28/07/2022

Morning

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Memory Based
Answers & Solutions

Time : 3 hrs. *for* M.M. : 300

JEE (Main)-2022 (Online) Phase-2

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Half life of a radioactive sample is 0.3010 min. The ratio of initial activity to final activity after 2 min is equal to

(1) 10 (2) 1
(3) 100 (4) 20

Answer (3)

Sol. $A = A_0 e^{-\frac{\ln 2}{t_{1/2}} t}$

$$\frac{A_0}{A} = e^{\frac{0.693}{0.3010} \times 2}$$

= 100

2. A radioactive sample has concentration 64 times the allowed level. If half life of radioactive material in sample is 2 hours 30 min then after how much time the sample is safe for human exposure.

(1) 15 hrs (2) 9 hrs
(3) 6 hrs (4) 7.5 hrs

Answer (1)

Sol. $t = t_{1/2} \log_2 \left(\frac{A_0}{A}\right)$

= 2.5 $\log_2$ (64)

= 2.5 × 6 = 15 hrs

3. A wave has equation given as $y = 0.5\sin\left[\frac{2\pi}{3}(400t - x)\right]$. With x in meters and t in seconds the wave speed is equal to

(1) 100 m/s (2) 100π m/s
(3) 400 m/s (4) 200π m/s

Answer (3)

Sol. From equation of wave, $\omega = \frac{800\pi}{3}$ & $k = \frac{2\pi}{3}$

So, $v = \frac{\omega}{k} = 400$ m/s

4. In a medium with relative permittivity 1 and relative permeability 4 the speed of light is equal to

(1) 0.5 × 10^8 m/s (2) 1.5 × 10^8 m/s
(3) 2 × 10^8 m/s (4) 1 × 10^8 m/s

Answer (2)

Sol. $\mu = \sqrt{\varepsilon_r \times \mu_r}$

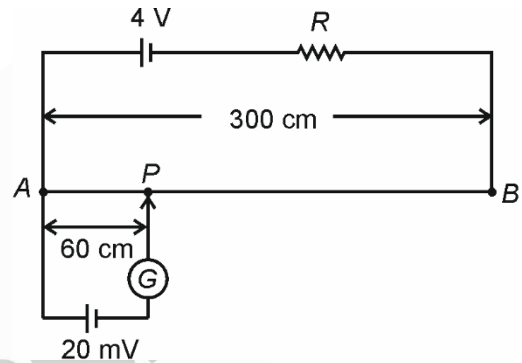
$= \sqrt{1 \times 4}$

= 2

So, $v = \frac{c}{\mu}$

= 1.5 × 10^8 m/s

5. In a potentiometer setup shown point P is point of null deflection. If wire AB has a resistance of 20 Ω then value of resistance R is



(1) 380 Ω (2) 480 Ω
(3) 680 Ω (4) 780 Ω

Answer (4)

Sol. $V_{AP} = 20$ mV

So, $\left(-\frac{dV}{dl}\right) = \frac{20 \times 10^{-3}}{60}$ V/cm

Thus, $V_{AB} = \left(-\frac{dV}{dl}\right) \times 300$

= 0.1 V

$$V_{AB} = \frac{R_{AB}E}{R_{AB} + R}$$

$$0.1 = \frac{20 \times 4}{20 + R}$$

R = 780 Ω

6. The force required to increase the length twice of a wire is (Given that cross section of wire is 1 cm^2 and Young modulus = 2 × 10^{11} N.m)

(1) 2 × 10^7 N (2) 1.5 × 10^7 N
(3) 1 × 10^7 N (4) 2.5 × 10^7 N

Answer (1)

Sol. $\Delta L = \dfrac{FL}{YA}$

So $F = YA = 2 \times 10^{11} \times 10^{-4}$ N

$= 2 \times 10^{7}$ N

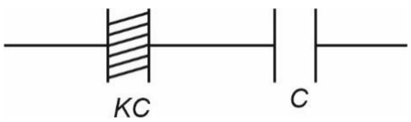
7. Two identical capacitors of 40 μF each are connected in series. Now a dielectric slab of dielectric constant *K* is inserted in one of them such the new equivalent capacitance come out 24 μF. Value of *K* is

(1) 2 (2) 1

(3) 1.5 (4) 3

Answer (3)

Sol.



$$C_{\text{net}} = \frac{KC \times C}{KC + C}$$

$$\frac{KC}{1+K} = 24\ \mu\text{F}$$

$$\frac{K}{1+K} \times 40\ \mu\text{F} = 24\ \mu\text{F}$$

$\Rightarrow$ K = 1.5

8. An apple falls from a height of 19.6 m from a tree and a man standing *s* distance away from the tree starts running with a speed of 2.5 m/s to catch the apple. To just catch the apple the distance *s* should be (g = 9.8 m/s^2)

(1) 5 m

(2) 10 m

(3) 15 m

(4) 20 m

Answer (1)

Sol. Time taken by apple to reach ground.

$$t = \sqrt{\frac{2h}{g}} = 2 \text{ sec}$$

So, $s = 2.5 \times 2 = 5$ m

9. Dimension formula of $\dfrac{Bo^2}{2\mu}$ is

(B_0 is magnetic field amplitude and μ is permeability)

(1) $[M^1L^2T^{-2}]$ (2) $[M^1L^{-1}T^{-2}]$

(3) $[M^1L^3T^{-1}]$ (4) $[ML^2T^{-1}]$

Answer (2)

Sol. $\dfrac{Bo^2}{2\mu}$ is the formula of energy density $\dfrac{dU}{dV}$

$$\text{So, } \frac{dU}{dV} = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}]$$

10. Efficiency of carnot cycle is 50%. Now the efficiency is increased by 30% of its earlier value when temperature of sink is reduced by 40°. So the temperature of source is

(1) 266.7 K (2) 300 K

(3) 366.7 K (4) 255 K

Answer (1)

