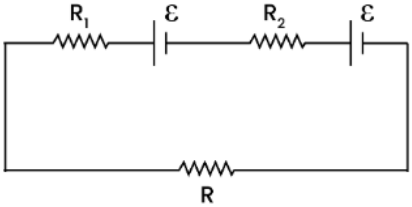
JEE-Main-27-07-2022-Shift-1 (Memory Based)

Physics

Question: Find R such that potential diff across 1st cell (on the left of the diagram) is zero.



Options:

(a) $R = R_1 + R_2$

(b) $R = R_1 - R_2$

(c) $R = R_2 - R_1$

(d) $R = R_2 + R_1$

Answer: (b)

Solution:

Current in the circuit

$$i = \frac{2\varepsilon}{R + R_1 + R_2}$$

P.D. across cell 1,

$$\varepsilon - iR_1 = 0$$

$$\varepsilon - \frac{2\varepsilon R_1}{R + R_1 + R_2} = 0$$

$$\varepsilon R + \varepsilon R_1 + \varepsilon R_2 = 2\varepsilon R_1$$

$$R = R_1 - R_2$$

Question: Two satellites of mass ratio 4:3 and radii ratio 3:4. Find the ratio of total mechanical energy.

Options:

(a) 1

(b) 3

(c) 5

(d) 2

Answer: (a)

Solution:

$$U + K = E$$

$$E = -\frac{GM_e m}{2r}$$

$$E\alpha\frac{m}{r} \Rightarrow \frac{E_1}{E_2} = \frac{m_1}{r_1}\frac{r_2}{m_2}$$

$$=\frac{4}{3}\times\frac{3}{4}=1$$

Question: Two charges Q each are placed at a distance of 2a. At midpoint, q is placed and is displaced slightly. Find time period.

Options:

(a) $T=4x\sqrt{\frac{a^3m}{4KQq}}$

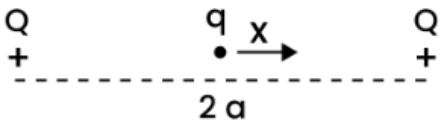
(b) $T=3x\sqrt{\frac{a^3m}{3KQq}}$

(c) $T=2x\sqrt{\frac{a^3m}{4KQq}}$

(d) $T=2x\sqrt{\frac{a^3m}{2KQq}}$

Answer: (c)

Solution:



$$a=\left(\frac{4KQq}{a^3m}\right)x$$

$$F_{Net}=\frac{KQq}{(a-x)^2}-\frac{KQq}{(a+x)^2}$$

$$=KQq\left[\frac{(a+x)^2-(a-x)^2}{(a-x)^2(a+x)^2}\right]$$

$$=KQq\frac{\left[(2a)(2x)\right]}{a^4}$$

$$\Rightarrow F=\frac{4KQq}{a^3}x$$

Question: A DC current of 4 A and AC current of peak value 4A passes through 3Ω and 2Ω resistors respectively. Find the ratio of heat generated.

Options:

(a) 3 : 1

(b) 3 : 2

(c) 3 : 4

(d) 1 : 1

Answer: (a)

Solution:

For DC current

$H_{DC}=i^2R_1t$

& for AC

$$H_{AC} = i_{rms}^{\ 2} R_2 t$$

$$\frac{H_{DC}}{H_{AC}} = \frac{i^2}{i_{rms}^{\ 2}} \frac{R_1}{R_2}$$

$$= \frac{(4)^2}{\left(\frac{4}{\sqrt{2}}\right)^2} \frac{3}{2} = 3:1$$

Question: Sand is falling on conveyer belt at rate of 0.5 kg is if conveyer is moving with 4 m/s. How much power is required maintain constant speed?

Options:

(a) 5 w

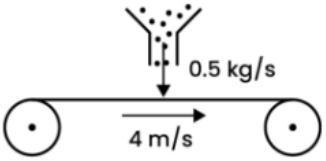
(b) 7 w

(c) 4 w

(d) 8 w

Answer: (d)

Solution:



$$\text{Force} = \frac{d}{dt}(p)$$

$$= \frac{d}{dt}(mv)$$

$$= v\frac{d}{dt}(m)$$

$$= v(0.5)$$

$$F = 4 \times 0.5 = 2$$

Power = Force x vel.

= 2 x 4

= 8W

Question: If activity of radioactive sample becomes $1/16^{th}$ of its initial value in 30 hrs. Find the half-life period.

