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JEE (Main)

PAPER-1 (B.E./B. TECH.)

2022

COMPUTER BASED TEST (CBT)

Memory Based Questions & Solutions

Date: 26 July, 2022 (SHIFT-1) | TIME : (9.00 a.m. to 12.00 p.m)

Duration: 3 Hours | Max. Marks: 300

SUBJECT: MATHEMATICS

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PART : MATHEMATICS

1. Value of $\tan\left(2\tan^{-1}\left(\frac{1}{5}\right)+\sec^{-1}\left(\frac{\sqrt{5}}{2}\right)+2\tan^{-1}\left(\frac{1}{8}\right)\right)$ is

(1) $\frac{1}{2}$ (2) $\frac{1}{3}$ (3) 2 (4) 4

Ans. (3)

Sol. $\tan\left(2\tan^{-1}\left(\dfrac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{40}}\right)+\sec^{-1}\left(\dfrac{\sqrt{5}}{2}\right)\right)$

$= \tan\left(2\tan^{-1}\left(\dfrac{1}{3}\right)+\tan^{-1}\left(\dfrac{1}{2}\right)\right)$

$= \tan\left(\tan^{-1}\left(\dfrac{3}{4}\right)+\tan^{-1}\left(\dfrac{1}{2}\right)\right)$

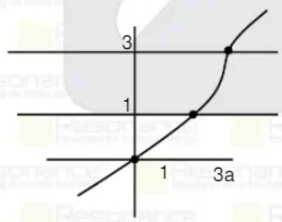
$= = \tan\left(\tan^{-1}\left(\dfrac{\frac{3}{4}+\frac{1}{2}}{1-\frac{3}{8}}\right)\right) = 2$

2. If the area enclosed by $y = 1$, $y = 3$, $x = 0$ and $x = y^a$ is $\dfrac{364}{3}$, then the value of a is

(1) 3 (2) 5 (3) 7 (4) 9

Ans. (2)

Sol. Area $= \int_1^3 y^a dy$



$= \left(\dfrac{y^{a+1}}{a+1}\right)_1^3 = \dfrac{364}{3}$

$= \dfrac{3^{a+1}-1}{a+1} = \dfrac{364}{3}$

$= a = 5$

3. Suppose $a_1, a_2, a_3, \ldots\ldots\ldots, a_n$ be a given sequence such that

$a_1 = \{3\}$

$a_2 = \{6,9,12\}$

$a_3 = \{15,18,21,24,27\}$

$a_4 = \{30,33,\ldots\ldots\ldots,48\}$ and so on. Now the sum of all the elements of a_{11} is

Ans. (6993)

Sol. Number of elements in $a_{11} = 1+(11-1)2=21$

First term of $a_{11} = 303$

Sum of all elements of $a_{11} = \dfrac{21}{2}[2\times 303+(21-1)3]$

$= 21 \times 303 + 21 \times 30$

$= 21 \times 333 = 6993$

4. If f(x) is a continuous and differentiable function satisfying $f(3x) - f(x) = x\ \forall\ x \in R - \{0\}$ and $f(8) = 7$ then the value of f(14) is

(1) 3 (2) 7 (3) 10 (4) 14

Ans. (3)

Sol. $\dfrac{f(3x)-f(x)}{3x-x} = \dfrac{1}{2}$

By LMVT

$\Rightarrow f'(x) = \dfrac{1}{2} \Rightarrow f(x) = \dfrac{x}{2}+C$

f(8) = 4 + 5 = 7 ⇒ C = 3

f(14) = 7 + 3 = 10

5. In the expansion of $(1+x)^p (1-x)^q$, when p > 0 , q > 0 the coefficient of x and x^2 are –3 and –5 respectively, then the coefficient of x^3 is

Ans. **(23)**

Sol. $(1+x)^p (1-x)^q = \left(1+px+\frac{p(p-1)}{2!}x^2+\frac{p(p-1)(p-2)}{3!}x^3+....\right)\left(1-qx+\frac{q(q-1)}{2!}x^2----\right)$

$p - q = -3$....(1) and $-pq + \frac{p(p-1)}{2} + \frac{q(q-1)}{2} = -5$ ....(2)

$\Rightarrow p^2 + q^2 - 2pq - p - q = -10$

$\Rightarrow (p-q)^2 - (p+q) = -10$

$\Rightarrow -(p+q) = -10-9 = -19$

$\Rightarrow p + q = 19$......(3)

$\Rightarrow p - q = -3$ ......(1)

