27/07/2022
Evening

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Memory Based
Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2022 (Online) Phase-2

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. The activity of radioactive material be 6.4×10^{-4} curie. Its half life is 5 days, then the activity is 5×10^{-6} curie after

(1) 7 days (2) 15 days

(3) 25 days (4) 35 days

Answer (4)

Sol. $A_0 = 6.4 \times 10^{-4}$

$A = 5 \times 10^{-6}$

$$A = \left(\frac{1}{2}\right)^{\frac{n}{t_{1/2}}} A_0$$

$$\Rightarrow \quad \frac{5}{640} = \left(\frac{1}{2}\right)^{\frac{n}{t_{1/2}}}$$

$$\frac{n}{t_{1/2}} = 7$$

or $x = 5 \times 7 = 35$ days

2. Potential energy density U can be given as

$$U = \frac{\alpha}{\beta}\sin\frac{\alpha x}{kt}$$

Where $x \Rightarrow$ length, $t \Rightarrow$ temperature, $k \Rightarrow$ Boltzmann constant, α and β are constant. Then the dimension of β is

(1) $M^0L^0T^0$ (2) $M^0L^2T^{-2}$

(3) $ML^2T^{-2}\theta^{-1}$ (4) $M^0L^2T^0$

Answer (4)

Sol. Dimensions of αx and kt should be same

$\Rightarrow \quad [\alpha]\,[x] = [k]\,[t]$

$$\text{Now } [U] = \frac{[\alpha]}{[\beta]}$$

$$\Rightarrow \quad [\beta] = \frac{[\alpha]}{[U]} = \frac{[k][t]}{[x][U]}$$

$$= \frac{[L^3]}{[L]} = [L^2]$$

$[\beta] = [M^0L^2T^0]$

3. Block moves down an inclined rough plane with constant velocity. The contact force experienced by the block is. (Mass of block is M)

(1) Mg (2) $Mg\sin\theta$

(3) $Mg(\sin\theta + \cos\theta)$ (4) $\sqrt{Mg}$

Answer (1)

Sol. As the block moves with constant velocity so forces on the block should be balanced. That is the contact force from the incline should balance the weight of the block.

$\Rightarrow$ contact force = Mg

4. If a charge $4\mu C$ is divided into two and kept apart at some distance. The magnitude of charges so that the force between them is maximum is

(1) $1\mu C$ and $3\mu C$ (2) $2\mu C$ and $2\mu C$

(3) $0\mu C$ and $4\mu C$ (4) $1.5\mu C$ and $2.5\mu C$

Answer (2)

Sol. $$F = \frac{K(4-x)(x)}{a^2}$$

For F to be maximum

$$\Rightarrow \quad \frac{dF}{dx} = 0$$

$\Rightarrow \quad 4 - 2x = 0$

or $x = 2$

$\Rightarrow \quad 4 - x = x = 2\mu C$

5. An object is thrown vertically upwards with velocity equal to $\lambda v_e (\lambda < 1)$ from the surface of earth. Maximum height achieved by the object from surface of earth is

(1) $\frac{R_e}{1-\lambda^2}$ (2) $\frac{(1+\lambda^2)R_e}{1-\lambda^2}$

(3) $\frac{\lambda^2 R_e}{1-\lambda^2}$ (4) $(1-\lambda^2)R_e$

Answer (3)

Sol. Using energy conservation

$$\frac{1}{2}m\left(\lambda\sqrt{\frac{2GM_e}{R_e}}\right)^2 - \frac{GM_em}{R_e} = \frac{-GM_em}{R_e+h}$$

$$\lambda^2\frac{GM_em}{R_e} - \frac{GM_em}{R_e} = \frac{-GM_em}{R_e+h}$$

$$(R_e+h) = \frac{R_e}{1-\lambda^2}$$

$$h = \frac{R_e\lambda^2}{1-\lambda^2}$$

6. A proton (m = 1.6 × 10^{-27} kg) moves in a circle of radius 60 cm in uniform magnetic field 1T in transverse direction. Kinetic energy of the proton (in MeV) is equal to

(1) 18 (2) 12

(3) 10 (4) 6

Answer (1)

Sol. $R = \frac{\sqrt{2mK}}{9B}$

$0.6 = \frac{\sqrt{2\times1.6\times10^{-27}K}}{1.6\times10^{-19}\times1}$

$\Rightarrow$ K = 18 MeV

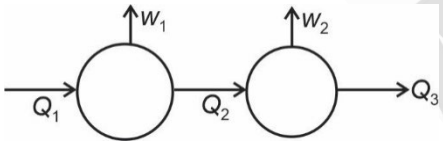
7. A Carnot engine works between temperature 300 K and 100 K. Now another Carnot engine is put in series to the earlier one such that first engine works between temperatures 300 K and 200 K while the second works between 200 K and 100 K. It η_1 is the efficiency in first case and η_2 is efficiency in second case then relation in η_1 and η_2 is

(1) $\eta_2 = \eta_1 = \frac{1}{3}$ (2) $\eta_2 = \eta_1 = \frac{2}{3}$

(3) $\eta_2 = 2\eta_1$ (4) $2\eta_2 = \eta_1$

Answer (2)

Sol. $\eta_1 = \left(1-\frac{1}{3}\right) = \frac{2}{3}$



$\eta_2 = \left(1-\frac{Q_3}{Q_1}\right) = \left(1-\frac{Q_3}{Q_2}\times\frac{Q_2}{Q_1}\right)$

$1-\eta_2 = (1-\eta_b)(1-\eta_a)$

$1-\eta_2 = \frac{200}{300}\times\frac{100}{200} = \frac{1}{3}$

$\Rightarrow$ $\eta_2 = \frac{2}{3}$

8. In an amplitude modulation, amplitude of carrier wave is 5 V while amplitude of modulating signal is 1V. The modulation index of the message signal is

