Answer: $\frac{1}{8}$

Solution:

$$\frac{dy}{dx} + (2 \tan x)y = \sin x$$

$$\text{IF} = e^{\int 2 \tan x dx} = e^{-2 \ln \cos x} = \frac{1}{\cos^2 x}$$

$$\frac{y}{\cos^2 x} = \int \frac{\sin x}{\cos^2 x} dx$$

$$\frac{y}{\cos^2 x} = \frac{1}{\cos x} + C$$

$$0 = \frac{1}{\cos \frac{\pi}{3}} + C$$

$$C = -2$$

$$y = \cos x - 2 \cos^2 x$$

$$= -2 \left[\cos^2 x - \frac{1}{2} \cos x \right]$$

$$= -2 \left[\cos^2 x - \frac{1}{2} \cos x + \frac{1}{16} - \frac{1}{16} \right]$$

$$= -2 \left[\cos x - \frac{1}{4} \right]^2 + \frac{1}{8}$$

$$y_{\max} = \frac{1}{8}$$

Question: Find sum of elements in 11th term: (3);(6,9,12);(15,18,21,24,27);.....

Answer: 6993.00

Solution:

(3);(6,9,12);(15,18,21,24,27);.....

$$1 + 3 + 5 + \dots + 10 \text{ terms} = \frac{10}{2} [2 \times 1 + (10 - 1)2]$$

$$= 100$$

$$a_{146} = 3 + (99)3 = 300$$

11th term = (303 + 21 terms)

$$= \frac{21}{2} [2 \times 303 + (20)3]$$

$$= 6993$$

Question: $\tan \left[2 \tan^{-1} \left(\frac{1}{5} \right) + \sec^{-1} \left(\frac{\sqrt{5}}{2} \right) + 2 \tan^{-1} \left(\frac{1}{8} \right) \right] = ?$

Answer: 2.00

Solution:

$$2 \left(\tan^{-1} \left(\frac{1}{8} \right) + \tan^{-1} \left(\frac{1}{5} \right) \right) = 2 \tan^{-1} \left(\frac{\frac{1}{8} + \frac{1}{5}}{1 - \frac{1}{40}} \right)$$

$$= 2 \tan^{-1} \left(\frac{1}{3} \right)$$

$$= \tan^{-1} \left(\frac{\frac{2}{3}}{1 - \frac{1}{9}} \right)$$

$$= \tan^{-1} \left(\frac{3}{4} \right)$$

∴ The given terms reduces to

$$\tan \left(\tan^{-1} \left(\frac{3}{4} \right) + \tan^{-1} \left(\frac{1}{2} \right) \right)$$

$$= \tan \left(\tan^{-1} \frac{\frac{3}{4} + \frac{1}{2}}{1 - \frac{3}{8}} \right)$$

$$= 2$$

Question: $x + y - z = 0 = x - 2y + 3z - 5$. There is a line parallel to this & passing through $(1, -2, 3)$. Find distance of this line from $(1, 4, 5)$.

Answer: ()

Solution:

$$x + y - z = 0, x - 2y + 3z - 5 = 0$$

$$n = \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 1 & -2 & 3 \end{vmatrix} = i(3-2) - j(3+1) + k(-2-1)$$

$$= i - 4j - 3k$$

$$\text{Line} \Rightarrow \frac{x-1}{1} = \frac{y+2}{-4} = \frac{z-3}{-3} = \lambda$$

$$P \equiv x = \lambda + 1, y = -4\lambda + 2, z = -3\lambda + 3$$

$$PQ = (\lambda, -4\lambda - 2, -3\lambda - 2)$$

$$\text{Now, } \lambda - 4(-4\lambda - 2) - 3(-3\lambda - 2) = 0$$

$$\lambda + 16\lambda + 8 + 9\lambda + 6 = 0$$

$$26\lambda + 14 = 0$$

$$\lambda = \frac{-14}{26} = \frac{-7}{13}$$

$$PQ = \sqrt{\lambda^2 + (-4\lambda - 2)^2 + (-3\lambda - 2)^2}$$

$$= \sqrt{\lambda^2 + 16\lambda^2 + 16\lambda + 4 + 9\lambda^2 + 12\lambda + 4}$$

$$= \sqrt{26\lambda^2 + 18\lambda + 4}$$

$$= \sqrt{26\left(\frac{-7}{13}\right)^2 + 18\left(\frac{-7}{13}\right) + 4}$$

$$= \sqrt{\frac{2 \times 49}{13} - \frac{7 \times 18}{13} + 4}$$

$$= \sqrt{\frac{24}{13}}$$

Question: Area under $y = f(x)$ from 3 to x (where $x > 3$) is $\left(\frac{y}{x}\right)^3 \cdot f(3) = 3$ then for

$y = 6\sqrt{10}$, what will be x ?

