JEE-Main-26-07-2022-Shift-2 (Memory Based)

Physics

Question: Two projectiles are thrown with same initial velocity at angle 30° & 45° with horizontal. Find ratio of their ranges.

Options:

(a) $\frac{\sqrt{5}}{2}$

(b) $\frac{\sqrt{7}}{2}$

(c) $\frac{\sqrt{3}}{2}$

(d) $\frac{\sqrt{6}}{2}$

Answer: (c)

Solution:

$$R = \frac{u^2 \sin 2\theta}{g}$$

$$\Rightarrow \frac{R_1}{R_2} = \frac{\sin 60°}{\sin 90°} = \frac{\sqrt{3}}{2}$$

Question: Find radius of gyration of solid cylinder about an axis perpendicular to cylinder axis & passing through centre is

Options:

(a) $\sqrt{\frac{L^2}{2^2} + \frac{R^2}{4}}$

(b) $\sqrt{\frac{L^2}{3^2} + \frac{R^2}{4}}$

(c) $\sqrt{\frac{L^2}{1^2} + \frac{R^2}{4}}$

(d) $\sqrt{\frac{L^2}{1^2} - \frac{R^2}{2}}$

Answer: (c)

Solution:

Moment of inertia of a solid cylinder about transverse axis.

$$= \frac{1}{4}MR^2 + \frac{1}{12}ML^2$$

$$\therefore K = \sqrt{\frac{I}{M}} = \sqrt{\frac{R^2}{4} + \frac{L^2}{12}}$$

Question: Two bodies m_1 and m_2 are attracting each other with gravitational force. Acceleration of m_1 is a_1 when $m_1 = 2\ m_2$ and a_2 when $m_1 = 3\ m_2$. Find ratio of a_1 and a_2.

Options:

(a) $\frac{6}{2}$

(b) $\frac{4}{2}$

(c) $\frac{5}{2}$

(d) $\frac{3}{2}$

Answer: (d)

Solution:

$$a = \frac{F}{m_1} = \frac{Gm_2}{r^2}$$

So $a_1 = \frac{G(m_1/2)}{r^2}$ and $a_2 = \frac{G(m_1/3)}{r^2}$

$$\frac{a_1}{a_2} = \frac{3}{2}$$

Question: A mass 0.5 kg moving with 12 m/s collides with a wall elastically. Find time of collision if F = 100 N acts during collision.

Options:

(a) t = 0.16 s

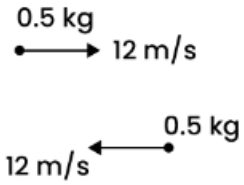
(b) t = 0.12 s

(c) t = 0.10 s

(d) t = 0.15 s

Answer: (b)

Solution:

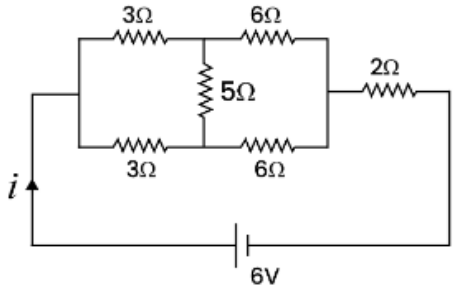


Change in momentum = 2mu = 2 x 0.5 x 12 = 12 kg m/s

$$F = \frac{dp}{dt} \quad 100 = \frac{12}{t}$$

$$\boxed{t = 0.12s}$$

Question: Find i



Options:

(a) 0.723 A

(b) 0.523 A

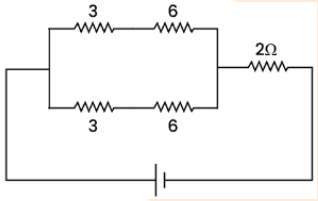
(c) 0.923 A

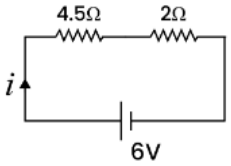
(d) 0.623 A

Answer: (c)

Solution:

wheat more bride





$$i = \frac{6}{4.5+2} = 0.923A$$

Question: Two springs connected with spring constant 3k and k in series have time period T_1, and in parallel have time period T_2. Ratio of T_1/T_2 is?

Options:

(a) 7.31

(b) 2.31

(c) 5.30

(d) 4.31

Answer: (b)

Solution:

$$\frac{1}{k_s} = \frac{1}{3k} + \frac{1}{k} = \frac{4}{3k} \Rightarrow k_s = \frac{3k}{4}$$

$$k_p = 3k + k = 4k \Rightarrow k_p = 4k$$

$$\because T = 2\pi\sqrt{\frac{m}{k}} \text{ so } T\alpha\frac{1}{\sqrt{k}}$$

So, $\frac{T_1}{T_2}=\sqrt{\frac{k_p}{k_s}}=\sqrt{\frac{4k}{3k/4}}=\frac{4}{\sqrt{3}}$

$=2.31$

Question: A coil having resistance 8Ω has flux varying with time as $\phi=\frac{2}{3}\left(9-t^2\right)$. Find heat produced in coil until flux becomes zero.

Options:

(a) H = 2J

(b) H = 5J

(c) H = 1J

(d) H = 3 J

Answer: (a)

Solution:

ϕ is zero at t = 3s

$\varepsilon=-\frac{d\phi}{dt}=\frac{2}{3}\times 2t=\frac{4}{3}t$

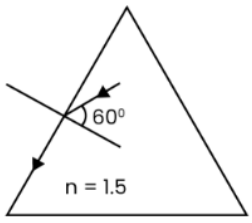
current $i=\frac{\varepsilon}{R}=\frac{4t}{3\times 8}=\frac{t}{6}A$

$H=\int_0^3 i^2R\,dt=\int_0^3\frac{t^2}{36}\times 8\,dt=\frac{8}{36}\times\frac{t^3}{3}\Big]_0^3$

$=\frac{8}{36}\times\frac{3^3}{3}=\frac{8}{36}\times 9$

$H=2J$

Question: A ray is incident inside glass prism, grazes after refraction as shown. Find refractive index of liquid.