(1) 10% (2) 20%

(3) 15% (4) 30%

Answer (2)

Sol. In amplitude modulation

$\mu = \frac{A_m}{A_c} = \frac{1}{5}$

= 20%

9. Two waves of same frequency having intensity k and $4k$ interfere with each other. The ratio of maximum to minimum intensity in interference pattern is

(1) 9 : 1 (2) 3 : 1

(3) 5 : 1 (4) 5 : 3

Answer (1)

Sol. $\frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2}$

$= \frac{9}{1}$

10. In common-emitter configuration when base current is varied from 20 μA to 25 μA the collector current varies from 6 mA to 8 mA. The current gain factor is equal to

(1) 150 (2) 200

(3) 400 (4) 40

Answer (3)

Sol. $\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{2\text{ mA}}{5\,\mu\text{A}}$

$= \frac{2000}{5} = 400$

11. A dielectric $(\varepsilon_r = k)$ is used to fill the space between plates of a capacitor of capacitance C. The new capacitance of capacitor is

(1) $\frac{C}{k}$ (2) $(k-1)C$

(3) kC (4) k^2C

Answer (3)

Sol. $C_0 = \frac{\varepsilon_0\varepsilon_r A}{d}$

When $\varepsilon_r = 1$

$C_0 = \frac{\varepsilon_0 A}{d} = C$

When $\varepsilon_r = k$

$C_0' = \frac{\varepsilon_0 A}{d}k = kC$

12. For An EM wave chose the correct option regarding following statements (ε denotes the electric field component, B denotes the magnetic field component and C denotes the velocity of EM wave at a point

(a) $\vec{E} \perp \vec{B}, \vec{E} \parallel \vec{c}$ (b) $\vec{E} \perp \vec{B}$

(c) $\vec{E} \parallel \vec{c}$ (d) $\hat{E} = \hat{c} \times \hat{B}$

(1) Only a and c are

(2) Only b, c, d are true

(3) Only a and b are true

(4) Only b and c are true

Answer (4)

Sol. $\vec{E}, \vec{B}$ and $\vec{c}$ are mutually perpendicular such that $\hat{B} \times \hat{c} = \hat{E}$ so only B and C are correct.

13. in photoelectric effect

(a) Photoelectric current increases by increasing the incident frequency.

(b) Photoelectric current increases by increasing incident intensity.

(c) Stopping potential increases in magnitude by increasing incident frequency.

(d) Stopping potential increases in magnitude by increasing incident intensity.

(1) Only (a) and (b) are true

(2) Only (a) and (c) are true

(3) Only (b) and (d) are true

(4) Only (b) and (c) are true

Answer (3)

Sol. In photoelectric effect stopping potential depends on frequency of light used and saturation current depends on intensity of light so (b) and (d) are correct.

14. Two wires having length ℓ_1, ℓ_2, and their young's modules Y_1 , Y_2 of same cross-sectional area A, are connected end to end and hanged with a load of some mass m suspended at lower end. If total elongation is $\Delta\ell$. Find load suspended.

(1) $\dfrac{A\Delta\ell Y_1Y_2}{g(\ell_1Y_2 + Y_1\ell_2)}$ (2) $\dfrac{\Delta\ell Y_1Y_2A}{g(\ell_1Y_1 + \ell_2Y_2)}$

(3) $\dfrac{\Delta\ell Y_1Y_2A}{g(\ell_1Y_2 - \ell_2Y_1)}$ (4) $\dfrac{\Delta\ell Y_1Y_2\ell_1\ell_2}{Ag(Y_2 - Y_1)}$

Answer (1)

Sol. $\Delta\ell = \Delta\ell_1 + \Delta\ell_2$

$$\Delta\ell = \frac{mg\ell_1}{Y_1A} + \frac{mg\ell_2}{Y_2A}$$

$$\Rightarrow m = \frac{\Delta\ell A}{g\left(\frac{\ell_1}{Y_1} + \frac{\ell_2}{Y_2}\right)}$$

$$\Rightarrow m = \frac{\Delta\ell AY_1Y_2}{g(\ell_1Y_2 + Y_1\ell_2)}$$

15. If frequency of n^{th} and $(n + 1)^{th}$ mode of string are 400 Hz and 450 Hz and length of string is 1 m then find is linear mass density with tension in string equal to 2700 N

(1) 0.125 kg/m (2) 0.27 kg/m

(3) 0.055 kg/m (4) 0.0675 kg/m

Answer (2)

Sol. $\dfrac{v}{2\ell} = 50$

$$\frac{\sqrt{\frac{T}{\mu}}}{2\ell} = 50$$

$$\Rightarrow \frac{T}{\mu} = 100 \times 100$$

$$\Rightarrow \mu = \frac{2700}{10^4} = 0.27 \text{ kg /m}$$

16.

17.

18.

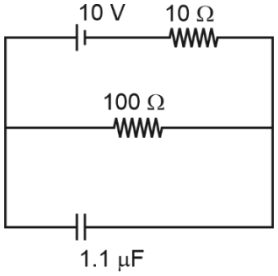
19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

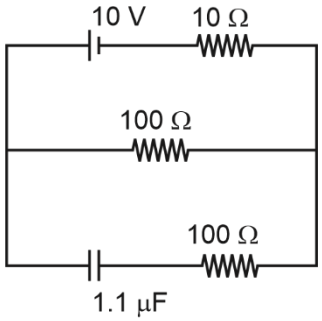
21.



For the given circuit. The charge on capacitor in steady state is N × 10^{-6} C. The value of N is

Answer (10)

Sol.