Options:

(a) 5.5 hrs

(b) 3.5 hrs

(c) 7.5 hrs

(d) 4.5 hrs

Answer: (c)

Solution:

Activity $N = N.e^{-\lambda t}$

$$\frac{N}{N_0} = \frac{1}{16} \text{after } 30 \text{ hrs}$$

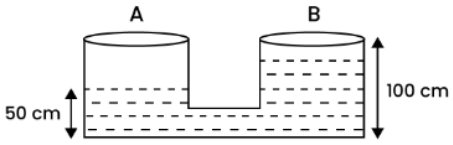
$$\frac{1}{16} = e^{-30\lambda}$$

$e^{30\lambda} = 16 \Rightarrow \lambda = \frac{\ln 16}{30}$

$\text{Also}, t\frac{1}{2} = \frac{\ln 2}{\lambda}$

$= \left(\frac{\ln 2}{\ln 16}\right) \times 30 = 7.5 hrs.$

Question: Two cylinders are joined as shown.



Water flows from B to A until water level becomes same. Find work done by gravity.

Options:

(a) $w = 625 A\rho g \times 10^{-4} J$

(b) $w = 225 A\rho g \times 10^{-4} J$

(c) $w = 425 A\rho g \times 10^{-4} J$

(d) $w = 125 A\rho g \times 10^{-4} J$

Answer: (a)

Solution:

$W = 625 \times 10^{-4} A\rho g$

Work done by gravity = $U_I - U_f$

$U_I = \left(A(50)\rho\right)(25) + A(100)\rho g(50) = A\rho g[6250]$

Common Height of cylinders $\Rightarrow h = 75 cm$

$U_f = \left(A(75)\rho g\right)\left(\frac{75}{2}\right) \times 2 = A\rho g[5625]$

$w = 625 A\rho g \times 10^{-4} J$

Question: A ball thrown vertically upwards. At same time another ball thrown at angle θ. If both remain in air for same time. Then ratio of maximum height.

Options:

(a) 2 : 3

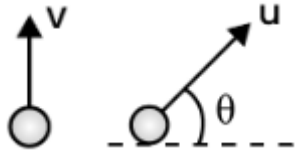
(b) 1 : 2

(c) 1 : 1

(d) 2 : 1

Answer: (c)

Solution:



$$T = \frac{2v}{g} \qquad T = \frac{2u\sin\theta}{g} \qquad \frac{(H_{ux})_1 = \frac{v^2}{2g}}{(H_{mx})_2 = \frac{u^2\sin 2\theta}{2g}}$$

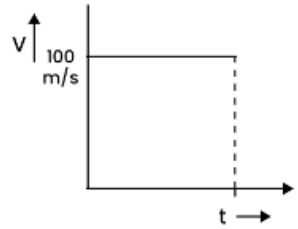
$$\frac{2v}{g} = \frac{2u\sin\theta}{g} \qquad \frac{H_1}{H_2} = \frac{v^2}{u^2\sin 2\theta}$$

$$v = u\sin\theta \qquad = \frac{1}{1}$$

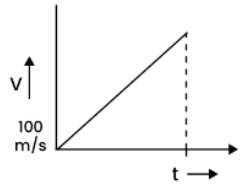
Question: A bullet is fired with velocity 100 m/s in vertically downward direction & on striking the ground it comes to rest. Draw v – t graph?

Options:

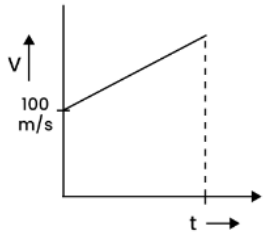
(a)



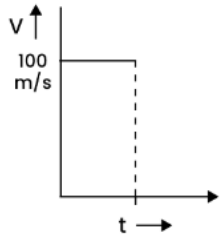
(b)



(c)

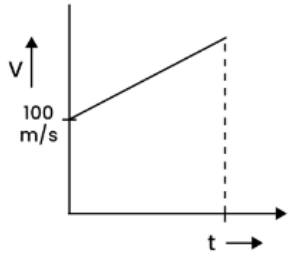


(d)



Answer: (c)

Solution:



$v = u + at$

$v = 100 + gt$

Question: The apparent angle of dip in a plane at an angle of 45° with magnetic meridian is 60° find true angle of dip

Options:

(a) $\tan^{-1}\sqrt{\frac{2}{1}}$

(b) $\tan^{-1}\sqrt{\frac{5}{2}}$

(c) $\tan^{-1}\sqrt{\frac{4}{2}}$

(d) $\tan^{-1}\sqrt{\frac{3}{2}}$

Answer: (d)

Solution:

Inclination of plane $(\alpha) = 45°$

Apparent dip $(\delta) = 60°$

Let true dip = ϕ

then we know

$\tan\delta = \frac{\tan\phi}{\cos\phi} \Rightarrow \tan\phi = \tan 60° \times \cos 45°$

$= \sqrt{\frac{3}{2}}$

Question: Intensity given I and 4I phase difference at A and B are 90 and 60. Then find the difference of resultant intensity at A and B

Options:

(a) 2I

(b) 5I

(c) 7I

(d) 9I

Answer: (a)

Solution:

Intensity at A

$I_A = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$\phi = 90°$

$I_A = I_1 + I_2$

$I_A = I + 4I = 5I \ ...(1)$

Intensity at B

$I_B = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

$\phi = 60°$

$I_B = I + 4I + 2\sqrt{I \times 4I} \times \frac{1}{2}$

$I_B = 7I$

Difference in Intensity

$\Delta I = I_B - I_A$

$= 7I - 5I = 2I$

Question: A tower of height 100m is used to transmit the signal. What is the increase in height of tower required to triple the range of transmitting signals.