Sol. Let the temperature of source is T_0 and sink is T

$$1 - \frac{T}{T_0} = \frac{1}{2} \text{ and } 1 - \frac{T-40}{T_0} = \frac{13}{20}$$

$$\text{So } \frac{1}{2} + \frac{40}{T_0} = \frac{13}{20}$$

$$\frac{40}{T_0} = \frac{3}{20}$$

$$T_0 = \frac{800}{3} = 266.67 \text{ K}$$

11. With wavelength 560 nm fringe width comes out to be 72 mm in a YDSE. Now if wavelength is charged so that new fringe width is 81 mm, then the new wavelength is equal to

(1) 490 nm (2) 630 nm

(3) 800 nm (4) 700 nm

Answer (2)

Sol. $\omega = \dfrac{\lambda D}{d}$

$\omega \propto \lambda$

$$\text{so } \frac{\omega_1}{\omega_2} = \frac{\lambda_1}{\lambda_2}$$

$$\frac{72}{81} = \frac{560}{\lambda_2}$$

λ_2 = 630 nm

12. Radius of earth is shrink by 2%. Now new acceleration due to gravity is

(1) Increases by 4% (2) Decreases by 4%

(3) Increases by 2% (4) Decreases by 2%

Answer (1)

Sol. $g = \dfrac{GM_e}{R^2}$

So $\dfrac{dg}{g} \times 100 = -2\dfrac{dR}{R} \times 100$

Given $\dfrac{dR}{R} \times 100 = -2\%$

So g will increase by 4%

13. Value of modulation index so that modulated signal does not get distorted

(1) $\mu \le 1$ (2) $\mu > 1$

(3) $\mu = 0$ (4) $\mu = 2$

Answer (1)

Sol. For signal does not get distorted

$A_m \le A_C$

So $\dfrac{A_m}{A_C} \le 1$

$\mu \le 1$

14. A train moves with a speed of 30 km/h with whistle frequency of 320 Hz, towards a wall. The frequency of echo heard by the driver is equal to

(Sound speed = 330 m/s)

(1) 336 (2) 350

(3) 300 (4) 280

Answer (1)

Sol. $f = \dfrac{v + v_t}{v - v_t} f_0$

$= \dfrac{330 + \dfrac{25}{3}}{330 - \dfrac{25}{3}} \times 320$

= 336 Hz

15. **Statement 1:** Net momentum of a ideal gas depends on temperature.

Statement 2: If initial V_{rms} of oxygen is V. Now temperature is double and oxygen atoms dissociate in two parts, so new V_{rms} is 2 V

(1) S1 $\rightarrow$ True, S2 $\rightarrow$ True

(2) S1 $\rightarrow$ True, S2 $\rightarrow$ False

(3) S1 $\rightarrow$ False, S2 $\rightarrow$ False

(4) S1 $\rightarrow$ False, S2 $\rightarrow$ True

Answer (4)

Sol. Net momentum of the ideal gas system is 0. So statement 1 is false.

For statement 2 using $V_{\text{rms}} = \sqrt{\dfrac{3RT}{M_0}}$

Temperature is doubled and molar mass is halved so rms speed is doubled.

16. A refracting telescope has angular magnification of 2 with distance between objective and eye piece equal to 30 cm. Find the focal length of the objective of telescope?

(1) 20 (2) 30

(3) 40 (4) 10

Answer (1)

Sol. $\dfrac{f_o}{f_e} = 2$

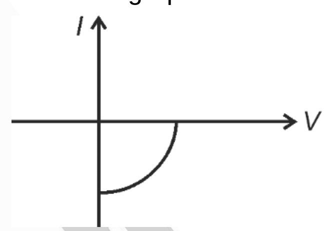
$f_o + f_e = 30$

$\Rightarrow \dfrac{3f_o}{2} = 30$

f_o = 20 cm, f_e = 10 cm

17. Which of the following represents the I-V characteristic graph of a solar cell

(1)



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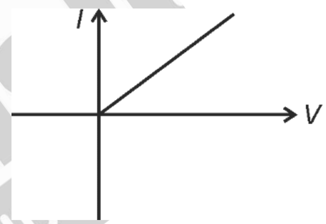

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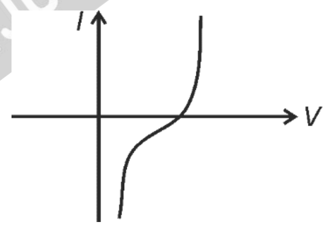

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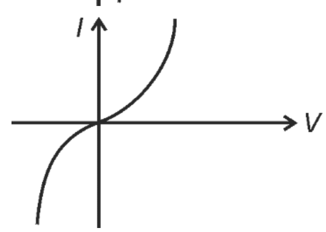
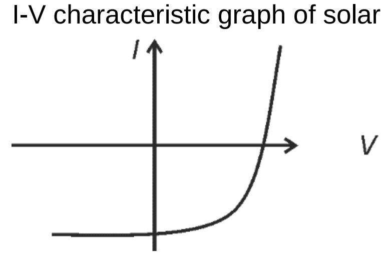


Answer (1)

Sol. I-V characteristic graph of solar cell is given as



A part of it resembles to option 1.

18. In expression of wavelength $\lambda_e = \frac{12.27}{K}$ Å, the value of K is (λ_e denotes the de-Broglie wavelength of electron)

(1) $\sqrt{V}$ (2) $\sqrt{p}$

(3) $\sqrt{\text{KE}}$ (4) $\sqrt{m}$

Answer (1)

Sol. de-Broglie wavelength of an electron accelerated through a potential difference V is given by

$$\lambda_e = \frac{12.27}{\sqrt{V}} Å$$

$$\Rightarrow \quad K = \sqrt{V}$$

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Projection of $\bar{a} = \hat{i} + 2\hat{j} + \alpha\hat{k}$ along $\bar{b} = 2\hat{i} + 2\hat{j} - 3\hat{k}$ is zero. So the value of α is equal to ______.