Options:

(a) $\frac{4\sqrt{4}}{3}$

(b) $\frac{4\sqrt{3}}{4}$

(c) $\frac{3\sqrt{3}}{4}$

(d) $\frac{3\sqrt{5}}{4}$

Answer: (c)

Solution:

Snell's law

$v_1 \times \sin 60^o = v_2 \sin 90^o$

$1.5 \times \frac{\sqrt{3}}{2} = v_2 \times 1$

$v_2 = \frac{3\sqrt{3}}{4}$

Question: Two Nuclei have masses in ratio 4 : 3. Find ratio of there Nuclear Density?

Options:

(a) 4 : 1

(b) 1 : 1

(c) 6 : 1

(d) 2 : 2

Answer: (b)

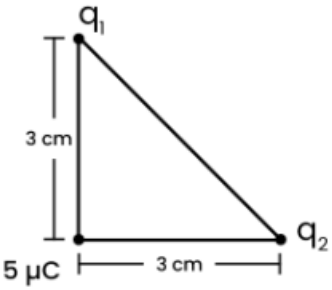
Solution:

Nuclear density is independent of nuclear mass.

Question: In the given figure, find the magnitude of force on 5μC charge is:

q_1 is 0.16μC

q_2 is 0.3 μC



Options:

(a) 14 N

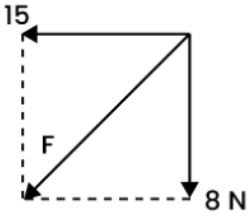
(b) 12 N

(c) 17 N

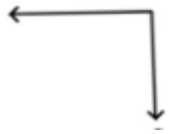
(d) 10 N

Answer: (c)

Solution:



$$\frac{9\times10^{9}\times5\times0.3\times10^{-12}}{9\times10^{-4}}$$



$$\frac{9\times10^{9}\times5\times0.16\times10^{-12}}{9\times10^{-4}}$$

$$F=\sqrt{8^{2}+15^{2}}=17N$$

Question: A body is projected from surface of earth with velocity $\frac{1}{3}rd$ of escape velocity.

Find maximum height achieved.

Options:

(a) $\frac{R}{2}$

(b) $\frac{R}{6}$

(c) $\frac{R}{8}$

(d) $\frac{R}{10}$

Answer: (c)

Solution:

$$\text{Escape velocity }=\sqrt{\frac{2GM}{R}}$$

$$\text{Velocity of projection }=\frac{1}{3}v_{e}=\frac{1}{3}\sqrt{\frac{2GM}{R}}$$

$$E_{i}=K+U$$

$$=\frac{1}{2}m\frac{1}{9}\left(\frac{2GM}{R}\right)-\frac{GmM}{R}$$

$$=\frac{GmM}{9R}-\frac{GmM}{R}=\frac{-8}{9}\frac{GmM}{R}$$

At maximum height (h)

$$E_{f}=0-\frac{GmM}{(R+h)}$$

$$E_i = E_f \Rightarrow -\frac{8}{9}\frac{GmM}{R} = -\frac{GmM}{(R+h)}$$

$$\Rightarrow (R+h) = \frac{9R}{8}$$

$$\Rightarrow 8R + 8h = 9R$$

$$h = \frac{R}{8}$$

Question: Maximum amplitude of AM modulated wave is 6 and minimum amplitude of AM modulated wave is 2, modulation index in percentage is x% find x.

Options:

(a) 10%

(b) 25%

(c) 35%

(d) 50%

Answer: (d)

Solution:

$$m = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}} = \frac{6-2}{6+2} = \frac{1}{2} = 50\%$$

Question: Two bodies with mass m and 8m have same kinetic energy. The ratio of their momentum is?

Options:

(a) 0.5

(b) 0.8

(c) 0.25

(d) 0.35

Answer: (d)

Solution:

$$P = \sqrt{2mKE}$$

$$\frac{P_1}{P_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{m}{9m}} = \frac{1}{2\sqrt{2}}$$

$$\frac{P_1}{P_2} = 0.35 \text{ (Approx.)}$$

Question: 0.5A nucleus of mass M splits into daughter nuclei $\frac{m'}{3}$ and $\frac{2m'}{3}$ $(m' < M)$. Find the ratio of de-Broglie wavelength of two daughter nuclei.

Options:

(a) λ are same

(b) λ of smaller part is more

(c) λ of bigger part is more

(d) Data insufficient

Answer: (a)

Solution:

As initially m' at rest,

$\frac{m'}{3}, \frac{2m'}{3}$ will have some momentum.

$\lambda = \frac{h}{p}$

Hence, λ are same.

Question: Find γ in terms of degree of freedom f.

Options:

(a) $1+\frac{2}{f}$

(b) $\frac{2}{f}$

(c) $1-\frac{2}{f}$

(d) $1-f$

Answer: (a)

Solution:

$\gamma = 1+\frac{2}{f}$

Question: $y = 2\sin(\omega t - kx)$ find λ such that wave velocity = maximum velocity of particle

Options:

(a) 2π

(b) 4π

(c) 7π

(d) 10π

Answer: (b)

Solution:

Wave velocity $= \frac{\omega}{k}$

Particle velocity (maximum) $= \omega A$

$\frac{\omega}{k} = \omega A$

$\frac{1}{k} = A = 2$

$k = \frac{1}{2}$

$$\therefore \frac{2\pi}{\lambda} = \frac{1}{2}$$

$$\boxed{\lambda = 4\pi}$$

Question: Breaking stress of a wire is increased by 2.5 times and tensile force is increased from 10 to 25 metric tonnes. If initial minimum cross section is $2.5\times10^{-4}m^2$ the minimum area to sustain the new load is

