2016

MATHEMATICS

Full Marks : 100

Pass Marks : 33

Time : Three Hours and *Fifteen Minutes
(*15 minutes are given as extra time for reading questions)

Attempt all Questions.

The figures in the right margin indicate full marks for the questions.

For Question Nos. *1-6*, write the letter associated with the correct answer.

1. The value of $tan\left(2\,tan^{-1}\dfrac{1}{5}\right)$ is

    A. $\dfrac{12}{5}$

    B. $\dfrac{5}{12}$

    C. $\dfrac{5}{17}$

    D. $\dfrac{17}{5}$

2. If f and g are functions defined from R to R by $f(x)=\sin x$ and $g(x)=x^2$, then fog is   1

    A. $x^2 \sin x$

    B. $(\sin x)^2$

    C. $\sin (x^2)$

    D. $(x \sin x)^2$

3. If matrix A is of order $m \times n$ and matrix B is of order 3×4, then AB is defined only if   1

    A. $n=3$

    B. $n=4$

    C. $m=3$

    D. $m=4$

4. The value of $\lim\limits_{x \to 0} (1+x)^{\frac{1}{x}}$ is   1

    A.   0

    B.   1

    C. e

    D. $\log a$

5. If $x = t^2$, $y = t^3$, then $\dfrac{d^2y}{dx^2}$ is equal to 　　　　　　　　1

A. $\dfrac{4t}{3}$

B. $\dfrac{2t}{3}$

C. $\dfrac{-3}{2t}$

D. $\dfrac{3}{4t}$

6. If the line $\dfrac{x+R}{3p} = \dfrac{y-3}{2} = \dfrac{z-4}{1}$ is parallel to the plane $2x + 3y - 4z + 7 = 0$, then the value of p is 　　　　　　　　1

A. $\dfrac{1}{3}$

B. $\dfrac{-1}{3}$

C. $\dfrac{-4}{3}$

D. $\dfrac{2}{3}$

7. What is an equivalence relation ? 　　　　　　　　1

8. Find the principal value of $sin^{-1}\left(sin\dfrac{4\pi}{3}\right)$.   1

9. When is a function $f(x)$ said to be differentiable at a given point ?   1

10. The radius of a circle is increasing at the rate of $0\cdot7\,cm/sec$. What is the rate of increase of its circumference?   1

11. When is a function $f(x)$ said to have a relative minimum at a point $x=c$?   1

12. Express $\int_a^b f(x)dx$ as the limit of a sum.   1

13. Write the differential equation obtained by eliminating the arbitrary constant c in the equation $xy=c^2$.   1

14. State the triangle law of vectors.   1

15. Write the distance of a point with position vector $\vec{a}$ from the plane $\vec{r}\cdot\vec{n}=q$.   1

16. Write the vector equation of a line passing through the point (1, 3, 7) and parallel to the line $\dfrac{x-5}{3}=\dfrac{y+4}{5}=\dfrac{z-1}{4}$.   1

17. A binary operation $*$ on Z is defined by $a*b=a+b-2$, for $a, b\in Z$.

    i)   Find the identity element in Z.

    ii)   Find inverse of x if $x*3=7$.   3

18. Prove that $cos^{-1} x = sin^{-1} \sqrt{1-x^2} = tan^{-1} \dfrac{\sqrt{1-x^2}}{x}$ 3

19. If $x^y = 5^{x-y}$, find $\dfrac{dy}{dx}$. 3

20. Prove that $\displaystyle\int \dfrac{dx}{\sqrt{a^2 + x^2}} = \log\left|x + \sqrt{a^2 + x^2}\right| + c$. 3

21. Solve the differential equation $y \dfrac{dx}{dy} + x = y$. 3

22. Show that the projection vector of $\vec{a}$ on $\vec{b}\left(\neq \vec{0}\right)$ is $\left(\dfrac{\vec{a} \cdot \vec{b}}{|\vec{b}|^2}\right)\vec{b}$. 3

23. The function f defined by $f(x) = \begin{cases} x^2 + 3x + a, & x \leq 1 \\ bx + 2 & , & x > 1 \end{cases}$ 4

 is given to be derivable for every x. Find a and b.