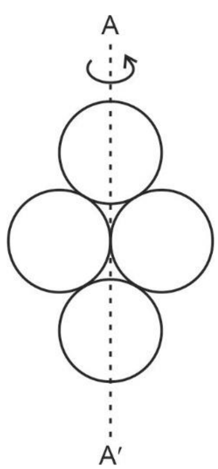
Answer (2)

Sol. $\vec{a} \cdot \vec{b} = 0$

$2 + 4 - 3\alpha = 0$

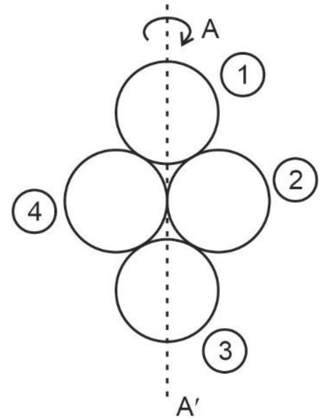
$\Rightarrow \quad \alpha = 2$

22. Four identical discs each of mass M and diameter a are rotating about the axis AA' as shown. The net moment of inertia of the system of discs about AA' is $\frac{3Ma^2}{x}$ then value of x is _____.



Answer (4)

Sol. $I = I_1 + I_2 + I_3 + I_4$



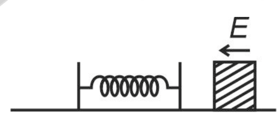
$$= \frac{m\left(\frac{a}{2}\right)^2}{4} + \frac{5m\left(\frac{a}{2}\right)^2}{4} + \frac{m\left(\frac{a}{2}\right)^2}{4} + \frac{5m\left(\frac{a}{2}\right)^2}{4}$$

$$= ma^2\left[\frac{1}{16} + \frac{5}{16} + \frac{1}{16} + \frac{5}{16}\right]$$

$$= \frac{3ma^2}{4}$$

So, $x = 4$

23. A block with initial kinetic energy E Joule compresses a spring as shown. The compression is 25 cm when the velocity of block becomes half of its initial value. If spring constant is $k = nE$ then the value of n is _______ (in SI units)



Answer (24)

Sol. $\frac{1}{2}k(0.25)^2 = E - \frac{E}{4}$

$$k = \frac{2 \times 3E}{4 \times 0.0625}$$

$= 24E$

So, $n = 24$

24.

25.

26.

27.

28.

29.

30.

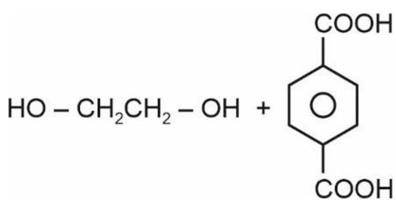
CHEMISTRY

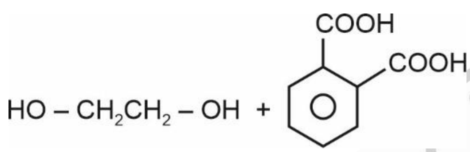
SECTION – A

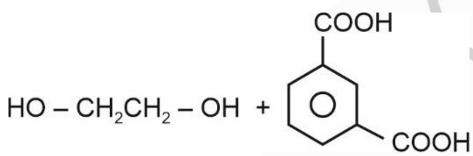
Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

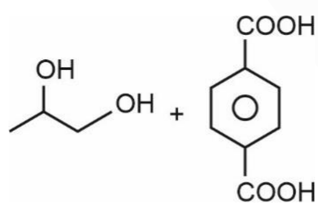
Choose the correct answer :

1. Which of the following options represent monomer of terylene?

(1) 

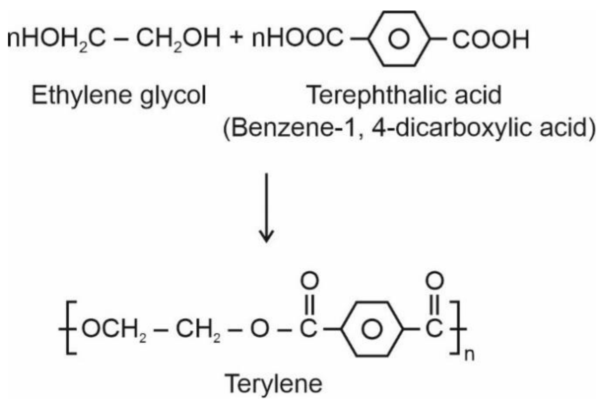
(2) 

(3) 

(4) 

Answer (1)

Sol. Formation of Terylene (also known as dacron) takes place by interaction of ethylene glycol and Terepthalic acid.



2. Which of the following metal has least tendency to evolve H_2 from acid?

(1) Cu

(2) Zn

(3) Mn

(4) Ni

Answer (1)

Sol. $E^{o}_{Cu^{2+}/Cu} = 0.34\text{ V}$

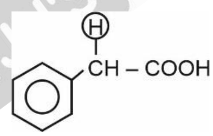
$Cu\,|\,Cu^{2+}\,||\,H^{+}\,|\,H_2\,|\,Pt$

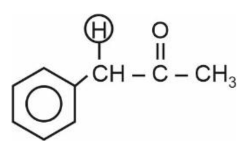
$E^{o}_{cell} = 0 - 0.34 = -0.34\text{ V}$

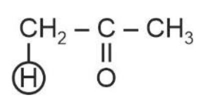
$\because$ E^{o}_{cell} is negative

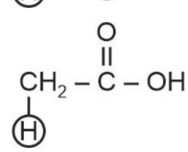
Hence Cu has least tendency to evolve H_2 from acid.

3. Which of the following indicated H-atom has lowest value of pK_a?

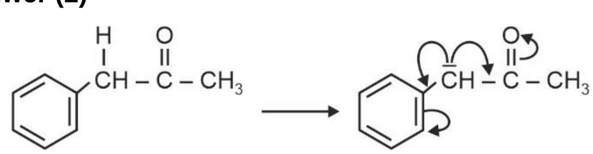
(1) 

(2) 

(3) 

(4) 

Answer (2)

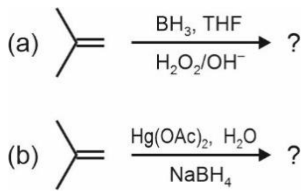
Sol. 