Options:

(a) $2.5\times10^{-4}m^2$

(b) $1.5\times10^{-4}m^2$

(c) $2.5\times10^{4}m^2$

(d) $2.5\times10^{2}m^2$

Answer: (a)

Solution:

$$\sigma = \frac{F}{A} \Rightarrow A = \left(\frac{F}{\sigma}\right)$$

$$\frac{A_1}{A_2} = \frac{2.5\times10^{-4}}{A_2} = \frac{\frac{10\times10^3 g}{\sigma}}{\frac{25\times10^3 g}{2.5\sigma}} = \frac{10}{25}\times 2.5$$

$$\frac{A_1}{A_2} = 1 \text{ so } A_2 = A_1 = 2.5\times10^{-4}m^2$$

Question: A light ray has speed $1.5\times10^8 m/s$ in medium 1 and $2\times10^8 m/s$ in medium 2. Find critical angle for system

Options:

(a) $\sin^{-1}\left(\frac{1}{2}\right)$

(b) $\sin^{-1}\left(\frac{2}{3}\right)$

(c) $\sin^{-1}\left(\frac{3}{4}\right)$

(d) $\cos^{-1}\left(\frac{1}{2}\right)$

Answer: (c)

Solution:

$$\mu = \frac{C}{V}; \frac{\mu_1}{\mu_2} = \frac{v_2}{v_1} = \frac{2\times10^8}{1.5\times10^8} = \frac{4}{3}$$

Also, $\sin i_c = \frac{\mu_2}{\mu_1}$

$\therefore i_c = \sin^{-1}\left(\frac{3}{4}\right)$

Question: The magnitude of magnetic field associated with an EM wave is 5×10^{-6}. The electric field magnitude is going to be;

Options:

(a) $1\frac{KN}{C}$

(b) $1.5\frac{KN}{C}$

(c) $2.5\frac{KN}{C}$

(d) $3.5\frac{KN}{C}$

Answer: (b)

Solution:

$E = BC$

$E = 5\times10^{-6}\times3\times10^{8}$

$= 15\times10^{2}$

$= 1.5\frac{KN}{C}$

Question: Projection of vector $\vec{A}$ on vector $\vec{B}$ is:

Options:

(a) $\left(\vec{A}.\hat{B}\right)\hat{A}$

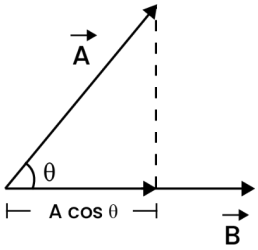
(b) $\left(\vec{A}.\hat{B}\right)\hat{B}$

(c) $\hat{A}$

(d) $|A|\hat{A}$

Answer: (b)

Solution:



$\vec{A}.\vec{B} = AB\cos\theta$

$A\cos\theta = \frac{\vec{A}.\vec{B}}{B}$

$A\cos\theta = \vec{A}.\hat{B}$

In vector from $\left(\vec{A}.\hat{B}\right)\hat{B}$

JEE-Main-26-07-2022-Shift-2 (Memory Based)

Chemistry

Question: Which of the following undergoes Vulcanization?
Options:
(a) Neoprene and sulphur
(b) Isoprene and sulphur
(c) Neoprene and styrene
(d) Isoprene and styrene
Answer: (b)
Solution: Vulcanization process consists of heating a mixture of natural rubber (Isoprene) with sulphur and an appropriate additive at a temperature range between 373 K to 415 K

Question: Correct order of covalent character of the following compound.
CaF_2, $CaBr_2$, $CaCl_2$, CaI_2
Options:
(a) $CaF_2 > CaBr_2 > CaCl_2 > CaI_2$
(b) $CaI_2 > CaBr_2 > CaCl_2 > CaF_2$
(c) $CaCl_2 > CaBr_2 > CaF_2 > CaI_2$
(d) $CaBr_2 > CaF_2 > CaCl_2 > CaI_2$
Answer: (b)
Solution: Cation is same in all the given compounds, while anions are different.
According to Fajan's rule, the larger the size of the anion, greater is the covalent character of the bond.
Therefore, correct order of covalent character is
$CaI_2 > CaBr_2 > CaCl_2 > CaF_2$

Question: Which of the following is not extracted from its sulphide ore?
Options:
(a) Aluminium
(b) Zinc
(c) Copper
(d) None of these
Answer: (a)
Solution: Aluminium is extracted from its oxide ore (Bauxite) by electrolysis

Question: Which of the following is other name of animal starch?
Options:
(a) Amylose
(b) Amylopectin
(c) Glycogen
(d) Maltose
Answer: (c)

Solution: Glycogen is the other name of animal starch.

Question: 0.34 percent iron by mass in hemoglobin find number of particles of iron in 3.3 g of hemoglobin.

Options:

(a) 2×10^{-4}

(b) 4×10^{-3}

(c) 2×10^{-2}

(d) 2

Answer: (a)

Solution: 0.34% iron by mass in hemoglobin

Weight of hemoglobin = 3.3 g

Mass of Fe in 3.3 g hemoglobin = $\frac{0.34}{100} \times 33 = 0.01122$ g

Number of particles of Fe atom = $\frac{0.01122}{56} = 0.000200 = 2 \times 10^{-4}$

Question: MnF_4, MnF_3, MnF_2 find magnetic moment of strongest oxidising agent.

Options:

(a) $2\sqrt{6}$

(b) $\sqrt{15}$

(c) $2\sqrt{2}$

(d) $\sqrt{35}$

Answer: (a)

Solution: Among MnF_4, MnF_3, MnF_2

MnF_3 is the strongest oxidizing agent

Mn^{3+} - Electronic configuration $3d^4$

Mn^{3+} | 1 | 1 | 1 | 1 | |

n = 4

$\mu = \sqrt{n(n+2)} = \sqrt{4(4+2)} = \sqrt{24} = 2\sqrt{6}$

Question: Which of the following has least melting point nearest to a metalloid?