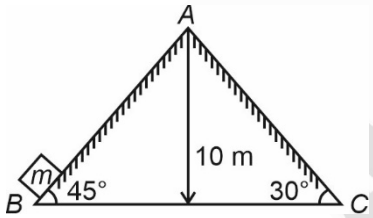


At steady state potential difference across capacitor $= \frac{10\times100}{110}$ V

$\Rightarrow$ Charge on capacitor $= \frac{10^3}{110}\times1.1\times10^{-6}$ C

$= 10\ \mu C$

22. A block of mass m is pushed above the plane from B such that it just crossed A and moves to C. Ignore friction everywhere if time taken to move from B to A then to C is $t\left(\sqrt{2}+1\right)$ s, then the value of t is _____.



Answer (2)

Sol. By energy conservation

$v_B^2 = v_C^2$

$v_B = \sqrt{2\times10\sqrt{2}\times10\times\frac{1}{\sqrt{2}}} = 10\sqrt{2}$

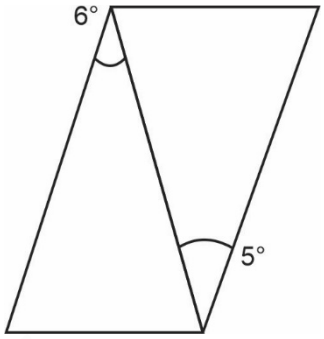
$t_{BA} = \frac{10\sqrt{2}}{10\sqrt{2}} = 2$ s

$t_{AC} = \frac{10\sqrt{2}}{5} = 2\sqrt{2}$

$\Rightarrow$ Total time $= 2\sqrt{2}+2 = 2\left(\sqrt{2}+1\right)$ s

$\Rightarrow$ $t = 2$

23. A prism of angle 6° and refractive index 1.5 and another prism of angle 5° and refractive index 1.55 are merged together as shown. It the angle of deviation is $\left(\frac{1}{x}\right)^\circ$ then the value of x is ___



Answer (4)

Sol. $\delta_1 = 6(\mu - 1)$

$= 3°$

$\delta_1 = -5(0.55)$

$= -2.75°$

$\Rightarrow$ $\delta_{net} = 3° - 2.75°$

$= 0.25°$

$= \frac{1}{4}^\circ$

$\Rightarrow$ $x = 4$

24. Soap bubble 6 cm is enclosing another bubble of radius 3 cm. The inside bubble is experiencing an internal pressure. The same internal pressure is experienced by another bubble of radius R inside. Find the value of R in cm.

Answer (2)

Sol. $P_0 + \frac{4T}{6} + \frac{4T}{3} = P_0 + \frac{4T}{R}$

$\Rightarrow$ $\frac{1}{R} = \frac{1}{6} + \frac{1}{3}$

$= R = 2$ cm

25.

26.

27.

28.

29.

30.

CHEMISTRY

SECTION – A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. **Assertion:** Boron is unable to form BF_6^{3-}.

   Reason: Boron is very small in size.

   (1) Both Assertion and Reason are correct and Reason is correct explanation of Assertion

   (2) Both Assertion and Reason are correct but Reason is not correct explanation of Assertion

   (3) Assertion is correct and Reason is incorrect

   (4) Assertion is incorrect and Reason is correct

Answer (2)

Sol. The outermost shell of boron is 2 and its maximum covalency is 4. Therefore, boron cannot form BF_6^{3-}. Hence, Assertion is true.

Boron is the first element of Group-13 of modern periodic table. It is very small in size. But it does not provide correct explanation of assertion.

2. $KMnO_4$ reacts in alkaline medium with $S_2O_3^{-2}$ to form which of the following ionic species:

   (1) SO_4^{-2} (2) SO_3^{-2}

   (3) $S_2O_7^{-2}$ (4) $S_2O_8^{-2}$

Answer (1)

Sol. $H_2O + 8MnO_4^{\ominus} + 3S_2O_3^{-2} \rightarrow 8MnO_2 + 6SO_4^{-2} + 2OH^{\ominus}$

3.

	Polymer	Monomer
(A)	Neoprene	(1) Acrylonitrile
(B)	Natural rubber	(2) Chloroprene
(C)	Teflon	(3) Isoprene
(D)	Orlon	(4) Tetrafluoroethene

(1) A(2); B(3); C(4); D(1)

(2) A(1); B(2); C(3); D(4)

(3) A(4); B(3); C(1); D(2)

(4) A(3); B(1); C(4); D(2)

Answer (1)

Sol. Neoprene is the polymer of chloroprene. Natural rubber is the polymer of isoprene. Teflon is the polymer of tetrafluoroethene. Orlon is the polymer of acrylonitrile.

4. Consider the following orbitals (A to D) containing electron with following set of quantum numbers (n, l, m, s).

   A. $3, 2, 1, \frac{1}{2}$ B. $3, 1, 1, \frac{1}{2}$

   C. $4, 0, 0, \frac{-1}{2}$ D. $3, 0, 0, \frac{1}{2}$

   The highest energy orbital among the above set of orbitals for a multielectron atom will be

   (1) A (2) B

   (3) C (4) D

Answer (1)

Sol. A has highest energy as (n + l) value for this orbital has the maximum value.

n + l for A = 5

n + l for B = 4

n + l for C = 4

n + l for D = 3

5.