Options:

(a) 200 m

(b) 300 m

(c) 500 m

(d) 800 m

Answer: (d)

Solution:

Range $= \sqrt{2Rh_T}$

For large to be 3 times

3 ×times $= \sqrt{2Rh_{T'}}$

$3 \times \sqrt{2 \times R \times 100} = \sqrt{2Rh_{T'}}$

$\sqrt{h_{T'}} = 30$

$h_{T'} = 900m$

So increase in length of the tower $= 900 - 100 = 800m$

Question: Two bar magnets oscillate in earth magnetic field with time period 3 : 4 and its moment of inertia is 3 : 2 then magnetic moment ratio.

Options:

(a) $\frac{8}{3}$

(b) $\frac{3}{8}$

(c) $\frac{5}{3}$

(d) $\frac{3}{5}$

Answer: (a)

Solution:

We know, Time period is given at

$T = 2H\sqrt{\frac{I}{\mu B}}$

Hence, $\frac{T_1}{T_2}=\sqrt{\left(\frac{I_1}{I_2}\right)\left(\frac{\mu_2}{\mu_1}\right)}$

$\frac{3}{4}=\sqrt{\frac{3}{2}\times\left(\frac{\mu_2}{\mu_1}\right)}$

$\frac{9}{16}=\frac{3}{2}\left(\frac{\mu_2}{\mu_1}\right)$

$\frac{\mu_2}{\mu_1}=\frac{3}{8}$

$\frac{\mu_1}{\mu_2}=\frac{8}{3}$

Question: If a compound microscope is taken from air to liquid with RI = 2, % change in resolving power is

Options:

(a) 50%

(b) 100%

(c) 150%

(d) 250%

Answer: (b)

Solution:

$R.P=\frac{1.22d}{\lambda}$

$\left(R.P\right)_1=\frac{1.22d}{\lambda}$

$\left(R.P\right)_2=\frac{2\times1.22d}{\lambda}$

% crave $=\frac{\left(R.P\right)_2-\left(R.P\right)_1}{\left(R.P\right)_T}\times100=\frac{2-1}{1}\times100=100\%$

Question: A block is placed on conveyor belt gently, which is moving with constant velocity 2 m/s. Coefficient of friction between belt and block is 0.4. Calculate the distance travelled by block till it comes at rest w.r.t. belt.

Options:

(a) 0.1 m

(b) 0.3 m

(c) 0.5 m

(d) 0.7 m

Answer: (c)

Solution:

Deceleration due to friction $=\mu g$

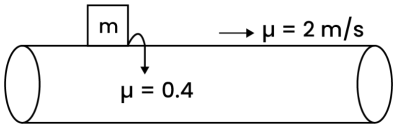
$=0.4\left(10\right)=4m/s^2$

Final speed w.r.t. belt = 0

Initial speed w.r.t. belt = -2 m/s

$v^2-u^2=2as\Rightarrow0-4=2\left(-4\right)s$

$s = 0.5m$



For block to be in rest w.r.t belt, both should give together so
$a = \mu g$ (maximum possible acceleration for them to move together)

Hence, $v^2 = u^2 + 2as$

$0 = u^2 - 2as$

$$s = \frac{u^2}{2a}$$

$$s = \frac{4}{2 \times 0.4 \times 10}$$

$$s = \frac{1}{2} = 0.5m$$

Question: In a meter bridge, balancing is achieved when jockey is at mark of 30 cm, where a known resistance of $5.6k\Omega$ is used in the right gap. Value of unknown resistance in $k\Omega$ is,

Options:

(a) 1.2
(b) 3.2
(c) 2.4
(d) 5.4

Answer: (c)

Solution:

$$\frac{R_1}{l_1} = \frac{R_2}{(100 - l_1)}$$

$$\frac{R_1}{30} = \frac{5.6}{(100 - 30)}$$

$$R_1 = \frac{5.6 \times 30}{79 \times 10}$$

$$R_1 = 2.4\Omega$$

Question: If mass, length and time each has 5% error then what is the error in reading of torque?