Due to maximum stabilisation of negative charge by cross-conjugation, the indicated H will be most acidic and will have least pK_a value.

4. Which of the following will give

(i) Markovnikov-product

(ii) Anti-Markovnikov product?



(1) a – Markovnikov, b – Markovnikov

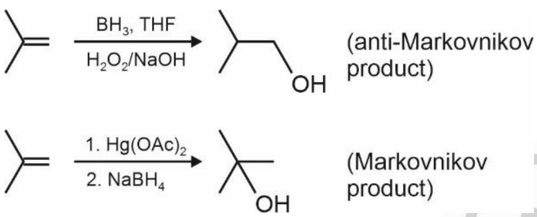
(2) a – Markovnikov, b – Anti-Markovnikov

(3) a – Anti-Markovnikov, b – Anti-Markovnikov

(4) a – Anti-Markovnikov, b – Markovnikov

Answer (4)

Sol.



5. Match the following

List-I	List-II
(A) $Cd(s) + 2NiO(OH)(s) + 2H_2O(l) \longrightarrow Cd(OH)_2 + 2Ni(OH)_2(s)$	(P) Fuel Cell
(B) $Zn(Hg) + 2\bar{O}H \longrightarrow ZnO(s) + H_2O + 2e^-$	(Q) Primary Battery
(C) $PbSO_4 + 2e^- \longrightarrow Pb(s) + SO_4^{2-}$	(R) Discharging of secondary Battery
(D) $2H_2(g) + O_2(g) \longrightarrow 2H_2O(l)$	(S) Charging of secondary battery

(1) A-(Q); B-(R); C-(P); D-(S)

(2) A-(Q); B-(P); C-(R); D-(S)

(3) A-(R); B-(Q); C-(S); D-(P)

(4) A-(P); B-(Q); C-(R); D-(S)

Answer (3)

Sol. Nickel Cadmium Battery is an example of secondary Battery

$Cd(s) + 2NiO(OH)(s) + 2H_2O(i) \longrightarrow Cd(OH)_2 + 2Ni(OH)_2(s)$

Below is equation of mercury cell, a primary Battery

$Zn(Hg) + 2\bar{O}H \longrightarrow ZnO(s) + H_2O + 2e^-$

Lead storage battery is an example of secondary cell

$PbSO_4 + 2e^- \longrightarrow Pb(s) + SO_4^{2-}$

Equation of Fuel Cell is,

$2H_2(g) + O_2(g) \longrightarrow 2H_2O(l)$

6. Match column (I) with Column (II)

Column I	Column II
(A) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta}$	(P) Cl_2
(B) $KMnO_4 + HCl \longrightarrow$	(Q) N_2
(C) $Al + NaOH \longrightarrow$	(R) O_2
(D) $NaNO_3 \xrightarrow{\Delta}$	(S) H_2

(1) A-(P); B-(Q); C-(R); D-(S)

(2) A-(Q); B-(P); C-(S); D-(R)

(3) A-(P); B-(R); C-(S); D-(Q)

(4) A-(Q); B-(R); C-(P); D-(S)

Answer (2)

Sol. The correct match is

Column (I)	Column (II)
(A) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta}$	(Q) $N_2\,[N_2 + Cr_2O_3 + H_2O]$
(B) $KMnO_4 + HCl \longrightarrow$	(P) $Cl_2\,[MnCl_2 + KCl + H_2O + Cl_2]$
(C) $Al + NaOH \xrightarrow{-H_2O}$	(S) $H_2\,[NaAlO_2 + H_2]$
(D) $NaNO_3 \xrightarrow{\Delta}$	(R) $O_2\,[NaNO_2 + O_2]$

Hence, the correct option is (2)

7. The correct decreasing order for magnitude of electron gain enthalpy is

(1) S > Se > Te > O

(2) Te > Se > S > O

(3) O > S > Se > Te

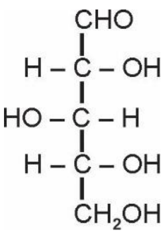
(4) S > O > Se > Te

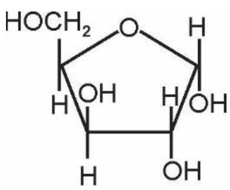
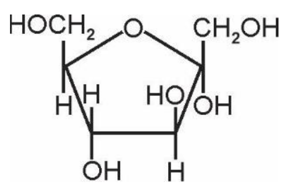
Answer (1)

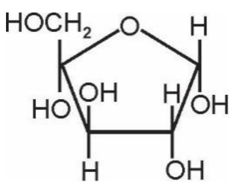
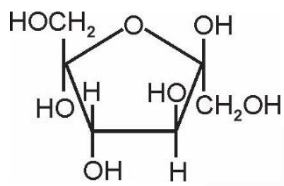
Sol. The correct decreasing order for magnitude of electron gain enthalpy for group 16 elements is,

S > Se > Te > O

8. Find the cyclic structure of pentose given below

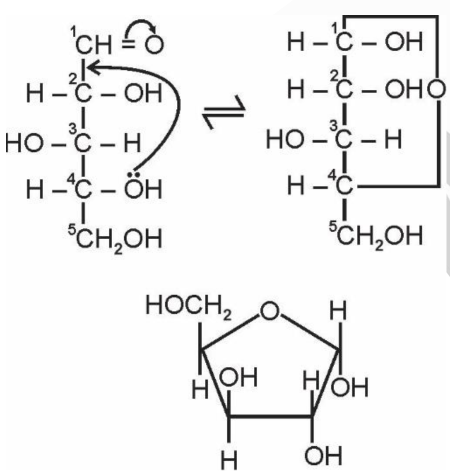


(1)  (2) 