	Column-I	Column-II
I.	$\Psi_{AB} = \Psi_A + \Psi_B$	(P) Bonding Molecular orbital (LCAO)
II.	$\mu = q \times d$	(Q) Antibonding Molecular orbital (LCAO)
III.	$\Psi^*_{AB} = \Psi_A - \Psi_B$	(R) dipole moment
IV.	$\frac{1}{2}$ (Number of bonding e^- – number of anti bonding e^-)	(S)Bond order

(1) I-(P); II-(R); III-Q; IV-S

(2) I-(P); II-(Q); III-R; IV-S

(3) I-(Q); II-(P); III-R; IV-S

(4) I-(R); II-(P); III-Q; IV-S

Answer (1)

Sol. $\Psi_{AB} = \Psi_A + \Psi_B$ — Bonding molecular orbital

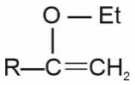
$\Psi^*_{AB} = \Psi_A - \Psi_B$ — Antibonding molecular orbital

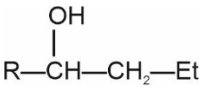
$\mu = q \times d$ — dipole moment

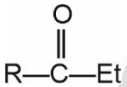
$\frac{n_b - n_a}{2}$ — Bond order, where n_b represents number of bonding electrons and n_a represents number of antibonding electrons

6. $R - COCH_3 \xrightarrow[EtBr]{NaOH}$ Final product (P) . The final product P is

(1) 

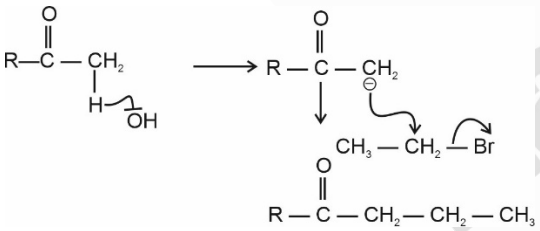
(2) 

(3) 

(4) 

Answer (1)

Sol.



7. Lathering property of soap is due to which of the following?

(1) Sodium stearate (2) Sodium carbonate

(3) Sodium rosinate (4) Glycerol

Answer (3)

Sol. Due to formation of sodium rosinate soap lathers well

8. Consider the following statements :

Statement I : on dilution, molar conductivity for KI (aqueous) increase steeply

Statement II : On dilution, molar conductivity for carbonic acid (aqueous) slowly increases till infinite dilution.

(1) Statement I is correct & statement II is incorrect

(2) Both statement (I) and (II) are correct

(3) Statement (I) is incorrect and statement (II) is correct

(4) Both statement (I) and (II) are incorrect

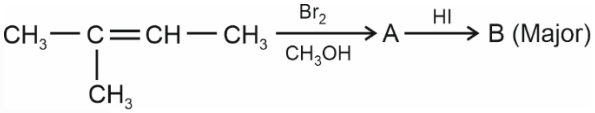
Answer (4)

Sol. On dilution, molar conductivity for strong electrolyte increases gradually till infinite dilution.

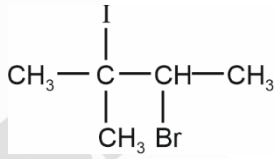
As KI is a strong electrolyte, molar conductivity increases gradually.

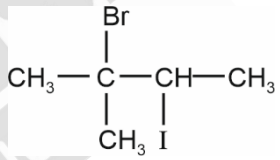
Carbonic acid is a weak electrolyte. So, molar conductivity increases steeply on excess dilution.

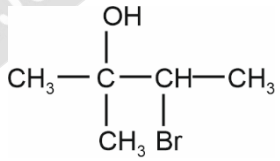
$\therefore$, statement (I) and statement (II) are incorrect.

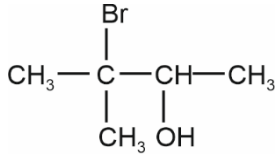
9. 

The product B is

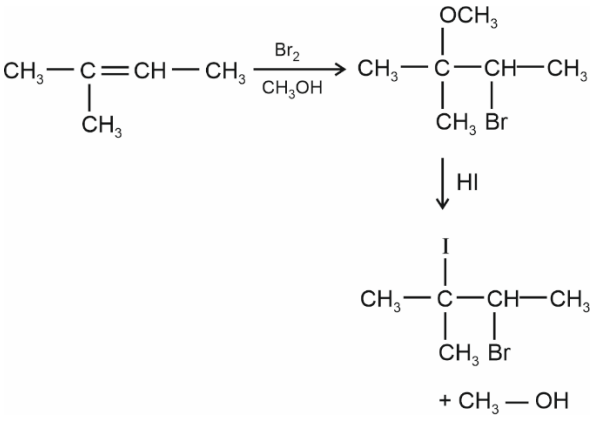
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

10. Match column-I with column-II.

Colum-I	Column-II
(A) Impure Aniline + water	(P) Crystallisation
(B) Aniline + Chloroform	(Q) Steam distillation followed by use of separating funnel
(C) Benzoic acid + naphthalene	(R) Sublimation
(D) Naphthalene + nonvolatile salts	(S) Distillation

(1) A(Q), B(S), C(P), D(R)

(2) A(P), B(Q), C(R), D(S)

(3) A(S), B(R), C(Q), D(P)

(4) A(Q), B(P), C(R), D(S)

Answer (1)

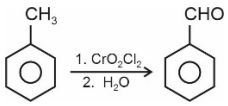
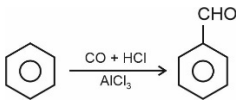
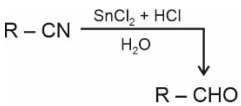
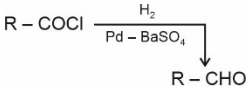
Sol. Chloroform (B.P. = 334 K) and Aniline (B.P. = 457) – Distillation

Imp Aniline and water – Steam distillation to purify water and aniline are separated by using separating funnel

Benzoic acid and Naphthalene – Crystallisation

Naphthalene and non volatile impurities – sublimation since naphthalene sublime while impure is non volatile

11. Match the reactions given in Column-I with their corresponding names in Column-II.

Column-I	Column-II
(A) 	(P) Etard's Reaction
(B) 	(Q) Gattermann Koch reaction
(C) 	(R) Stephen's reduction
(D) 	(S) Rosenmund reduction

The correct match is :

(1) A(P); B(Q); C(R); D(S)

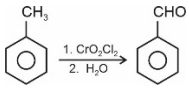
(2) A(Q); B(P); C(R); D(S)

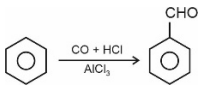
(3) A(Q); B(R); C(P); D(S)

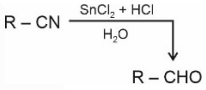
(4) A(R); B(Q); C(P); D(S)

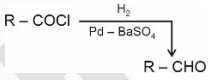
Answer (1)

Sol. The correct match is :

(A)  → (P) Etard's Reaction

(B)  → (Q) Gattermann Koch reaction

(C)  → (R) Stephen's reduction

(D)  → (S) Rosenmund reduction

12. Consider the following Reaction :

$Fe^{+3} + A \longrightarrow$ Prussian blue

Which of the following represents A?