Options:

(a) Al

(b) Ga

(c) Se

(d) B

Answer: (b)

Solution: Ga has the least melting point and B has the highest Melting point among the given elements.

Question: Assertion: LiF insoluble in water

Reason: LiF has low hydration enthalpy

Options:
(a) Both assertion and reason are true, reason is correct explanation of assertion
(b) Both assertion and reason are true, but reason is not a correct explanation of the assertion.
(c) Assertion is true, but reason is false.
(d) Assertion is false, but reason is true
Answer: (c)
Solution: The low solubility of LiF in water is due to its high lattice enthalpy
The hydration enthalpies of alkali metal ions decrease with increase in ionic sizes.
$Le^+ > Na^+ > K^+ > Rb^+ > Cs^+$

Question: Match the following.

Column-I	Column-II
(A) Micro organisms	(i) Strip mining
(B) Plants nutrients	(ii) Domestic sewage
(C) Toxic Heavy metals	(iii) Chemical fertilizers
(D) Sediment	(iv) Chemical industry

Options:
(a) A → (i); B → (iv); C → (iii); D → (ii)
(b) A → (ii); B → (iii); C → (iv); D → (i)
(c) A → (iii); B → (ii); C → (i); D → (iv)
(d) A → (iv); B → (i); C → (iii); D → (ii)
Answer: (b)
Solution:
(A) Micro organisms ⇒ Domestic sewage
(B) Plants nutrients ⇒ Chemical fertilizers
(C) Toxic Heavy metals ⇒ Chemical industry
(D) Sediment ⇒ Strip mining

Question: Which of the following are broad spectrum antibiotic?
Options:
(a) Penicillin
(b) Salvarsan
(c) Furacine
(d) Chloramphenicol
Answer: (d)
Solution: Chloramphenicol is broad spectrum antibiotic

Question: 100 ml of CH_3CH_2MgBr react with methanol to produce a gas of 2.24ml. The mass of the gas produced is
Options:
(a) 0.003 g
(b) 30 g
(c) 0.03 g
(d) 3 g
Answer: (a)
Solution:

$$CH_3OH + CH_3CH_2MgBr \rightarrow \underset{2.24\,mL}{CH_3 - CH_3} + CH_3OMgBr$$

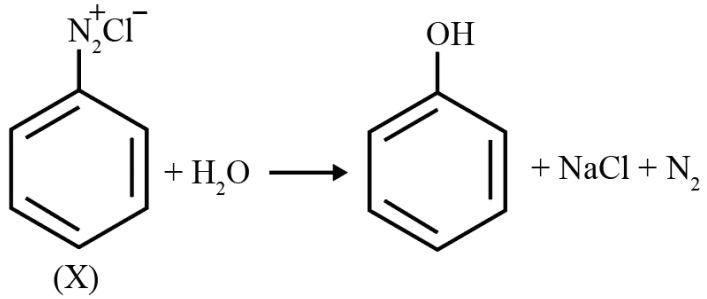
Gas formed is ethane

Number of moles of ethane formed = $\frac{2.24}{22400}$ = 0.0001 mole

Mass of ethane produced = 0.0001 × 30 = $\frac{30}{10000}$ = 0.003 g

Question: Assertion: Boric acid is a weak acid.
Reason: It is not able to release H^+ on its own. It receives OH^- ion from water molecule to complete its octet and in turn releases H^+ ions
Options:
(a) Both assertion and reason are true, reason is correct explanation of assertion
(b) Both assertion and reason are true, but reason is not a correct explanation of the assertion.
(c) Assertion is true, but reason is false.
(d) Assertion is false, but reason is true
Answer: (a)
Solution: Boric acid is a weak acid because it does not completely ionize in water or other aqueous solution. It is not able to release H^+ ions on its own because firstly it receives hydroxide ions (OH^-) from water molecule in order to complete its octet and then it releases H^+ ions.
Therefore, both the assertion and reason are true.

Question: Hydrolysis of X gives carbolic acid. Identify X
Options:
(a) Nitrobenzene
(b) Benzene diazonium chloride
(c) Benzene
(d) Benzyl chloride
Answer: (b)
Solution:

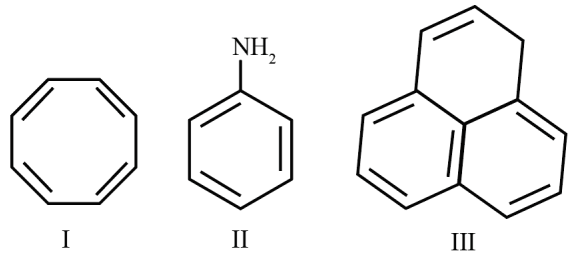


(X)

Question: Assertion: Phenolphthalein is a pH based indicator it is colourless in acidic solution and shows colour in basic solution.
Reason: Phenolphthalein is a weak base which do not dissociate.
Options:
(a) Both assertion and reason are true, reason is correct explanation of assertion
(b) Both assertion and reason are true, but reason is not a correct explanation of the assertion

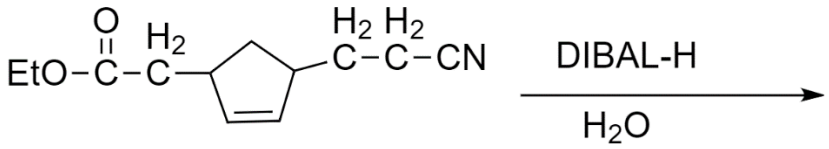
(c) Assertion is true, but reason is false
(d) Assertion is false, but reason is true
Answer: (c)
Solution: Phenolphthalein is colorless in acidic solution and shows pink color in basic solution. Phenolphthalein is a weak acid which dissociates in water.