(3)  (4) 

Answer (1)

Sol. The five membered cyclic structure of the given pentose is



9. Which of the following reactions represent leaching

(1) $Al_2O_3 + NaOH \rightarrow$ Products

(2) $Cu_2S + O_2 \rightarrow$ Products

(3) $Fe_2O_3 + CO \rightarrow$ Products

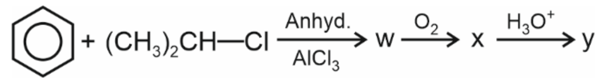
(4) $FeS_2 + O_2 \rightarrow$ Products

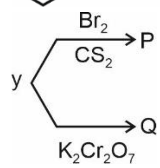
Answer (1)

Sol. $Al_2O_3 + NaOH \rightarrow$ Products

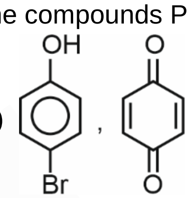
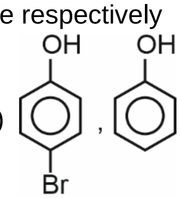
This reaction is used for concentration of bauxite ore and represents leaching.

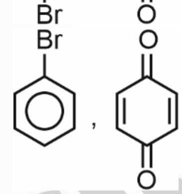
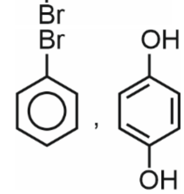
Reactions given in option (2) and (4) represent roasting and reaction given in option (3) represents reduction of Haematite ore.

10. 



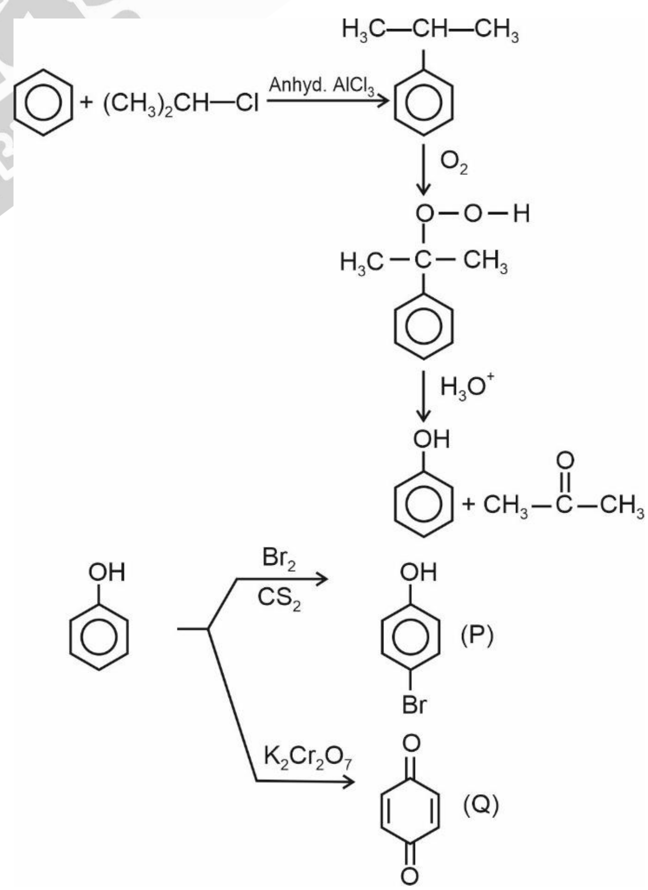
The compounds P and Q are respectively

(1)  (2) 

(3)  (4) 

Answer (1)

Sol.



11. Choose the incorrect statement

Regarding Bohr's theory

(1) The electron move around the nucleus in certain circular stable orbits without emitting radiations

(2) The angular momentum of electron in the Bohr orbit always remains constant

(3) The angular momentum in 3rd orbit is more than that of 1st orbit

(4) The energy difference between consecutive levels remain same as n-increases.

Answer (4)

Sol. According to Bohr's theory, an electron moves in certain permissible circular orbits around the nucleus without emitting any radiation. Only those orbits are permissible for which mvr (angular momentum) is integral multiple of $\left(\frac{h}{2\pi}\right)$.

$$m v r = n\frac{h}{2\pi}$$

As 'n' increases, angular momentum increases. Energy difference between two consecutive levels decreases as 'n' increases.

$$\Delta E = 13.6Z^2\left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$$

12. Match column I with Column II.

	Column I (Reactions)		**Column II (Catalyst)**
(i)	$N_2 + 3H_2 \rightarrow 2NH_3$	(p)	Fe
(ii)	$2SO_2 + O_2 \rightarrow 2SO_3$	(q)	Pt
(iii)	Sucrose $\rightarrow$ glucose + fructose	(r)	H_2SO_4
(iv)	$NH_3+O_2 \rightarrow NO+H_2O$	(s)	NO

(1) (i)-(p), (ii)-(q), (iii)-(r), (iv)-(s)

(2) (i)-(p), (ii)-(r), (iii)-(q), (iv)-(s)

(3) (i)-(p), (ii)-(s), (iii)-(r), (iv)-(q)

(4) (i)-(p), (ii)-(s), (iii)-(q), (iv)-(r)

Answer (3)

Sol. The correct match is

	Reaction		**Catalyst**
(i)	$N_2 + 3H_2 \rightarrow 2NH_3$	(p)	Fe (Haber's process)
(ii)	$2SO_2 + O_2 \rightarrow 2SO_3$	(s)	NO (Lead chamber process)
(iii)	Sucrose $\rightarrow$ glucose +fructose	(r)	H_2SO_4 (Hydrolysis of sucrose)
(iv)	$NH_3 + O_2 \rightarrow NO+H_2O$	(q)	Pt (Ostwald process)

13. Which of the following salts are formed in Clark's method.

(1) $Ca(OH)_2$, $Mg(OH)_2$

(2) $CaCO_3$, $Mg(OH)_2$

(3) $Ca(OH)_2$, $MgCO_3$

(4) $CaCO_3$, $MgCO_3$

Answer (2)

Sol. Clark's method is used to remove Temporary Hardness of water. In this method calculated amount of lime is added to hard water precipitating calcium carbonate and magnesium hydroxide.

$$Mg(HCO_3)_2 + 2Ca(OH)_2 \longrightarrow 2CaCO_3\downarrow + Mg(OH)_2\downarrow + 2H_2O$$

14. Which of the following is a incorrect statement?

(1) LiF is less soluble in water due to low hydration enthalpy

(2) KO_2 is Paramagnetic

(3) Density of sodium is greater than Potassium

(4) K_2O_2 is diamagnetic

Answer (1)

Sol. Hydration enthalpy of Li^+ and F^- is very high due to high charge density.