(1) A is $[Fe(CN)_6]^{-4}$

(2) A is $[Fe(CN)_6]^{-3}$

(3) A is $[FeCl_4]^{\ominus}$

(4) A is $FeSO_4$

Answer (1)

Sol. $4Fe^{+3} + 3\left[Fe(CN)_6\right]^{-4} \longrightarrow \underset{\text{Prussian Blue}}{Fe_4\left[Fe(CN)_6\right]_3}$

Hence A is $\left[Fe(CN)_6\right]^{-4}$

13. Correct order of first ionisation energy for the elements with given electronic configuration.

(i) $3s^2$

(ii) $3s^2\ 3p^1$

(iii) $3s^2\ 3p^3$

(iv) $3s^2\ 3p^4$

(1) (iii) > (iv) > (i) > (ii)

(2) (iv) > (iii) > (ii) > (i)

(3) (i) > (ii) > (iii) > (iv)

(4) (ii) > (iii) > (i) > (iv)

Answer (1)

Sol. Orbitals with fully filled and half filled are stable, and require more energy for ionisation.

Elements with greater electronegativity require more energy for ionisation.

Hence, the correct order is (iii) > (iv) > (i) > (ii).

14. Group-1 element (A) with maximum hydration enthalpy, shows similarity with group-2 element (B). A and B respectively are

(1) Li, Mg

(2) Be, Mg

(3) Na, Ca

(4) K, Be

Answer (1)

Sol. Among group-1 elements lithium has maximum hydration enthalpy and shows diagonal relationship with Mg.

15. Statement I : Parchment paper can be used to separate true solution from colloid.

Statement II : When we use parchment paper the particles of true solution cannot pass but colloids can pass.

(1) Both statement I and II are correct

(2) Statement I is correct but statement II is incorrect

(3) Statement I is incorrect, Statement II is correct

(4) Neither statement I, nor statement II is correct

Answer (2)

Sol. Colloidal particles cannot pass through parchment paper but they can pass through ordinary filter paper.

Particles of true solution can pass through ordinary filter paper as well as parchment paper. Hence, parchment paper can be used for separation.

Statement I is correct and statement II is incorrect.

16. Match the following :

	Column-I (Reactions)	**Column-II (Product formed)**
A.	Glucose + Br_2 water	(1) n-Hexane
B.	Glucose + conc. HNO_3	(2) Gluconic acid
C.	Glucose + acetic anhydride	(3) Saccharic acid
D.	Glucose + HI/red P_4	(4) Glucose pentaacetate

(1) A(3); B(2); C(4); D(1)

(2) A(2); B(3); C(4); D(1)

(3) A(4); B(2); C(1); D(3)

(4) A(1); B(2); C(3); D(4)

Answer (2)

Sol. Glucose + Br_2 water $\rightarrow$ Gluconic acid

Glucose + conc. $HNO_3 \rightarrow$ Saccharic acid

Glucose + acetic anhydride $\rightarrow$ Glucose pentaacetate

Glucose + HI/red $P_4 \rightarrow$ n-hexane

Correct match is A(2); B(3); C(4); D(1).

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. 100 ml of 0.1 M H_2SO_4 is reacted with 50 ml of 0.1 M NaOH. What is the normality of H_2SO_4 left in the solution?

Answer (00.10)

Sol.

	H_2SO_4	+ 2NaOH	$\rightarrow Na_2SO_4$	+ $2H_2O$
Initial $\rightarrow$	10 millimoles	5 millimoles	–	–
Final $\rightarrow$	7.5 millimoles	–	2.5 mm	–

$\therefore$ Normality of $H_2SO_4 = \frac{7.5}{150} \times 2 = \frac{7.5}{75} = 0.1$ N

22. Consider a first order reaction

$A \rightarrow B$

The concentration of A after 70 minutes becomes half. If the rate constant of the reaction is 'x' $\times 10^{-6}$ seconds^{-1}, then find x. [Take ln2 = 0.693]

Answer (165.00)

Sol. $t_{1/2} = \frac{0.693}{k}$

$\therefore \quad k = \frac{0.693}{70 \times 60} = 0.000165\ s^{-1}$

$= 165 \times 10^{-6}\ s^{-1}$

Hence, x = 165

23. How many among the following are primary ores of Fe (iron)?

Siderite, Malachite, Magnetite, Haematite, Cryolite, Cuprite, Limonite, Kaolinite, Sphalerite, Bauxite, Chalcopyrite

Answer (04)

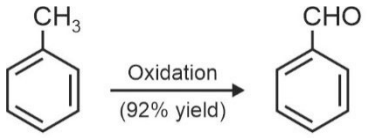
Sol. Primary ores of iron are,

Magnetite (Fe_3O_4), Haematite (Fe_2O_3), Siderite ($FeCO_3$) and Limonite ($FeO(OH)\cdot nH_2O$)

24. 5.0 gm of toluene is subjected to controlled oxidation to get benzaldehyde. The percentage yield of the product formed in the above reaction is 92%. Find the mass of benzaldehyde formed in gm.