Question: Which of the following is not a benzenoid structure?

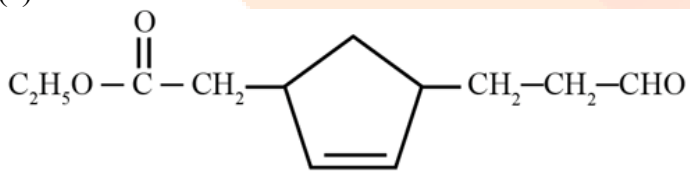


Options:
(a) I
(b) II
(c) III
(d) II and III
Answer: (a)
Solution: The compound which contains atleast 1 benzene ring in it is called benzenoid structure.

Question:



Options:
(a)



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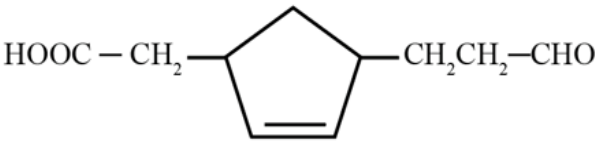
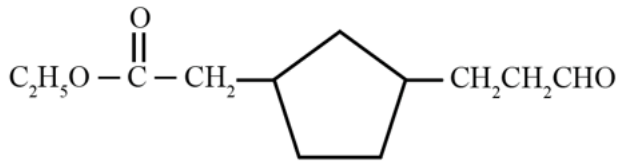
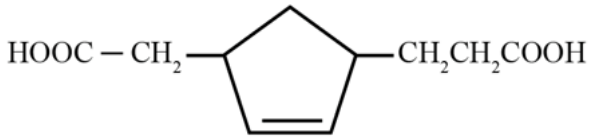


(c)

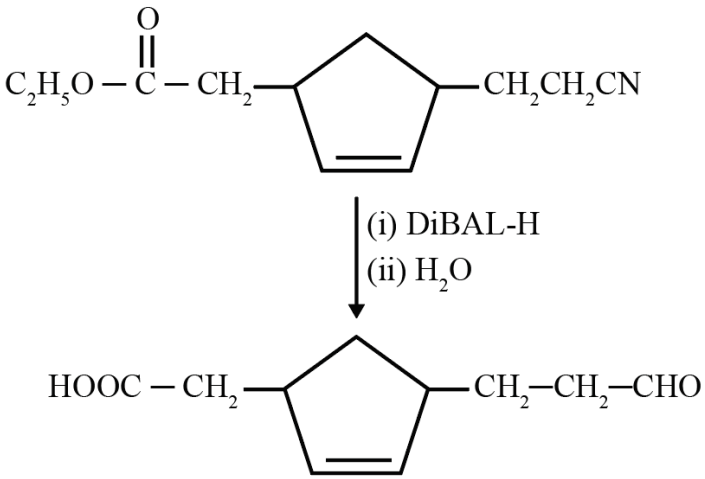


(d)

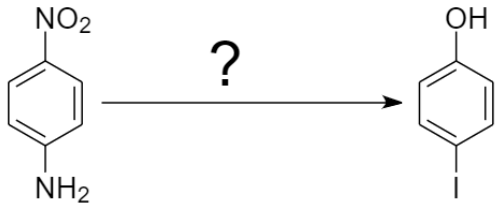


Answer: (b)

Solution:



Question:



Options:

(a) $NaNO_2$ + HCl, KI, Zn/HCl, $NaNO_2$ + HCl, H_2O (warm)

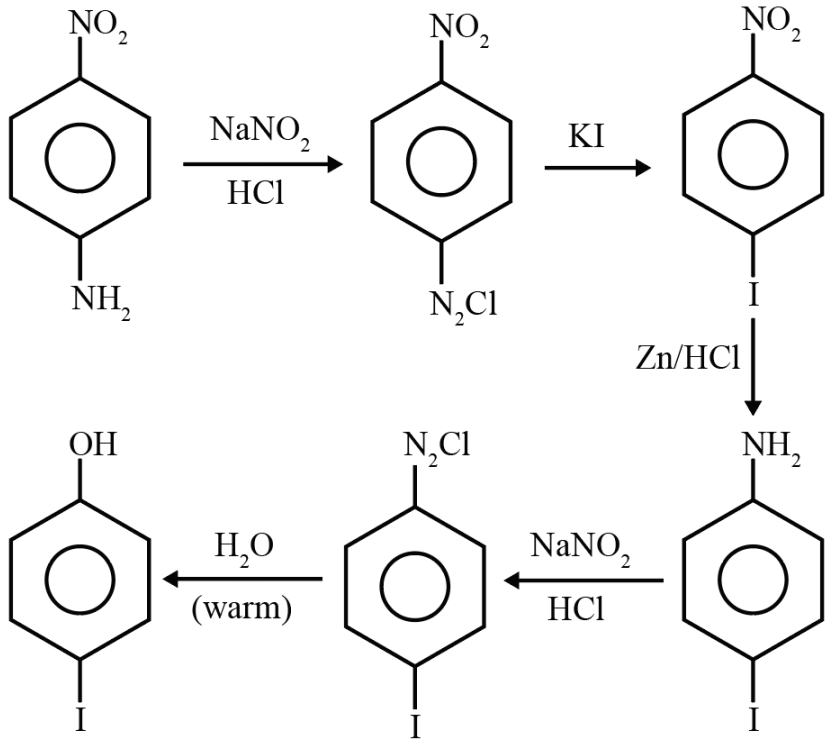
(b) $NaNO_2$ + HCl, H_2O (warm), Zn/HCl, $NaNO_2$ + HCl, KI