O_2^- is Paramagnetic

O_2^{2-} is diamagnetic

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Identity the number of paramagnetic species out of the following:

$Li_2, B_2, C_2, C_2^{\ominus}, O_2, O_2^{-2}, O_2^{\oplus}$

Answer (04.00)

Sol.

Species		**Magnetic character**
Li_2	$\rightarrow$	Diamagnetic
B_2	$\rightarrow$	Paramagnetic
C_2	$\rightarrow$	Diamagnetic
$C_2^{\ominus}$	$\rightarrow$	Paramagnetic
O_2	$\rightarrow$	Paramagnetic
O_2^{-2}	$\rightarrow$	Diamagnetic
$O_2^{\oplus}$	$\rightarrow$	Paramagnetic

22. MnO_4^{-2} disproportionates in acidic medium to form two compounds of manganese A and B. Oxidation state of Mn is less in B than A. Spin only magnetic moment of B is (in B.M)

(Round off nearest integer)

Answer (04.00)

Sol. $MnO_4^{-2} \xrightarrow[\text{disproportionates}]{H^{\oplus}} \underset{A}{MnO_4^{\ominus}} + \underset{B}{MnO_2}$

MnO_2 has d^3 configuration

$\therefore\ \mu(\text{magnetic moment of B}) = \sqrt{3(3+2)}$

$= \sqrt{15}$ B.M

$\simeq 4$

23. Consider the following reaction

$\underset{1mol}{X} + \underset{1mol}{Y} + \underset{0.05mol}{3Z} \longrightarrow XYZ_3$

Calculate the mass of XYZ_3 formed at the end of the reaction (in grams).

[Given molar mass of X, Y and Z are 10, 20 and 30 g/mol respectively]

Answer (02.00)

Sol. $\underset{t=0}{} \quad \underset{1mol}{X} + \underset{1mol}{Y} + \underset{0.05mol}{3Z} \longrightarrow XYZ_3$

As Z is the limiting reagent, $\frac{0.05}{3}$ moles of XYZ_3 is formed.

$\therefore$ Mass of $XYZ_3 = \frac{0.05}{3} \times (10+20+90)$

$= \frac{0.05}{3} \times 120$

$= 0.05 \times 40$

$= 2$ g

24. 10.2 g of ascorbic acid is dissolved in 150 g of CH_3COOH. The depression of freezing point of the resulting solution is 'x' × 10^{-1} K.

[Given k_f = 3.9 K kg mol^{-1} and Molar mass of ascorbic acid = 176 g mol^{-1}]

[Consider ascorbic acid and acetic acid remain undissociated] (Round off to nearest integer)

Answer (15.00)

Sol. $\Delta T_f = k_f\, m$

$= 3.9 \times \frac{10.2}{176 \times 150} \times 1000$

$= 1.51$ K

$\simeq 15 \times 10^{-1}$ K

$\therefore\ x = 15$

25. Calculate the pH of butyric acid with $K_a = 2 \times 10^{-5}$ & concentration 0.2 M.

[Given log2 = 0.3]

Answer (02.70)

Sol. As α (degree of dissociation) is very less

$1 - \alpha \approx 1$

$[H^{\oplus}] = \sqrt{cK_a}$

$= \sqrt{0.2 \times 2 \times 10^{-5}}$

$= \sqrt{4 \times 10^{-6}}$

$= 2 \times 10^{-3}$ M

pH = 3 – log2

= 2.7

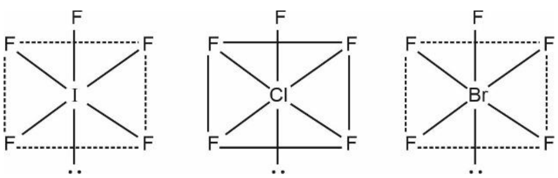
26. How many of the following interhalogen compound(s) have square pyramidal shape.

ClF_2, BrF_3, IF_5, IF_7, ClF_5, BrF_5, IF_3

Answer (03.00)

Sol. Out of the given interhalogen compounds

IF_5, ClF_5 and BrF_5 have square pyramidal shape.



27. For a first order reaction,

$A \longrightarrow B$

Half life is 0.3010 minutes. What will be the value of $\frac{[A_0]}{[A]}$ after 2 minutes

$[A_0]$ is initial concentration, [A] is concentration after 2 minutes]

Answer (100)

Sol. $A \longrightarrow B$

For a first order reaction, $t_{1/2} = 0.3010$ min

$\therefore$ Rate constant, $K = \frac{\ln 2}{t_{1/2}} = 2.303\,\text{mm}^{-1}$

$$K = \frac{2.303}{t}\log\frac{[A]_0}{[A]}$$

At t = 2 min

$$2.303 = \frac{2.303}{2}\log\frac{[A]_0}{[A]}$$

On solving, $\frac{[A]_0}{[A]} = 100$

28. What is the spin only magnetic moment of the manganese containing product formed at the end of the following reaction: $\rightarrow$ (in B.M.)

$$MnO_2 + KOH + O_2 \xrightarrow{\Delta} \text{Products}$$

(Round off to nearest integer)

Answer (02.00)