Options:

(a) 10%
(b) 5%
(c) 20%
(d) 25%

Answer: (d)

Solution:

Torque $= ML^2T^{-2}$

$\therefore$ Percentage error in torque
= % error in mass
2 (% error in length)

2(% error in time)
$= 5 + 2(5) + 2(5) = 25\%$

Question: Two containers contains identical at same temperature and volume. Number of moles of gas in each container are 1 and 3 respectively. Ratios of v_{rms} and pressure of gas in two containers respectively are

Options:
(a) 1 : 1, 3 : 1
(b) 3 : 1, 1 : 1
(c) 1 : 3, 1 : 1
(d) 1 : 1, 1 : 3

Answer: (d)

Solution:

$$v_{rms} = \sqrt{\frac{3k_B T}{m}}$$

As T and m are same $\frac{v_{rms},1}{v_{rms},2} = 1$

$$P = \frac{1}{3}\rho v_{rms}^2 = \frac{1}{3}\frac{nM}{v}v_{rms}^2$$

$$\therefore \frac{P_1}{P_2} = \frac{n_1}{n_2} = \frac{1}{3}$$

Question: A charge is moving with the velocity $3\times10^7\, m/s$ along y axis in an Em wave moving along x axis. Find the ratio of electric force and magnetic force exerted by the EM wave

Options:
(a) 10 : 1
(b) 1 : 10
(c) 1 : 5
(d) 1 : 6

Answer: (a)

Solution:

Magnetic force on a charge particle $F_B = qvB$

Electric force on a charge particle $F_E = qE = qcB$

So, $\frac{F_E}{F_B} = \frac{c}{v} \Rightarrow \boxed{\frac{F_E}{F_B} = \frac{10}{1}}$

Question: A cylinder having volume charge density ρ is uniformity charged. Find electric field at inside point $r = \frac{2 \in_0}{\rho}$

Options:

(a) $0NC^1$

(b) $1NC^{-1}$

(c) $3NC^{-1}$

(d) $2NC^{-1}$

Answer: (b)

Solution:

Electric field at any point inside the cylinder

$$E = \frac{\rho r}{2 \in_0}$$

Given: $r = \frac{2 \in_0}{\rho}$

So, $E = \frac{\rho}{2 \in_0} \times \frac{2 \in_0}{\rho} = 1N/C$

JEE-Main-27-07-2022-Shift-1 (Memory Based)

Chemistry

Question: Match the following.

(Column I) Ions	**(Column II)** Disease
(A) Fluoride	**(i)** Damage kidney
(B) Lead	**(ii)** Brown mottling of teeth
(C) Sulphate	**(iii)** Blue Baby syndrome
(D) Nitrate	**(iv)** Laxative effect

Options:

(a) A → (i); B → (iii); C → (ii); D → (iv)

(b) A → (iv); B → (iii); C → (i); D → (ii)

(c) A → (iii); B → (ii); C → (iv); D → (i)

(d) A → (ii); B → (i); C → (iv); D → (iii)

Answer: (d)

Solution:

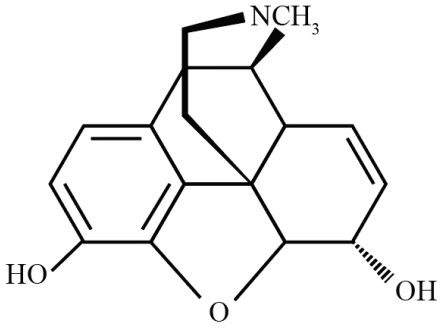
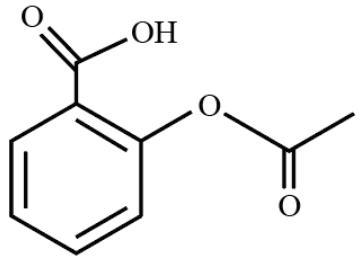
Fluoride ⇒ Brown mottling of teeth

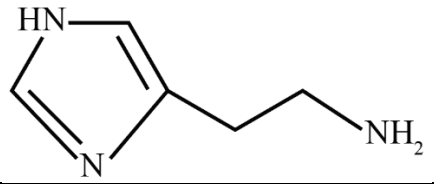
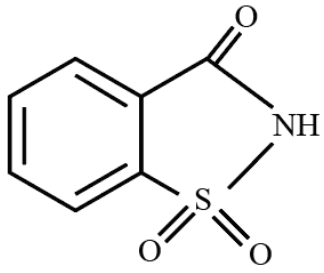
Lead ⇒ Damage kidney

Sulphate ⇒ Laxative effect

Nitrate ⇒ Blue Baby syndrome

Question: Match the following.