(c) $NaNO_2$ + HCl, Zn/HCl, H_2O (warm), $NaNO_2$ + HCl, KI

(d) Zn/HCl, $NaNO_2$ + HCl, H_2O (warm), $NaNO_2$ + HCl, KI

Answer: (a)

Solution:



Question: $CH_3COOH + PCl_3 \rightarrow A$
$A + PCl_3 \rightarrow B$
No of ionisable H^+ in B is
Answer: 2.00
Solution:

$$CH_3COOH + PCl_3 \rightarrow 3CH_3COCl + \underset{(A)}{H_3PO_3}$$

$$\underset{(A)}{H_3PO_3} + + PCl_3 + 2H_2O \rightarrow \underset{(B)}{H_4P_2O_5} + 3HCl$$

Number of ionisable hydrogens in $H_4P_2O_5$ is 2

Question: ΔT_b of 1 molal solution is 3K, ΔT_f of 2 molal solution is 6K. Find K_b/k_f =?
Answer: 1.00
Solution:
$\Delta T_b = 3$ K, $m_b = 1$ molal
$\Delta T_f = 6$ K, $m_f = 2$ molal
$\Delta T_b = K_b m_b$ ….. (1)
$\Delta T_f = K_f m_f$ ….. (2)
Dividing eq. (1) and (2)

$$\frac{K_b}{K_f} = \frac{\Delta T_b}{\Delta T_f} \times \frac{m_f}{m_b}$$

$$= \frac{3}{6} \times \frac{2}{1} = 1$$

Question: $t_{1/2}$ of reaction is 200 sec, find the time taken for 80 percent completion of reaction (Round off to nearest integer)

Answer: 464.00
Solution: $t_{1/2}$ = 200 sec

$$t_{1/2} = \frac{0.693}{k}$$

$$k = \frac{0.693}{200} s^{-1}$$

$$k = \frac{2.303}{t} \log \frac{[R]_o}{[R]}$$

$$\frac{0.693}{200} = \frac{2.303}{t} \log \frac{100}{100-80}$$

$$\frac{0.693}{200} = \frac{2.303}{t} \log \frac{100}{20}$$

$$t = 463.9 \approx 464.00 \text{ sec}$$

Question: Number of compounds including stereoisomers formed on monochlorination of cyclohexane
Answer: 1.00
Solution: Only one compound including stereoisomers is formed by monochlorination of cyclohexane as all the carbon atoms are exactly same.

Question: How many of the following are diamagnetic species?
I. $K_3[Fe(F_6)]$
II. $K_4[Fe(CN_6)]$
III. $K_3[Cu(CN)_4]$
IV. $K_2[Cu(CN)_4]$
Answer: 2.00
Solution:
$K_4[Fe(CN_6)]$ is diamagnetic
$K_3[Cu(CN)_4]$ is diamagnetic
$K_2[Cu(CN)_4]$ is paramagnetic as it has one unpaired electron
$K_3[Fe(F_6)]$ is paramagnetic as it has 5 unpaired electrons.
Therefore, only 2 compounds $K_4[Fe(CN_6)]$ and $K_3[Cu(CN)_4]$ are diamagnetic

JEE-Main-26-07-2022-Shift-2 (Memory Based)

MATHEMATICS

Question: The interval in which abscissa of point P on $y=x^2$ lies such that its distance from $(x-1)^2+(y+1)^2=1$ is minimum is:

Options:

(a) $0<x<\frac{1}{4}$

(b) $\frac{1}{4}<x<\frac{1}{2}$

(c) $\frac{1}{2}<x<\frac{3}{4}$

(d) $\frac{3}{4}<x<1$

Answer: (b)

Solution:

Let $P(x,x^2)$

Distance of P from given circle:

$l=\sqrt{(x-1)^2+(x^2+1)^2}-1$

For least value of l, we need to minimize:

$f(x)=(x-1)^2+(x^2+1)^2$

$f'(x)=2(x-1)+4x(x^2+1)$

$=2\left[2x^3+3x-1\right]=0$

$\because f'\left(\frac{1}{4}\right)$ is -ve and $f'\left(\frac{1}{2}\right)$ is +ve

So, $f'(x)=0$ for some $x\in\left(\frac{1}{4},\frac{1}{2}\right)$

Question: If $z=x+iy, |z|-2=0$ and $|z-i|-|z+5i|=0,$ then which of the following is TRUE:

Options:

(a) $x^2+2y+4=0$

(b) $x^2-2y+4=0$

(c) $x+y=0$

(d) $x^2-y+4=0$

Answer: (a)

Solution:

As $z = x + iy$

$x^2 + y^2 = 4 \quad ....(1)$

And $y = -2 \quad ....(2)$

So, $x = 0$

Hence, only $x^2 + 2y + 4 = 0$ is true.

Question: $x \sim B(n, p)$, mean $= 4$, variance $= \frac{4}{3}$, find $P(x \le 2)$.

Answer: $\frac{73}{729}$

Solution:

Given, $np = 4$

$npq = \frac{4}{3}$

$\therefore q = \frac{1}{3}$

$q = \frac{2}{3}$

Thus, $n = 6$

Now, $P(x \le 2) = P(x = 0) + p(x = 1) + P(x = 2)$

$= {}^6C_0 p^0 q^6 + {}^6C_1 p^1 q^5 + {}^6C_2 p^2 q^4$

$= \left(\frac{1}{3}\right)^6 + 6\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^5 + 15\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4$

Question: Find area between $y = |x^2 - 1|$ & $y = 1$.