Sol. $2MnO_2 + 4KOH + O_2 \rightarrow 2K_2MnO_4 + 2H_2O$

Magnetic moment of K_2MnO_4 (d configuration of Mn)

$= \sqrt{1(1+2)}$

$= \sqrt{3}$

$\simeq 1.73$ B.M

$\simeq 2$

29. In a bcc lattice, edge length of lattice is 300 pm. Density of the solid is 60 gm/cm^3. If the number of atoms in 180 gm of sample is 'x' × 10^{23}, then x is:

[Take $N_A = 6 \times 10^{23}$]

Answer (2.22)

Sol. $d = \frac{z \times M}{N_A \times a^3}$

$$60 = \frac{2 \times M}{N_A \times (3)^3 \times 10^{-24}}$$

$$60 = \frac{2 \times M}{27 \times 6} \times 10$$

M = 486 gm/mol

$\therefore$ 180 g of the sample contains $\frac{180}{486} \times 6 \times 10^{23}$

$= 2.22 \times 10^{23}$ atoms

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. Sum of $(7^{2022} + 3^{2022})$ is divided by 5, then remainder is

(1) 0 (2) 3

(3) 2 (4) 1

Answer (2)

Sol. $7^{2022} + 3^{2022}$

$= (50 - 1)^{1011} + (10 - 1)^{1011}$

$= {}^{1011}C_0 50^{1011}(-1)^0 + ...1011C_{1010}5 + 1011C_{1011}(-1)^{1011}$

$+ {}^{1011}C_0 10^{1011}(-1)^0 + ... + {}^{1011}C_{1011}(-1)^{1011}$

$= 5\lambda - 2$

$\therefore$ Remainder when divided by 5 is 3

2. The sum of all the value of x satisfying

$\cos^{-1} x - 2\sin^{-1} x = \cos^{-1} 2x$

(1) 0 (2) 1

(3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

Answer (1)

Sol. $\cos^{-1} x - 2\sin^{-1} x = \cos^{-1} 2x$

$\Rightarrow \frac{\pi}{2} - 3\sin^{-1} x = \cos^{-1} 2x$

$\Rightarrow \cos\left(\frac{\pi}{2} - 3\sin^{-1} x\right) = \cos\left(\cos^{-1} 2x\right)$

$\Rightarrow \sin(3\sin^{-1} x) = \cos(\cos^{-1} 2x)$

$\Rightarrow 3x - 4x^3 = 2x$

$\Rightarrow 4x^3 = x$

$\Rightarrow x = 0, \pm\frac{1}{2}$

$\therefore$ Sum of values of $x = 0$

3. If $xdy = \left(\sqrt{x^2 + y^2} + y\right)dx$ and $y(1) = 0$ then $y(2)$ equals

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$

(3) $\frac{3}{2}$ (4) $\frac{5}{2}$

Answer (3)

Sol. $\frac{dy}{dx} = \frac{\sqrt{x^2 + y^2} + y}{x}$

Put $y = vx$

$\Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore v + x\frac{dv}{dx} = \sqrt{1 + v^2} + v$

$\Rightarrow \int \frac{dv}{\sqrt{1 + v^2}} = \int \frac{dx}{x}$

$\Rightarrow \ln\left(\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}\right) = \ln x + c$

as $y(1) = 0$ We get $c = 0$

for $y(2)$ Put $x = 2$

$\Rightarrow \ln\left(y + \sqrt{4 + y^2}\right) = 2\ln 2$

$\Rightarrow y + \sqrt{4 + y^2} = 4$ $\Rightarrow 1 + y^2 = 16 - 8y + y^2$

$\Rightarrow y = \frac{3}{2}$

4. If $a_{n+2} = \frac{2}{a_{n+1}} + a_n$, $a_1 = 1$, $a_2 = 2$, then

$$\left(\frac{a_1 + \frac{1}{a_2}}{a_3}\right) \cdot \left(\frac{a_2 + \frac{1}{a_3}}{a_4}\right) ..... \frac{a_{30} + \frac{1}{a_{31}}}{a_{32}} = 2^{\alpha}\ {}^{61}C_{31}$$

then $\alpha =$

(1) – 30 (2) – 31

(3) – 60 (4) – 61

Answer (3)

Sol. $\because$ $a_{n+2} \cdot a_{n+1} - a_{n+1} \cdot a_n = 2$ where $a_1 = 1$, $a_2 = 2$

and $a_3 = 2$

Let $T_r = a_{r+1} \cdot a_r$

So $< T_r >$ is A.P. with common difference 2 and first term 2.

Clearly $T_r = 2r$

Now, $\displaystyle\prod_{i=1}^{30}\left(\frac{a_i + \frac{1}{a_{i+1}}}{a_{i+2}}\right) = \prod_{r=1}^{30}\left(\frac{T_r+1}{T_{r+1}}\right)$

$$= \prod_{i=1}^{30}\left(\frac{2r+1}{2r+2}\right) = \frac{3\cdot5\cdot7\cdot....\cdot61}{4\cdot6\cdot8\cdot....\cdot62} = \frac{|\underline{62}}{2(4\cdot6\cdot8\cdot...\cdot62)^2}$$

$$= \frac{|\underline{62}}{2^{61}\cdot(|\underline{31})^2} = 2^{-61}\,.\,{}^{62}C_{31} = 2^{-60}\,.\,{}^{61}C_{30}$$

$$= 2^{-60}\,.\,{}^{61}C_{31}$$

5. Let $z_1 \in C$, $|z_1 - 3| = \frac{1}{2}$,

$z_2 \in C, \left|z_2 + |z_2 - 1|\right| = \left|z_2 - |z_2 + 1|\right|$, then least value of $|z_1 - z_2|$ is

(1) 0

(2) $\frac{1}{2}$

(3) $\frac{3}{2}$

(4) $\frac{5}{2}$

Answer (3)

Sol. $\because$ $\left|z_2 + |z_2 - 1|\right|^2 = \left|z_2 - |z_2 + 1|\right|^2$