Answer: 6.00

Solution:

$$\int_3^x f(t) dt = \left(\frac{y}{x}\right)^3$$

$$f(x) = 3\left(\frac{y}{x}\right)^2 \left(\frac{y'x - y}{x^2}\right)$$

$$y = \frac{3y^2}{x^2} \left(\frac{y'x - y}{x^2}\right)$$

$$\Rightarrow x^4 = 3y(y'x - y)$$

$$\Rightarrow x^2 dx = 3y \left(\frac{xdy - ydx}{x^2}\right)$$

$$\Rightarrow x^2 dx = 3y d\left(\frac{y}{x}\right)$$

$$\Rightarrow x dx = 3\frac{y}{x} d\left(\frac{y}{x}\right)$$

$$\Rightarrow \frac{x^2}{2} = \frac{3}{2} \left(\frac{y}{x}\right)^2 + C$$

$$\Rightarrow x^2 = \frac{3y^2}{x^2} + C$$

$$\Rightarrow 3^2 = 3 + C$$

$$\Rightarrow C = 6$$

$$\Rightarrow x^2 = \frac{3y^2}{x^2} + 6$$

$$y = 6\sqrt{10}$$

$$\Rightarrow x^2 = \frac{3 \cdot 36 \times 10}{x^2} + 6$$

$$\Rightarrow x = 6$$

Question: From a group of 10 boys B_1, B_2, \dots, B_{10} and 5 girls G_1, G_2, \dots, G_5 , the number of ways of selection of group of 3 boys and 3 girls, such that B_1 & B_2 are not together in group is ____.

Answer: 1120.00

Solution:

Number of ways to select 3 boys = Total ways – No. of ways when both B_1 & B_2 are selected
 $= {}^{10}C_3 - {}^8C_1 = 112$

Number of ways to select 3 girls = ${}^5C_3 = 10$

Required number of ways = $112 \times 10 = 1120$

Question: $f(x) = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$, find relation between $f\left(\frac{a}{2}\right)$ & $f'\left(\frac{a}{2}\right)$

Options:

(a) $\sqrt{2f\left(\frac{a}{2}\right)} = f'\left(\frac{a}{2}\right)$

(b)

(c)

(d)

Answer: (d)

Solution:

$$f(x) = \sqrt{\frac{1 - \cos 2x}{1 + \cos 2x}}$$

$$f(x) = \sqrt{\frac{\sin^2 x}{\cos^2 x}} = \tan x$$

$$f'(x) = \sec^2 x = 1 + \tan^2 x$$

$$f'(x) = 1 + f^2(x)$$

$$f'\left(\frac{a}{2}\right) = 1 + f^2\left(\frac{a}{2}\right)$$

Question: $f(x) = \begin{cases} \frac{\ln(1-x+x^2) + \ln(1+x+x^2)}{\sec x - \cos x} & ; x < 0 \\ k & ; x \geq 0 \end{cases}$ is continuous, find k .

Answer: 1.00

Solution:

$$\lim_{x \rightarrow 0} \frac{\ln(1-x+x^2) + \ln(1+x+x^2)}{\sec x - \cos x}$$

$$\lim_{x \rightarrow 0} \frac{\frac{1}{1-x+x^2}(-1+2x) + \frac{1}{1+x+x^2}(1+2x)}{\sec x \tan x + \sin x}$$

$$\lim_{x \rightarrow 0} \frac{(2x-1)(1+x+x^2) + (2x+1)(1-x+x^2)}{(1+x^2+x^4)(\sec x \tan x + \sin x)}$$

$$= \lim_{x \rightarrow 0} \frac{2x + 2x^2 + 2x^3 - 1 - x - x^2 + 2x - 2x^2 + 2x^3 + 1 - x + x^2}{(1+x^2+x^4)(\sec x \tan x + \sin x)}$$

$$= \lim_{x \rightarrow 0} \frac{4x^3 + 2x}{(1+x^2+x^4)(\sec^2 x \sin x + \sin x)}$$

$$= \lim_{x \rightarrow 0} \frac{4x^2 + 2}{(1+x^2+x^4)(\sec^2 x + 1)}$$

$$= \frac{2}{1 \times 2} = 1$$

Question: Normal to $y^2 = 24x$ at (α, β) is perpendicular to $2x + 2t = 5$. Find equation of normal to $\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1$ at $(\alpha + 4, \beta + 10)$.

Answer: ()

Solution:

$$\frac{x^2}{\alpha^2} - \frac{y^2}{\beta^2} = 1 \quad (\alpha + 4, \beta + 10)$$

$$\beta^2 = 24\alpha$$

$$2yy' = 24$$

$$y' = \frac{12}{y} = \frac{12}{\beta}$$

$$\Rightarrow \frac{12}{\beta} = 1 \Rightarrow \beta = 12$$

$$12^2 = 24 \cdot \alpha$$

$$\Rightarrow \alpha = 6$$

$$\frac{x^2}{36} - \frac{y^2}{144} = 1$$

$$\frac{2x}{36} - \frac{2yy'}{144} = 0$$

$$\frac{x}{36} = \frac{yy'}{144}$$

$$\Rightarrow y' = \frac{144}{136} \cdot \frac{x}{y} = \frac{4x}{y} = \frac{4 \times 10}{22}$$

$$= \frac{20}{11}$$

$$y - 22 = \frac{20}{11}(x - 10)$$

$$11y - 242 = 20x - 200$$

$$\Rightarrow 20x - 11y + 42 = 0$$