Answer: $\frac{4}{3}\left(\sqrt{2} - 1\right)$

Solution:

$\int_0^1 -\sqrt{1-y} + \sqrt{1+y}$

$\frac{2}{3}(1-y)^{\frac{3}{2}} + \frac{2}{3}(1+y)^{\frac{3}{2}}\Big|_0^1$

$\frac{2}{3} \times 2^{\frac{3}{2}} - \left(\frac{4}{3}\right)$

$\frac{4\sqrt{2}}{3} - \frac{4}{3} = \frac{4}{3}\left(\sqrt{2} - 1\right)$

Question: How 4 digit numbers lying between 1000 & 3000 can be made which are divisible by 4, using digits 1, 2, 3, 4, 5, 6 with no repetition.

Answer: 30.00

Solution:

We will solve the Question in two cases.

Case I: When first digit is 1.

Then last two digits can be 24, 32, 36, 52, 56 and 64.

Number of such numbers $= 6 \times 3 = 18$

Case II: When first digit is 2

Then last two digits can be 16, 36, 56 or 64

Number of such numbers $= 4 \times 3 = 12$

Total numbers of numbers $= 18 + 12 = 30$

Question: $\int_0^{20\pi} \left(|\sin x| + |\cos x|\right)^2 dx$

Answer: $20\pi + 40$

Solution:

$$\int_0^{20\pi} \left(|\sin x| + |\cos x|\right)^2 dx$$

$$\Rightarrow \int_0^{20\pi} \left(\left(\sin^2 x + \cos^2 x\right) + |\sin 2x|\right) dx$$

$$\Rightarrow \int_0^{20\pi} 1\, dx + \int_0^{20\pi} |\sin 2x|\, dx$$

$$\Rightarrow 20\pi + 40\int_0^{\frac{\pi}{2}} \sin 2x\, dx$$

$$\Rightarrow 20\pi + 40\left[\frac{-\cos 2x}{2}\right]_0^{\frac{\pi}{2}}$$

$$\Rightarrow 20\pi + 20(1+1)$$

$$\Rightarrow 20\pi + 40$$

Question: Find equation of common tangent to $y = x^2$ & $y = -(x-2)^2$.

Answer: ()

Solution:

Given, $y = x^2$ & $y = -(x-2)^2$

Tangent for $y = x^2$

$y = mx - \frac{1}{4}m^2$

Tangent for $y = -(x-2)^2$

$y = m(x-2) + \frac{1}{4}m^2$

$y = mx - 2m + \frac{1}{4}m^2$

$-\frac{1}{4}m^2 = -2m + \frac{1}{4}m^2$ (For common tangents)

$2m = \frac{1}{2}m^2$

$m^2 - 4m = 0$

$m = 0, m = 4$

Thus, equation of tangents are $y = 0$ or $y = 4x - 4$

Question: If $\sin^{-1}\left(\frac{x}{\alpha}\right) = \cos^{-1}\left(\frac{x}{\beta}\right)$ then find value of $\sin\left(\frac{2\pi}{\alpha+\beta}\right)$.

Answer: ()

Solution:

Given, $\sin^{-1}\left(\frac{x}{\alpha}\right) = \cos^{-1}\left(\frac{x}{\beta}\right) = k$

$\Rightarrow \alpha = \frac{\sin^{-1}x}{k}, \beta = \frac{\cos^{-1}x}{k}$

$\therefore \sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right) = \sin\left(\frac{2\pi\frac{\sin^{-1}x}{k}}{\frac{\sin^{-1}x+\cos^{-1}x}{k}}\right)$

$\Rightarrow \sin\left(\frac{2\pi\left(\sin^{-1}x\right)}{\frac{\pi}{2}}\right)$

$\Rightarrow \sin\left(4\sin^{-1}x\right)$

$\Rightarrow \sin\left(2\left(\sin^{-1}\left(2x\sqrt{1-x^2}\right)\right)\right)$

$\Rightarrow 2\left(2x\sqrt{1-x^2}\right)\sqrt{1-\left(2x\sqrt{1-x^2}\right)^2}$

$\Rightarrow 4x\sqrt{1-x^2}\sqrt{1-4x^2\left(1-x^2\right)}$

$\Rightarrow 4x\sqrt{1-x^2}\left(2x^2-1\right)$

Question: $\ln 2 \times \dfrac{d}{dx}\left(\dfrac{\log \operatorname{cosec} x}{\log \cos x}\right)\Bigg|_{\frac{\pi}{4}}$

Answer: 4.00

Solution:

$$\ln 2 \times \frac{d}{dx}\left(\frac{\log \operatorname{cosec} x}{\log \cos x}\right)$$

$$\ln 2 \times \frac{d}{dx}\left(-\frac{\log \sin x}{\log \cos x}\right)$$

$$\ln 2\left(\frac{(\log \cos x)\left(-\dfrac{\cos x}{\sin x}\right)+\log \sin x\left(-\dfrac{\sin x}{\cos x}\right)}{(\log \cos x)^2}\right) \text{ at } x=\frac{\pi}{4}$$

$$\ln 2\left(\frac{-2\log \dfrac{1}{\sqrt{2}}}{\left(\log \dfrac{1}{\sqrt{2}}\right)^2}\right)$$

$$\frac{-2\ln 2}{\log 2^{-\frac{1}{2}}}=\frac{-2}{-\frac{1}{2}}=4$$

Question: If $\beta=\lim\limits_{x\to 0}\dfrac{\alpha x-\left(e^{3x}-1\right)}{\alpha x\left(e^{3x}-1\right)}$ then $\alpha+\beta=?$

Answer: $\dfrac{5}{2}$

Solution:

Given, $\beta=\lim\limits_{x\to 0}\dfrac{\alpha x-\left(e^{3x}-1\right)}{\alpha x\left(e^{3x}-1\right)}$

$$\beta=\lim_{x\to 0}\frac{\alpha x-\left(1+3x+\dfrac{9x^2}{2}-1\right)}{\alpha x\left(1+3x+\dfrac{9x^2}{2}-1\right)}$$

$$\beta=\lim_{x\to 0}\frac{x(\alpha-3)-\dfrac{9}{2}x^2}{\alpha x(3x)}$$

$$\beta=\frac{1}{3\alpha}\lim_{x\to 0}\frac{x(\alpha-3)-\dfrac{9}{2}x^2}{x^2}$$

$\therefore \alpha = 3,\ \beta = \frac{1}{3\times 3}\times\left(-\frac{9}{2}\right) = \frac{-1}{2}$