OR

 If $\sqrt{1-x^2} + \sqrt{1-y^2} = a(x-y)$, prove that $\dfrac{dy}{dx} = \sqrt{\dfrac{1-y^2}{1-x^2}}$.

24. Using integration, find the area of the region bounded by the parabola $y = x^2 + 2$ and the lines $y = x$, $x = 0$ and $x = 3$. 4

25. Evaluate $\displaystyle\int \dfrac{5x - 2}{3x^2 + 2x + 1} dx$ 4

Evaluate $\int \cos^{-1} \sqrt{x}\, dx$.

26. What type of differential equation is said to be homogeneous differential equation ? Write a stepwise method for solving a homogeneous differential equation.   4

27. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\hat{b} = \hat{j} - \hat{k}$, find a vector $\hat{c}$, such that $\vec{a} \times \vec{c} = \vec{b}$ and $\vec{a} \cdot \vec{c} = 3$.   4

28. If $P(A) = \dfrac{3}{8}$, $P(B) = \dfrac{1}{2}$ and $P(A \cap B) = \dfrac{1}{4}$, find $P\left(\dfrac{\overline{A}}{B}\right)$.   4

29. Define a symmetric matrix. Prove that every square matrix can be uniquely expressed as the sum of a symmetric and a skew-symmetric matrix.   6

30. Define direction consines of a line. If θ be the angle between two lines with direction cosines l_1, m_1, n_1 and l_2, m_2, n_2 , respectively.   6

Prove that   i) $\cos \theta = l_1 l_2 + m_1 m_2 + n_1 n_2$

ii) $\sin \theta \sqrt{(m_1 n_2 - n_1 m_2)^2 + (n_1 l_2 - l_1 n_2)^2 + (m_1 l_2 - l_1 m_2)^2}$

OR

Define a plane. Prove that every first degree equation $ax + by + cz + d = 0$ represents a plane.

31. Using matrices solve the system of equations   6

$x + 2y + z = 1$

$2x - y + z = 5$

$3x + y - z = 0.$

32. Show that the height of the right circular cylinder of maximum volume that
can be inscribed in a given right circular cone of height h is $\dfrac{h}{3}$.   6

33. Prove that $\displaystyle\int_0^{\frac{\pi}{4}} \left(\sqrt{\tan x} + \sqrt{\cot x}\right)dx = \dfrac{\pi}{2}.$ 6

OR

Show that $\displaystyle\int_0^{\frac{\pi}{2}} \dfrac{dx}{\left(a^2 \cos^2 x + b^2 \sin^2 x\right)^2} = \dfrac{\pi\left(a^2 + b^2\right)}{4a^3 b^3}.$

34. A company makes 3 model of calculators A, B and C at factory I and factory
II. The company has orders for at least 6400 calculators of model A, 4000
calculators of model B and 4800 calculators of model C. At factory I, 50
calculators of model A, 50 of model B and 30 of model C are made everyday,
at factory II, 40 calculators of model A, 20 of model B and 40 of model C
are made everyday. It costs Rs. 12,000 and Rs. 15,000 each day to operate
factory I and II respectively. Find the number of days each factory should
operate to minimise the operating costs and still meet the demand. Solve the
L.P.P. graphically. (No graph paper will be supplied).   6

35. There are three candidates for the position of principal A, B and C. The chances of their selection are in the proportion $4:2:3$ respectively. The probabilities that A, if selected, will introduce co-education in the college is $0 \cdot 3$. The probability of B and C doing the same are respectively $0 \cdot 5$ and $0 \cdot 8$. What is the probability that there will be co-education in the college ? Also, find the probability that principal B introduces co-education in the college. 6

OR

In a test, an examinee either guesses or copies or knows the answer to a multiple choice question with four choices. The probability that he makes a guess is $\dfrac{1}{3}$ and the probability that he copies the answer is $\dfrac{1}{6}$. The probability that his answer is correct, given that he copied it, is $\dfrac{1}{8}$. Find the probability that he knew the answer to the question, given that he correctly answered it.