(Column I)	**(Column II)** Structures
(A) Antacids	**(i)** 
(B) Analgesic	**(ii)** 
(C) 500 times greater than sugar	**(iii)**

	
(D) Narcotics	**(iv)** 

Options:

(a) A → (i); B → (iii); C → (ii); D → (iv)

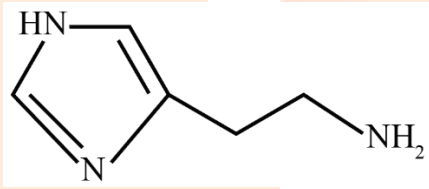
(b) A → (iv); B → (iii); C → (i); D → (ii)

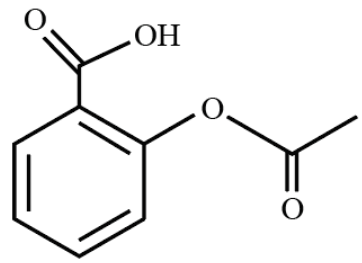
(c) A → (iii); B → (ii); C → (iv); D → (i)

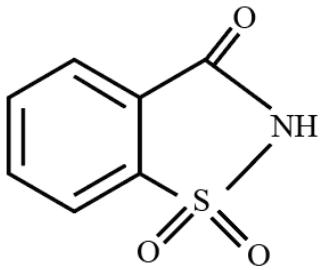
(d) A → (ii); B → (i); C → (iv); D → (iii)

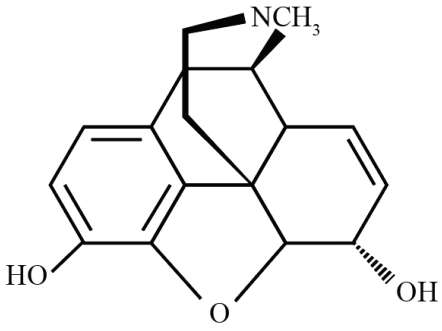
Answer: (c)

Solution:

(A) Antacids ⇒ 

(B) Analgesic ⇒ 

(C) 500 times greater than sugar ⇒ 

(D) Narcotics ⇒ 

Question: Statement -1: H_2O_2 can act as oxidizing agent in both acidic and basic medium
Statement-2: Density of H_2O at 298 K is less than D_2O
Options:
(a) Both statement 1 and 2 are correct
(b) Statement 1 is correct but statement 2 is incorrect
(c) Statement 1 is incorrect but statement 2 is correct
(d) Both statement 1 and 2 are incorrect.
Answer: (a)
Solution: H_2O_2 can act as oxidizing agent in both acidic and basic medium.
Density of H_2O at 298 K is less than that of D_2O
Therefore, both the statements are true.

Question: Sugar X is reacted with furfural and further reacted with Resorcinol to give a colored compound. Sugar X is
Options:
(a) Aldopentose
(b) Aldotetrose
(c) Carboxylic acid
(d) Ketotetrose
Answer: (d)
Solution: Seliwanoff's reagent is a mixture of resorcinol and concentrated hydrochloric acid. Ketose sugars react with the Seliwanoff's reagent to give immediately a deep cherry red color.

Question: In D-glucose find molality of Glucose if mass % is 10.8% and weight of solution is 250g.
Options:
(a) 0.6 m
(b) 0.06 m
(c) 6 m
(d) 2.5 m
Answer: (a)
Solution: Weight of solution = 250 g
Mass % = 10.8%

$$\frac{\text{Mass of glucose}}{\text{Mass of solution}} \times 100 = 10.8$$

$$\text{Mass of glucose} = \frac{10.8}{100} \times 250 = 27 \text{ g}$$

$$\text{Molality} = \frac{\text{Mass of glucose}}{\text{Molar mass of glucose}} \times \frac{1000}{\text{wt.of solution}}$$

$$= \frac{27}{180} \times \frac{1000}{250} = 0.6 \text{ m}$$

Question: Boiling point of non volatile solution A and B are same. Mass percent of sol A is 2% and mass percent of sol B is 3%. What is the ratio of their Mol. mass?

Options:

(a) $M_A = 4M_B$

(b) $M_B = 4M_A$

(c) $3M_A = 2M_B$

(d) $3M_B = 2M_A$

Answer: (c)

Solution:

$(\Delta T_b)_A = (\Delta T_b)_B$

$K_b\, m_A = K_b\, m_B$

$$\frac{2}{M_A \times 98} = \frac{3}{M_B \times 97}$$

$$2M_B = 3M_A$$

Question: Statement-1: Cl^- of $Al(Al_2Cl_6)$ and $Be(BeCl_2)$ form bridged structure and acts as Lewis base

Statement-2: Hydrolysis of aluminium and beryllium reacts with excess of alkalis to give beryllate and aluminate ion

Options:

(a) Both statement 1 and 2 are correct

(b) Statement 1 is correct but statement 2 is incorrect

(c) Statement 1 is incorrect but statement 2 is correct

(d) Both statement 1 and 2 are incorrect

Answer: (c)

Solution: Beryllium hydroxide dissolves in excess of alkali to give a beryllate ion, $[Be(OH)_4]^{2-}$ just as aluminium hydroxide gives aluminate ion, $[Al(OH)_4]^-$.