$\therefore \alpha + \beta = 3 - \frac{1}{2} = \frac{5}{2}$

Question: Find minimum value of sum of squares of roots of $x^2 + (3-a)x = 2a - 1$

Answer: 6.00

Solution:

Let α, β be the roots of the equation

$x^2 + (3-a)x + 1 - 2a = 0$

Then, $\alpha + \beta = a - 3,\ \alpha\beta = 1 - 2a$

$\therefore \alpha^2 + \beta^2 = (a-3)^2 - 2(1-2a)$

$= a^2 - 2a + 7$

$= (a-1)^2 + 6$

$\therefore$ Minimum value of $\alpha^2 + \beta^2 = 6$

Question: If $\sum_{k=1}^{10} \frac{k}{\left(k^4 + k^2 + 1\right)} = \frac{m}{n}$, such that m and n are coprime, then $m+n$ is equal to ___

Answer: $\frac{55}{111}$

Solution:

$\sum_{k=1}^{10} \frac{k}{\left(k^4 + k^2 + 1\right)} = \sum_{k=1}^{10} \frac{k}{\left(k^2 + k + 1\right)\left(k^2 - k + 1\right)}$

$= \sum_{k=1}^{10} \frac{1}{2}\left(\frac{1}{k^2 - k + 1} - \frac{1}{k^2 + k + 1}\right)$

$= \frac{1}{2}\left[\left(1 - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{7}\right) + \ldots + \left(\frac{1}{91} - \frac{1}{111}\right)\right]$

$= \frac{1}{2}\left(1 - \frac{1}{111}\right)$

$\sum_{k=1}^{10} \frac{k}{k^4 + k^2 + 1} = \frac{55}{111}$

$\therefore m = 55,\ n = 111$

$\therefore m + n = 166$

Question: If $A=\begin{bmatrix}1\\1\\1\end{bmatrix}, B=\begin{bmatrix}9^2 & 10^2 & 11^2\\12^2 & -13^2 & 14^2\\15^2 & 16^2 & -17^2\end{bmatrix}$, then $A'BA$ is equal to:

Answer: 665.00

Solution:

$$A=\begin{bmatrix}1\\1\\1\end{bmatrix}, B=\begin{bmatrix}9^2 & 10^2 & 11^2\\12^2 & -13^2 & 14^2\\15^2 & 16^2 & -17^2\end{bmatrix}$$

$$A'=\begin{bmatrix}1 & 1 & 1\end{bmatrix}$$

$$A'B=\begin{bmatrix}9^2+12^2+15^2 & 10^2-13^2+16^2 & 11^2+14^2-17^2\end{bmatrix}$$

$$A'BA=\begin{bmatrix}9^2+12^2+15^2 & 10^2-13^2+16^2 & 11^2+14^2-17^2\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$A'BA=\begin{bmatrix}9^2+12^2+15^2+10^2-13^2+16^2+11^2+14^2-17^2\end{bmatrix}$$

$$=\begin{bmatrix}665\end{bmatrix}$$

Question: If $ax^2+by^2+2gx+2fy+c=0$ is a circle whose diametric end points are given by $x^2-4x-9=0$ & $y^2+2x-4=0$ then find $a+b-c$.

Answer: 15.00

Solution:

Diametric points be (x_1,y_1) & (x_2,y_2) and equation of circle will be

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

$$\Rightarrow x^2-x(x_1+x_2)+x_1x_2+y^2-y(y_1+y_2)+y_1y_2=0$$

$$\Rightarrow x^2-x(4)+(-9)+y^2-y(-2)-4=0$$

$$\Rightarrow x^2+y^2-4x+2y-13=0$$

Composing with $ax^2+by^2+2gx+2fy+c=0$

$$a=1, b=1, c=-13$$

$$a+b-c=1+1+13=15$$

Question: Let $A=\{1,2,3,4,5,6\}, B=\{3,4,6,7,9\}$ and $C=A\cup B$, then number of elements in cartesian product of $C\times B$ is _____.

Answer: 40.00

Solution:

$A=\{1,2,3,4,5,6\}, B=\{3,4,6,7,9\}$

$\therefore C=A\cup B=\{1,2,3,4,5,6,7,9\}$

$\therefore n(C\times B)=8\times 5=40$

Question: $2\sin^2\theta-\cos 2\theta=0$, $2\cos^2\theta+3\sin\theta=0$. If sum of all solutions of θ in $[0,2\pi]$ is $k\pi$, then find k.

Answer: 3.00

Solution:

Given, $2\sin^2\theta-\cos 2\theta=0$

$2\sin^2\theta-1+2\sin^2\theta=0$

$4\sin^2\theta=1$

$\sin^2\theta=\frac{1}{4}$

$\sin\theta=\pm\frac{1}{2}$

$\theta=\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}$

$2\cos^2\theta+3\sin\theta=0$

$2-2\sin^2\theta+3\sin\theta=0$

$2\sin^2\theta-3\sin\theta-2=0$

$2\sin^2\theta-4\sin\theta+\sin\theta-2=0$

$(\sin\theta-2)(2\sin\theta+1)=0$

$\sin\theta=-\frac{1}{2}$

$\theta=\frac{7\pi}{6},\frac{11\pi}{6}$

$\therefore$ Sum of all value of common θ

$\frac{7\pi}{6}+\frac{11\pi}{6}=\frac{18\pi}{6}=3\pi$

$\therefore k=3$