Answer (5.30)

Sol. Mass of toluene (molar mass = 92) given = 5.0 gm



(Molar mass = 106)

Number of moles of benzaldehyde formed

= 0.92 × Number of moles of toluene

Mass of benzaldehyde formed

$= 0.92 \times \frac{5}{92} \times 106 = 5.30$ gm

25. How many of the following molecules species are non planar?

$BF_3, NO_3^-, SF_4, XeF_4, XeO_3, PH_4^+, PCl_3, Al(OH)_4^-$

and H_2O_2

Answer (6)

Sol. Among the given molecules/species, the following species are non planar.

SF_4	see-saw
XeO_3	Pyramidal
PH_4^+	Tetrahedral
PCl_3	Pyramidal
$Al(OH)_4^-$	Tetrahedral
H_2O_2	Open book

$\therefore$ No. of non planar species = 6.

26. When $MnO_4^{\ominus}$ reacts with H_2O_2 in alkaline medium, the oxidation state of Mn in the product containing Mn will be:

Answer (04.00)

Sol. $2MnO_4^{\ominus} + 3H_2O_2 \longrightarrow 2MnO_2 + 3O_2 + 2OH^{\ominus} + 2H_2O$

Oxidation state of Mn in MnO_2 is (+4)

27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Let sum of an infinite G.P. be 5, let the sum of first five terms be $\frac{98}{25}$. Then what is the sum of first 21 terms of an A.P. whose first term is $10ar$, common difference is $10ar^2$ and n^{th} term is a_n

(1) $22a_{11}$ (2) $21a_{11}$

(3) $14a_{16}$ (4) $15a_{16}$

Answer (2)

Sol. Given $\frac{a}{1-r}=5$ and $a(1-r^5)=\frac{98}{25}$

Let sum of 21-terms of A.P. = S_{21}

$\therefore \quad s_{21}=\frac{21}{2}\left[2(10ar)+20(10ar^2)\right.$

$= 21(ar + 10ar^2)$

$= 21a_{11}$

2. Let $A=\begin{bmatrix}4 & -2\\ \alpha & \beta\end{bmatrix}$, If $A^2 + \gamma A + 18I = 0$ then det(A) equals

(1) – 18 (2) 18

(3) – 50 (4) 50

Answer (2)

Sol. Characteristic equation of matrix

$$\begin{vmatrix}4-\lambda & -2\\ \alpha & \beta-\lambda\end{vmatrix}=0$$

$\Rightarrow \quad 4\beta+\lambda^2-(\beta+4)\lambda+2\alpha=0$

$\therefore \quad A^2 - (\beta + 4)A + (2\alpha + 4\beta)I = 0$

$\Rightarrow \quad \gamma = -\beta + 4$ and $2\alpha + 4\beta = 18$

$\det(A) = 4\beta + 2\alpha = 18$

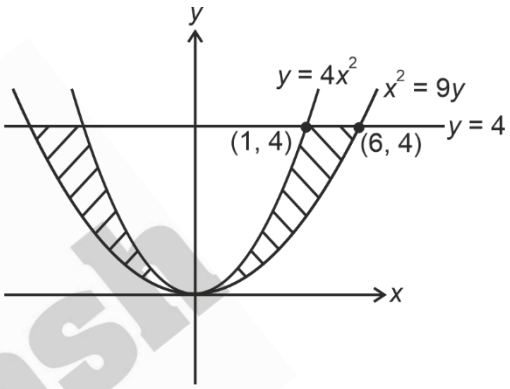
3. The area of region enclosed by $y \le 4x^2$, $x^2 \le 9y$, $y \le 4$ is equal to

(1) $\frac{40}{3}$ (2) $\frac{56}{3}$

(3) $\frac{112}{3}$ (4) $\frac{80}{3}$

Answer (1)

Sol.



$\therefore$ Required area

$$=\int_0^1\left(4x^2-\frac{x^2}{9}\right)dx+\int_1^6\left(4-\frac{x^2}{9}\right)dx$$

$$=\frac{4x^3}{3}-\frac{x^3}{27}\Bigg|_0^1+4x-\frac{x^3}{27}\Bigg|_1^6$$

$$=\left(\frac{4}{3}-\frac{1}{27}\right)+\left((24-8)-\left(4-\frac{1}{27}\right)\right)$$

$$=\frac{35}{27}+\left(16-\frac{107}{27}\right)=\frac{360}{27}=\frac{40}{3}$$

4. If the length of the latus rectum of a parabola whose focus is (a, a) and tangent at its vertex is $x + y = a$ is 16, then $|a|$ is equal to

(1) $2\sqrt{3}$

(2) $2\sqrt{2}$

(3) $4\sqrt{2}$

(4) 4

Answer (3)

Sol. Length of perpendicular from focus to tangent at vertex,

$$l = \left|\frac{a}{\sqrt{2}}\right|$$

So, length of latus rectum will be, $4l = 16$

$\Rightarrow \quad 2\sqrt{2}\,|a| = 16$

$\Rightarrow \quad |a| = 4\sqrt{2}$

5. Let $f(x) = \dfrac{(729p(1+x))^{\frac{1}{7}} - 3}{(729(1+qx))^{\frac{1}{3}} - 9}$, and $f(x)$ is continuous at $x = 0$, then

(1) $21qf(0) - p = 0$ (2) $21q^2 \cdot f(0) - p^3 = 0$

(3) $21p^2f(0) - q^3 = 0$ (4) $p^2f(0) - 7q^2 = 0$

Answer (1)