The chlorides of both beryllium and aluminium have Cl^- bridged chloride structure in vapour phase. Both the chlorides are soluble in organic solvents and are strong Lewis acids. They are used as Friedel Craft catalysts.

Question: The name of oxyacid of phosphorus having max no. of oxygen

Options:

(a) Hypophosphorous acid

(b) Pyrophosphoric acid

(c) Phosphorus acid

(d) Phosphoric acid

Answer: (b)

Solution:
Hypophosphorous acid ⇒ H_3PO_2
Pyrophosphoric acid ⇒ $H_4P_2O_7$
Phosphorus acid ⇒ H_3PO_3
Phosphoric acid ⇒ H_3PO_4

Question: What are the monomer of Glyptal, Buna-S?
Options:
(a) Glyptal – Phenol, Formaldehyde; Buna-S – Styrene
(b) Glyptal – Ethylene, Glycol, Phthalic acid; Buna-S – 1,3-Butadiene, Styrene
(c) Glyptal – Phenol, Phthalic acid; Buna-S – 1,3 Butadiene, Acrylonitrile
(d) Glyptal – Urea, Formaldehyde; Buna-S – 1,3 Butadiene, Propene
Answer: (b)
Solution: Monomers of Glyptal are Ethylene Glycol, Phthalic acid
Monomers of Buna-S are 1,3-Butadiene, Styrene

Question: Find out the solubility product of CaF_2 if solubility of CaF_2 is 2.34 g/100 mL
Options:
(a) 0.108 $(mol/L)^3$
(b) 0.072 $(mol/L)^3$
(c) 0.036 $(mol/L)^3$
(d) 0.032 $(mol/L)^3$
Answer: (a)
Solution: Solubility = $\frac{2.34}{100}$ g/ml
Molar mass of CaF_2 = 40 + 19 + 19 = 78 g/ml
Solubility in (mol/L) = $\frac{2.34}{78} \times \frac{1000}{100}$ mol / L = 0.3mol / L

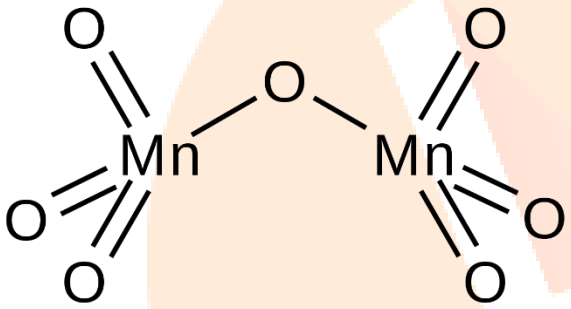
$CaF_2 \rightleftharpoons Ca^{2+} + 2F^-$
s 2s
$K_{sp} = s \times (2s)^2 = 4s^3$
$K_{sp} = 4 \times (0.3)^3 = 0.108\ (mol/L)^3$

Question: Change in oxidation state of C when oxalic acid reacts with acidic $KMnO_4$
Options:
(a) 5
(b) 2
(c) 1
(d) 3
Answer: (c)
Solution: $2MnO_4^- + 16H^+ + 5\overset{+3}{C}_2O_4^{2-} \rightarrow 2Mn^{2+} + 10\overset{+4}{C}O_2 + 8H_2O$

Question: Assertion: Hydrogen's 2s orbital has more energy than lithium's 2s orbital
Reason: For same orbital, as atomic number increases energy decreases
Options:

(a) Both assertion and reason are true, reason is correct explanation of assertion.
(b) Both assertion and reason are true, but reason is not a correct explanation of the assertion.
(c) Assertion is true, but reason is false
(d) Assertion is false, but reason is true
Answer: (a)
Solution: An increase of atomic number is in the order, H < Li
Energies of the orbitals in the same subshell decrease with an increase in the atomic number.
Hence, 2s orbital (energy level) of Li will be less than H

Question: The number of Mn = O bond in Mn_2O_7
Options:
(a) 7
(b) 4
(c) 5
(d) 6
Answer: (d)
Solution:



Question: After 30 sec, product was 1/16 of reactant. Calculate half life?
Answer: 330.00
Solution:

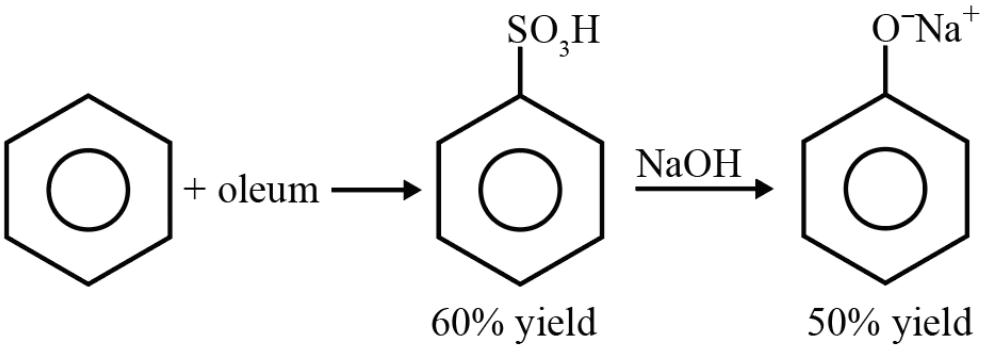
$$k = \frac{2.303}{t}\log\frac{[R]_o}{[R]}$$

$$k = \frac{2.303}{30}\log\frac{16}{15}$$

$k = 0.0021$

$$t_{1/2} = \frac{0.693}{0.0021} = 330\ s$$

Question: The percentage yield of the complete reaction is



Answer: 30.00

Solution: Percentage yield of complete reaction $= \frac{60}{100} \times \frac{50}{100} \times 100 = 30\%$

Question: Number of species having identical Bond order
CN^-, NO^+, O_2, O_2^+, O_2^{2+}
Answer: 3.00
Solution:
1) $O_2(16) = \sigma 1s^2\ \sigma^*1s^2\ \sigma 2s^2\ \sigma^*2s^2\ 2pz^2\ \pi 2px^2 = \pi 2py^2, \pi^*2px^1 = \pi^*2py^1$

Bond order $= \frac{10-6}{2} = 2$

2) $NO^+(14)$ = Bond order $= \frac{10-4}{2} = 3$

3) O_2^+ (15) = Bond order $= \frac{10-5}{2} = 2.5$

4) O_2^{2+} (14) = Bond order $= \frac{10-4}{2} = 3$

5) CN^- (14) = Bond order $= \frac{10-4}{2} = 3$

NO^+, CN^-, and O_2^{2+} has identical bond order

Question: How many of the following are not the ways to purify metal?
Distillation, Liquation, Electrolysis, Leaching, Calcination
Answer: 2.00
Solution: Leaching and calcination are not the method to purify metal.

Question: 20 ml of 0.02 M $K_2Cr_2O_7$ is titrated against 10 mL Fe^{2+} solution. The molarity of Fe^{2+} is ___ $\times 10^{-2}$.
Answer: 24.00
Solution: $Cr_2O_7^{2-} + Fe^{2+} + 14H^+ \rightarrow 2Cr^{3+} + Fe^{3+} + 7H_2O$
The redox changes involved are
i) $6e^- + Cr_2O_7^{2-} \rightarrow 2Cr^{3+}$ (n = 6)
ii) $Fe^{2+} \rightarrow Fe^{3+} + e^{-+}$ (n = 1)
Milliequivalent of $Cr_2O_7^{2-}$ = Milliequivalent of Fe^{2+}
$20 \times 0.02 \times 6 = 10 \times N$
N = 0.24

$$M = \frac{N}{'n'\text{factor}} = \frac{0.24}{1}$$

$= 0.24\ M$

$= 24 \times 10^{-2}\ M$

JEE-Main-27-07-2022-Shift-1 (Memory Based)

MATHEMATICS

Question: Let $a_1, a_2, a_3, ..., a_n$ be in A.P. The ratio of sum of first five term to the sum of first nine terms is $5:17$. Also $110 < a_{15} < 120$. Find the sum of first 10 terms of the A.P. (where all $a_i \left(i = 1, 2, 3, ..., n\right)$ are integers)

Options:

(a) 330

(b) 460

(c) 290

(d) 380

Answer: (d)

Solution:

Let first term be 'a' and common difference be 'd' for the A.P.

$$\frac{S_5}{S_9} = \frac{5}{17}$$

$$\Rightarrow \frac{\frac{5}{2}(2a + 4d)}{\frac{9}{2}(2a + 8d)} = \frac{5}{17}$$

$$\therefore 4a = d$$

$$a_{15} = a + 14d = 57a$$

It is given that $110 < a_{15} < 120$

$$\Rightarrow 110 < 57a < 120$$

For integral terms of the A.P, $a = 2$

Sum of 10 terms of A.P:

$$S_{10} = \frac{10}{2}(4 + 9 \times 8) = 380$$

Question: Let S be sample space for 5 digit numbers. If p is probability of a number being randomly selected which is multiple of 7 but not divisible by 5, then $9p$ is equal to:

Options:

(a) 1.0146

(b) 1.2085

(c) 1.0285

(d) 1.1521

Answer: (c)

Solution:

Five digit number line from 10000 to 99999

$\therefore S = 90000$

Number divisible by $7 = \frac{90000}{7}$

Number divisible by 7 and multiple by $5 = \frac{90000}{35}$

$\therefore$ Required Probability $= \frac{\frac{90000}{7} - \frac{90000}{35}}{90000}$

$\Rightarrow p = \frac{4}{35}$

$\therefore 9p = \frac{36}{35} = 1.02857$

Question: Let $A = \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix}$, α & β belongs to real numbers such that $\alpha A^2 + \beta A = 2I$, where I is an identity matrix of order 2×2. Then the value of $\alpha + \beta$ is equal to:

Options:

(a) -10

(b) -6

(c) 6

(d) 10

Answer: (d)

Solution:

$A = \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} = \begin{bmatrix} -3 & -8 \\ 8 & 21 \end{bmatrix}$

It is given that $\alpha A^2 + \beta A = 2I$

$\alpha \begin{bmatrix} -3 & -8 \\ 8 & 21 \end{bmatrix} + \beta \begin{bmatrix} 1 & 2 \\ -2 & -5 \end{bmatrix} = 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$-3\alpha + \beta = 2$ & $-8\alpha + 2\beta = 0$

$\alpha = 2,\ \beta = 8$

$\Rightarrow \alpha + \beta = 10$

Question: $(p \wedge r) \Leftrightarrow (p \wedge \sim q)$ which is equivalent to $\sim p$. Then r will be:

Options:

(a) p

(b) $\sim p$

(c) q

(d) $\sim q$

Answer: (c)

Solution:

The truth table is

p	q	$\sim p$	$\sim q$	$p \wedge q$	$p \wedge \sim q$	$p \wedge q \Leftrightarrow p \wedge \sim q$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	F	T
F	F	T	T	F	F	T

Clearly $(p \wedge r) \Leftrightarrow (p \wedge \sim q) \equiv \sim p$

$\therefore r = q$

Question: The remainder of $(2021)^{2022} + (2022)^{2021}$ when divided by 7 is:

Answer: 0.00

Solution:

Let $S = (2021)^{2022} + (2022)^{2021}$

$\Rightarrow (2023-2)^{2022} + (2023-1)^{2021}$

$= 7k_1 + 2^{2022} + 7k_2 - 1$

$= 7(k_1 + k_2) + 8^{674} - 1$

$= 7(k_1 + k_2) + (7+1)^{674} - 1$

$= 7(k_1 + k_2) + 7k_3 + 1 - 1$

$= 7(k_1 + k_2 + k_3)$

Therefore, S is divisible by 7

Question: The mean and variance of 10 observation was 15 and 15. The mistake was 25 instead of 15. The new standard deviation is:

Answer: ()

Solution:

$$\frac{x_1 + ... + x_9 + 25}{10} = 15$$

$$\frac{x_1 + ... + x_9 + 15}{10} = \text{ correct mean } = m$$

$$\frac{25}{10} - \frac{15}{10} = 15 - m$$

$m = 4$

Correct mean is 4.

$$\frac{x_1^2 + ... + x_9^2 + 25^2}{10} - 15^2 = 15$$

$\frac{x_1^2+....+x_9^2+15^2}{10}-14^2$ = correct variance = new SD

$\frac{25^2-15^2}{10}-\left(15^2-14^2\right)=15-v$

Variance is 4, SD is 2.

Question: Let $f(x)=2x^2-x-1$ and $S=\{n:|f(n)|\le 800\}$ where $n\in z$, then $\sum_{n\in s} f(n)=$

Answer: 10620.00

Solution:

$-800\le f(n)\le 800$

$-800\le 2n^2-n-1\le 800$

$2n^2-n+799\ge 0$

$a>0$

$D=1-4(2)(799)<0$

Always true

$n\in R$

$2n^2-n-801\le 0$

$n=\frac{1\pm\sqrt{1+4(2)(801)}}{4}$

$=\frac{1\pm\sqrt{6408}}{4}$

$n\approx\frac{1\pm 80}{4}$

$n=\frac{-79}{4},\frac{81}{4}$

$n\in[-19.75,\ 20.25]$

$n\in\{-19,-18,-17,....,-1,0,1,....,20\}$

$f(n)=2n^2-n-1$

$f(-19)=2(-19)^2-(-19)-1$

$\vdots$

$f(19)=2(19)^2-(19)-1$

$f(20)=2(20)^2-(20)-1$

$=2\left[(-19)^2+(-18)^2+...(-1)^2+0^2+(1)^2+...+19^2+20^2\right]$

$=-\left[(-19)+(-18)+...+(-1)+0+(1)+...+(19)+(20)\right]-40$

$$= 2\left[400 + 2\left(\frac{19 \times 20 \times 39}{6}\right)\right] - 20 - 40$$

$$= 10620$$