$\Rightarrow p = 8, \quad q = 11$

Now coefficient of x^3 in : $(1+x)^8 (1-x)^{11} = (1-x^2)^8 (1-x)^3 = (1-8x^2 + ...)(1-3x+3x^2-x^3)$

so coefficient of $x^3 = -1 + 24 = 23$

6. If the mean and variance of a binomial probability distribution are α and $\frac{\alpha}{3}$ respectively and

$P(x = 1) = \frac{4}{243}$, then the value of P(x = 4 or 5) is

(1) $\frac{16}{27}$ (2) $\frac{64}{243}$ (3) $\frac{1}{27}$ (4) $\frac{64}{81}$

Ans. **(1)**

Sol. $n.p. = \alpha$

$npq = \frac{\alpha}{3}$

$q = \frac{1}{3}, p = \frac{2}{3}$

$\Rightarrow n = \frac{3\alpha}{2}$

$P(x = 1) = {}^nC_1.p.q^{n-1} = \frac{4}{243}$

$= \frac{2n}{3^n} = \frac{4}{243} \Rightarrow n = 6$

$P(X = 4 \text{ or } 5) = {}^6C_4.\ P^4.q^2 + {}^6C_5.p^5.q$

$= 15\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^2 + 6\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right) = \frac{80}{3^5} + \frac{64}{3^5}$

$= \frac{144}{243} = \frac{16}{27}$

7. Statement $\sim(p \Leftrightarrow \sim q) \wedge q$ is equivalent to

(1) Tautology (2) contradiction (3) $(p \to q) \wedge q$ (4) $(p \to q) \wedge p$

Ans. **(4)**

Sol. $\sim(p \Leftrightarrow \sim q) \wedge q$

$= (p \Leftrightarrow q) \wedge q$

p	q	$p\leftrightarrow q$	$(p\leftrightarrow q)\wedge q$	$(p\rightarrow q)$	$(p\rightarrow q)\wedge q$	$(p\rightarrow q)\wedge p$
T	T	T	T	T	T	T
T	F	F	F	F	F	F
F	T	F	F	T	T	F
F	F	T	F	T	F	F

8. If y(x) is the solution of differential equation $\frac{dy}{dx}+2y\tan x=\sin x$, where $x\in\left(0,\frac{\pi}{2}\right)$ and $y\left(\frac{\pi}{2}\right)=0$, then the maximum value of y(x) is

(1) $\frac{1}{16}$ (2) $\frac{1}{8}$ (3) $\frac{1}{4}$ (4) $\frac{1}{6}$

Ans. **(2)**

Sol. I.F. = $e^{\int 2\tan x}=\sec^2 x$

$\Rightarrow y(\sec^2 x) = \int\frac{\sin x}{\cos^2 x}dx+C$ Let cosx = t

$\Rightarrow \frac{y}{\cos^2 x}=-\int\frac{1}{t^2}dt+C$

$\Rightarrow \frac{y}{\cos^2 x}=\frac{1}{\cos x}+C$

$\Rightarrow y(x) = \cos x + C\cos^2 x$

$\because\ y\left(\frac{\pi}{3}\right)=0$

$\Rightarrow 0=\frac{1}{2}+\frac{c}{4}\Rightarrow C=-2$

$\Rightarrow y(x) = \cos x - 2\cos^2 x$

$\Rightarrow y(x) = -2\left(\cos^2 x-\frac{1}{2}\cos x+\frac{1}{16}-\frac{1}{16}\right)$

$\Rightarrow y(x) = \frac{1}{8}-2\left(\cos x-\frac{1}{4}\right)$

$\Rightarrow y_{max.} = \frac{1}{8}$ at $\cos x = \frac{1}{4}$

9. A is 2 × 2 matrix such that det(A) = – 1, det((A + I) (adjA + I)) = 4. The sum of diagonal elements of matrix A is

(1) $\sqrt{2}$ (2) 1 (3) 2 (4) $-\sqrt{2}$

Ans. **(3)**

Sol. $|(A + I)(\text{adj}A + I)| = |A\text{adj}A + A + \text{adj}A + I|$

$\Rightarrow ||A|.I + A + \text{adj}A + I|$

$= |-I + A + \text{adj}A + I| = |A + \text{adj}A|$

Let $A=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ $|A| = ad - bc = -1$

$A + \text{adj}A = \begin{bmatrix}a & b\\ c & d\end{bmatrix}+\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}=\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}=(a+d)I$

$|A + \text{adj}A| = |(a+d)I| = (a+d)^2\ |I| = (a+d)^2$

$\Rightarrow (a + d)^2 = 4 \Rightarrow a + d = \pm 2$

10. If $f(x) = \begin{cases} \dfrac{\ell n(1-x+x^2)+\ell n(1+x+x^2)}{\sec x - \cos x} & x \neq 0 \\ k & x = 0 \end{cases}$ is continuous at x = 0 then k is equal to

(1) 2 (2) 1 (3) –1 (4) 0

Ans. **(2)**

Sol. $f(0) = \lim\limits_{x\to 0} \dfrac{\ell n(1-x+x^2)+\ell n(1+x+x^2)}{\sec x - \cos x}$