Sol. $\lim\limits_{x\to 0} f(x)$ exist if numerator of $f(x)$ is zero at x = 0

Clearly $p = 3$

Now $\lim\limits_{x\to 0} f(x) = \lim\limits_{x\to 0} \dfrac{3\left[(x+1)^{\frac{1}{7}} - 1\right]}{9\left[(1+qx)^{\frac{1}{3}} - 1\right]}$

$$= \frac{1}{3}\left(\frac{\frac{1}{7}}{\frac{q}{3}}\right) = \frac{1}{7q} = f(0)$$

So $21qf(0) = 3 = p$

6. Let $f(x) = \min\{[x], [x-1], [x-2], \ldots [x-10]\}$, where [ ] denotes greatest integer function, then $\int\limits_0^{10} \left(f(x) + |f(x)| + f^2(x)\right)dx$ is equal to

(1) 55 (2) 385

(3) 5050 (4) 270

Answer (2)

Sol. Clearly $f(x) = [x-10]$

Here, $f(x) \le 0 \ \ \forall x \in (0, 10)$

So, $\int\limits_0^{10} \left(f(x) + |f(x)|\right)dx = 0$

Now, $\int\limits_0^{10} f^2(x)dx = \int\limits_0^{10} ([x]-10)^2\,dx$

$$= \int_0^1 100\,dx + \int_1^2 81dx + \int_2^3 64\,dx + \ldots + \int_9^{10} 1\cdot dx$$

$$= \left(1^2 + 2^2 + 3^2 + \ldots + 10^2\right) = \frac{10\times 11\times 21}{6} = 385$$

7. The value of $\int_0^2 \left(\left|2x^3 - 3x\right| + \left[x - \frac{1}{2}\right]\right)dx$, where $[\cdot]$ is greatest integer function, is

(1) $\frac{7}{6}$ (2) $\frac{19}{12}$

(3) $\frac{17}{4}$ (4) $\frac{3}{2}$

Answer (3)

Sol. $\int_0^2 \left|2x^3 - 3x\right|dx + \int_0^2 \left[x - \frac{1}{2}\right]dx$

$$\int_0^{\sqrt{\frac{3}{2}}} \left(3x - 2x^3\right)dx + \int_{\sqrt{\frac{3}{2}}}^2 \left(2x^3 - 3x\right)dx$$

$$+\int_0^{\frac{1}{2}} \left[x - \frac{1}{2}\right]dx + \int_{1/2}^{3/2} \left[x - \frac{1}{2}\right]dx + \int_{3/2}^2 \left[x - \frac{1}{2}\right]dx$$

$$= \frac{3x^2 - x^4}{2}\Bigg|_0^{\sqrt{\frac{3}{2}}} + \frac{x^4 - 3x^2}{2}\Bigg|_{\sqrt{\frac{3}{2}}}^2 + \left(-\frac{1}{2}\right) + 0 + \left(\frac{1}{2}\right)$$

$$= \frac{9}{8} + 2 + \frac{9}{8} = \frac{17}{4}$$

8. The domain of

$$f(x) = \sin^{-1}[2x^2 - 3] + \log_2\left(\log_{\frac{1}{2}}(x^2 - 5x + 5)\right) \text{ is}$$

(1) $\left(1, \frac{5-\sqrt{5}}{2}\right)$ (2) $\left(1, \frac{\sqrt{5}}{2}\right)$

(3) $\left(-\sqrt{\frac{5}{2}}, \sqrt{\frac{5}{2}}\right)$ (4) (1, 4)

Answer (1)

Sol. $\because \quad [2x^2 - 3] = -1, 0 \text{ or } 1$

$\Rightarrow \quad 2x^2 - 3 \in [-1, 2) \Rightarrow 2x^2 \in [2, 5) \ \Rightarrow x^2 \in \left[1, \frac{5}{2}\right)$

$\Rightarrow \quad x \in \left(-\sqrt{\frac{5}{2}}, -1\right] \cup \left[1, \frac{5}{2}\right)$

Also, $\log_{\frac{1}{2}}(x^2-5x+5)>0$

$\Rightarrow\ 0 < x^2 - 5x + 5 < 1$

$\Rightarrow\ x \in \left(1, \frac{5-\sqrt{5}}{2}\right) \cup \left(\frac{5+\sqrt{5}}{2}, 4\right)$

Finally $x \in \left(1, \frac{5-\sqrt{5}}{2}\right)$

9. If the line of intersection of the planes $ax + by = 3$ and $ax + by + cz = 0$ makes an angle 30° with the plane $y - z + 2 = 0$, then the direction cosines of the line are

(1) $\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}$ (2) $\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0$

(3) $\frac{1}{\sqrt{5}}, \frac{-2}{\sqrt{5}}, 0$ (4) $\frac{1}{2}, \frac{-\sqrt{3}}{2}, 0$

Answer (2)

Sol. Direction ratios of line of intersection $<bc, -ac, 0>$

As angle between this line & $y - z + 2 = 0$ is 30°

$\therefore\ \sin\theta = \left|\frac{a}{\sqrt{a^2+b^2}\cdot\sqrt{2}}\right|$

$\Rightarrow\ a^2 = b^2$

$\therefore$ Possible combination is $\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}, 0$

10. Let PQ be a building of height 10 m and angle of elevation of point P from a point A on the ground is 45°. Let B be another point from where foot of perpendicular on ground is R and angle of elevation of B from A is 30°. If angle of elevation of P from B is 60°. Then area of trapezium $PQRB$ and length AB is

(1) $25, \frac{15(\sqrt{3}-1)}{2}$

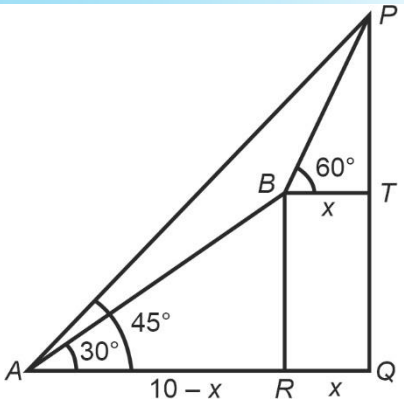
(2) $15(\sqrt{3}+1), \frac{5(\sqrt{3}+1)}{2}$

(3) $25, \frac{5(\sqrt{3}+1)}{2}$

(4) $15(\sqrt{3}+1), \frac{5(\sqrt{3}-1)}{2}$

Answer (1)

Sol.