$\Rightarrow k = \lim\limits_{x\to 0} \dfrac{\ell n\,(1+x^2+x^4)}{\sin^2 x} \times \cos x$

$\Rightarrow k = \lim\limits_{x\to 0} \dfrac{\ell n\,(1+x^2+x^4)}{x^2+x^4} \times \dfrac{(x^2+x^4)}{\sin^2 x} \times \cos x$

$\Rightarrow\ k = 1$

11. If $g(x) = \begin{cases} x+a & x \le 0 \\ |x-4| & x > 0 \end{cases}$ and $f(x) = \begin{cases} x+1 & x < 0 \\ (x-4)^2+b & x \ge 0 \end{cases}$ and g and f are continuous functions then the value of (gof)(2) + (fog)(–2) is equal to

(1) 18 (2) –18 (3) 10 (4) – 11

Ans. **(2)**

Sol. $g(0) = g(0^+)$

a = 4

$f(0) = f(0^-)$

16 + b = 1

b = – 15

Now $g(x) = \begin{cases} x+4 & x \le 0 \\ |x-4| & x > 0 \end{cases}$

$f(x) = \begin{cases} x+1 & x < 0 \\ (x-4)^2-15 & x \ge 0 \end{cases}$

Now gof(2) + fog(–2) = g(–11) + f(2)

= –7 + 4 – 15

= – 3 – 15 = – 18

12. Number of all 5 digit natural numbers such that the product of all digits is 36.

(1) 90 (2) 180 (3) 150 (4) 120

Ans. **(3)**

Sol. abcde = 36 a,b,c,d,e > 0

C-I a,b,c,d,e ∈ {1,2,2,3,3}

Total ways = $\dfrac{5!}{2!.2!} = 30$

C-II a,b,c,d,e ∈ {1,1,4,3,3}

Total ways = $\dfrac{5!}{\ldots} = 30$

Total ways = $\frac{5!}{2!.2!}$ = 30

C-III a,b,c,d,e $\in$ {1,1,6,2,3}

Number of ways = $\frac{5!}{2!}$ = 60

C-IV a,b,c,d,e $\in$ {1,1,2,2,9}

Total ways = $\frac{5!}{2!.2!}$ = 30

Total number of integers = 30 + 30 + 60 + 30 = 150

13. If $a = \lim_{n\to\infty} \sum_{r=1}^{n} \frac{2n}{r^2 + n^2}$ and a function $f(x) = \sqrt{\frac{1-\cos x}{1+\cos x}}$ then which of the following statement is correct

(1) $F'\left(\frac{a}{2}\right) = 2F\left(\frac{a}{2}\right)$ (2) $F'\left(\frac{a}{2}\right) + \sqrt{2}F\left(\frac{a}{2}\right) = 0$

(3) $F'\left(\frac{a}{2}\right) = \sqrt{2}F\left(\frac{a}{2}\right)$ (4) $F'\left(\frac{a}{2}\right).F\left(\frac{a}{2}\right) = \sqrt{2}$

Ans. (3)

Sol. $a = \lim_{n\to\infty} \frac{1}{n} \sum_{r=1}^{n} \frac{2}{\frac{r^2}{n^2} + 1}$

$= \int_0^1 \frac{2}{1+x^2} dx$

$a = 2\left[\tan^{-1}(x)\right]_0^1$

$\Rightarrow a = \frac{\pi}{2}$

now after doing rationalisation we get F(x) = cosecx – cotx

$F'(x) = -\text{cosecxcotx} + \text{cosec}^2x$

$F'(x) = -\text{cosecx.cotx} + \text{cosec}^2x$

F′(x) = cosecx (cosec – cotx)

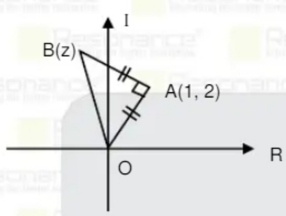
F′(x) = cosecx.F(x)

14. In complex plane, three points O,A,B are such that O is origin, A(1, 2) and Re(B) < 0. If ΔOAB is right angle isosceles with OB as hypotenuse then the point B is

(1) (–3, 4) (2) (–1, 3) (3) (–2, 3) (4) (–1, 4)

Ans. (2)

Sol. By rotation theorem



$\frac{z-(1+2i)}{0-(1+2i)} = \frac{|\overrightarrow{AB}|}{|\overrightarrow{OA}|} e^{-i\frac{\pi}{2}}$

$\because |\overrightarrow{AB}| = |\overrightarrow{OA}|$

$\frac{z-(1+2i)}{0-(1+2i)} = -i$

$\Rightarrow z = -1 + 3i$ so point B is (–1, 3)

27th June & 4th July

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