$\because\ AQ = PQ \cot 45° = 10$

Let $QR = x$

So $AR = 10 - x$

Now $BR = AR\tan 30° = \frac{10-x}{\sqrt{3}} = QT$

Also $PT = x\tan 60° = \sqrt{3}x$

$\therefore\ QT + PT = 10$

$\Rightarrow\ \frac{10-x}{\sqrt{3}} + \sqrt{3}x = 10 \Rightarrow x = 5\left(\sqrt{3}-1\right)$

Now $BR = 5\left(\sqrt{3}-1\right)$

Hence area of trapezium

$PQRB = \frac{1}{2}\left(5\left(\sqrt{3}-1\right)\right)\left(10+5\left(\sqrt{3}+1\right)\right)$

$= 25$

$AB = AR\cos 30° = \frac{15\left(\sqrt{3}-1\right)}{2}$

11.

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. If $A=\begin{bmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ \beta+\gamma & \alpha+\gamma & \alpha+\beta\end{bmatrix}$ and

$$\frac{|adj(adj(adj(adj(A))))|}{(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}}=2^{32}.3^{16}$$

where α, β, γ are distinct natural number, then number of triplets of (α, β, γ) is

Answer (55)

Sol. $A=\begin{bmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ \beta+\gamma & \alpha+\gamma & \alpha+\beta\end{bmatrix}$ $R_3 \to R_3 + R_1$

$$\Rightarrow |A|=(\alpha+\beta+\gamma)\begin{vmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ 1 & 1 & 1\end{vmatrix}$$

$$=(\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$$

$\because \quad \left|adj\left(adj\left(adj\left(adjA\right)\right)\right)\right|=|A|^{24}=|A|^{16}$

Clearly $(\alpha+\beta+\gamma)^{16}=2^{32}\cdot 3^{16}$

$\Rightarrow \alpha+\beta+\gamma=12$

Number of positive integral solutions $= {}^{11}C_2 = 55$

22. Let n^{th} term of any sequence is given by

$$T_n=\frac{-1^3+2^3-3^3+4^3.....+(2n)^3}{n(4n+3)}$$ then

$\sum_{n=1}^{15} T_n$ is equal to

Answer (120)

Sol. $$T_n=\frac{2\left[2^3+4^3+...+(2n)^3\right]-\left[1^3+2^3+3^3+...+(2n)^3\right]}{n(4n+3)}$$

$$\Rightarrow T_n=\frac{16\left(\frac{n(n+1)}{2}\right)^2-\left(\frac{2n(2n+1)}{2}\right)^2}{n(4n+3)}=\frac{n^2(4n+3)}{n(4n+3)}=n$$

So, $\sum_{n=1}^{15} T_n=\frac{15\times 16}{2}=120$

23. Let $\frac{1+i\sin\alpha}{1-2i\sin\alpha}$ is purely imaginary and $\frac{1+i\cos\beta}{1-2i\cos\beta}$ is purely real, where $\alpha, \beta \in [\pi, 2\pi]$ and $z = \sin2\alpha + i\cos 2\beta$, then $\sum\left(iz+\frac{1}{i\bar{z}}\right)$ is equal to

Answer (1)

Sol. $\because \quad \frac{1+i\sin\alpha}{1-2i\sin\alpha}$ is purely imaginary, so

$1-2\sin^2\alpha=0 \Rightarrow \sin^2\alpha=\frac{1}{2} \Rightarrow \alpha=\frac{5\pi}{4}$ and $\frac{7\pi}{4}$

Similarity $\frac{1+i\cos\beta}{1-2i\cos\beta}$ is purely real, so

$\cos\beta=0 \Rightarrow \beta=\frac{3\pi}{2}$

Now $z = \sin2\alpha + i\cos2\beta \Rightarrow z = 1 - i$ or $-1 - i$

$\because \quad \frac{1}{i\bar{z}}=\frac{-iz}{|z|^2}=-\frac{iz}{2}$

So $\sum\left(iz+\frac{1}{i\bar{z}}\right)=\sum\frac{iz}{2}=\frac{i}{2}\Sigma z=\frac{i}{2}(-2i)=1$

24. Let $\vec{a}, \vec{b}, \vec{c}$ be any three vectors such that $\vec{a}\times\vec{b}=4\vec{c}$, $\vec{b}\times\vec{c}=9\vec{a}$, $\vec{c}\times\vec{a}=\alpha\vec{b}$ and $|\vec{a}|+|\vec{b}|+|\vec{c}|=36$, then α is equal to

Answer (36)

Sol. Let a, b, c be the modulus of $\vec{a}$, $\vec{b}$ and $\vec{c}$ respectively.

$\therefore \quad \vec{a}, \vec{b}, \vec{c}$ are mutually perpendicular vectors.

$\because \quad |\vec{a}\times\vec{b}|=4|\vec{c}| \Rightarrow ab = 4c$

similarity $bc = 9a$

and $ca = \alpha b$

$abc = 36\alpha$

Then $c=3\sqrt{\alpha}$, $a=2\sqrt{\alpha}$ and $b=6$

$\therefore \quad a+b+c=36$

$\Rightarrow 5\sqrt{\alpha}+6=36$

$\Rightarrow \alpha = 36$

25.

26.

27.

28.

29.

30.