$\Rightarrow$ $\left(z_2 + |z_2 - 1|\right)\left(\bar{z}_2 + |z_2 - 1|\right)$

$=$ $\left(z_2 - |z_2 + 1|\right)\left(\bar{z}_2 - |z_2 + 1|\right)$

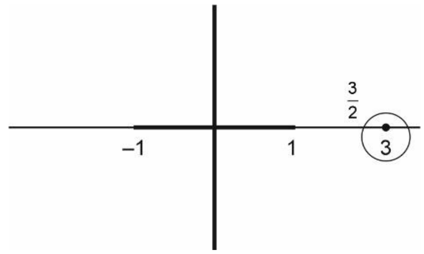
$\Rightarrow$ $z_2\left(|z_2 - 1| + |z_2 + 1|\right) + \bar{z}_2\left(|z_2 - 1| + |z_2 + 1|\right)$

$=$ $|z_2 + 1|^2 - |z_1 + 1|^2$

$\Rightarrow$ $\left(z_2 + \bar{z}_2\right)\left(|z_2 + 1| + |z_2 - 1|\right) = 2\left(z_2 + \bar{z}_2\right)$

$\Rightarrow$ Either $z_2 + \bar{z}_2 = 0$ or $|z_2 + 1| + |z_2 - 1| = 2$

So z_2 lies on imaginary axis or on real axis within $[-1, 1]$. Also $|z_1 - 3| = \frac{1}{2} \Rightarrow z_1$ lies on the circle having centre 3 and radius $\frac{1}{2}$.



Clearly $|z_1 - z_2|_{\min} = \frac{3}{2}$

6. If $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $B_0 = A^{98} + 2A^{49}$,

$B_n = \text{adj}\,(B_{n-1})$, then $|B_4|$ is equal to

(1) 3^{16}

(2) 3^{32}

(3) 2^{16}

(4) 2^{32}

Answer (2)

Sol. $A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$\therefore$ $B_0 = I + 2A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

As $B_n = \text{adj}\,(B_{n-1})$

$\therefore$ $B_4 = \text{adj}(B_3) = \text{adj}(B_2) = \text{adj}\,(\text{adj}\,(\text{adj}\,B_1))$

$= (\text{adj}\,(\text{adj}\,(\text{adj}\,(\text{adj}\,B_0))))$

$\therefore$ $|B_4|\;|B_0|^{2^4} = |B_0|^{16}$

$|B_0| = \begin{vmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 3 \end{vmatrix} = 3(-3)$

$\therefore$ $|B_0|^{16} = 9^{16} = 3^{32}$

7. The domain of $\cos^{-1}\left(\dfrac{x^2-4x+2}{x^2+3}\right)$ is

(1) $\left(-\infty, \dfrac{-1}{3}\right)$ (2) $\left[\dfrac{-1}{4}, \infty\right)$

(3) $\left(\dfrac{-1}{3}, \infty\right)$ (4) $\left(\dfrac{-1}{4}, \infty\right)$

Answer (2)

Sol. $f(x) = \cos^{-1}\left(\dfrac{x^2-4x+2}{x^2+3}\right)$

For D_f $-1 \le \dfrac{x^2-4x+2}{x^2+3} \le 1$

$\Rightarrow -x^2-3 \le x^2-4x+2 \le x^2+3$

$\Rightarrow 2x^2-4x+5 \ge 0$ and $4x \ge -1$

$\Rightarrow x \in R$ and $x \in \left[\dfrac{-1}{4}, \infty\right)$

$\therefore D_f = \left\{x : x \in \left[\dfrac{-1}{4}, \infty\right)\right\}$

8. Tangent at two point P and Q on parabola $y^2 = 2x - 3$ meet at $R(0, 1)$, then orthocentre of ΔPQR is

(1) (0, 1)

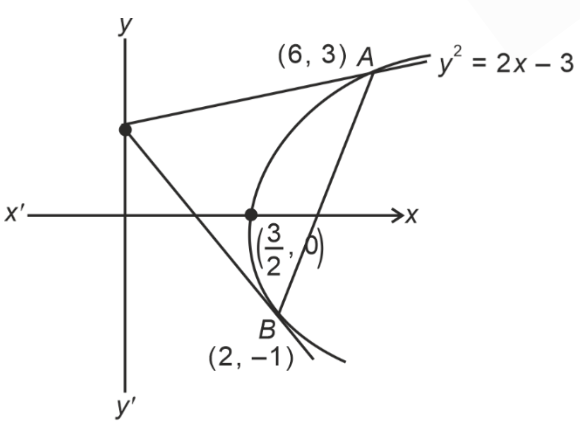
(2) (2, –1)

(3) (6, 3)

(4) (2, 1)

Answer (2)

Sol.



$y^2 = 2x - 3$

$y = 2\left(x - \dfrac{3}{2}\right)$

Let a point on parabola

$A = \left(\dfrac{3}{2} + \dfrac{t^2}{2}, t\right)$

Equation of tangent at A is

$yt = x + \dfrac{3x + t^2}{2} - 3$

$\because$ $R(0, 1)$ lies on it then $t = 3$ or -1

$\therefore$ $A = (6, 3)$ and $B = (2, -1)$

Here, ΔABC is right angled at $B(2, -1)$

$\therefore$ Orthocentre of ΔABC is $(2, -1)$

9. The solution of differential equation $\sin^2(2x)\dfrac{dy}{dx} + (8\sin^2 2x + 4\sin 2x)y = 0$ $y\left(\dfrac{\pi}{4}\right) = e^{-\pi}$, then $y\left(\dfrac{\pi}{6}\right)$ is

(1) $3e^{\frac{\pi}{3}}$

(2) $3e^{-\frac{\pi}{3}}$

(3) $\sqrt{3}e^{\frac{\pi}{3}}$

(4) $\sqrt{3}e^{-\frac{\pi}{3}}$

Answer (2)

Sol. $\sin^2 2x\dfrac{dy}{dx} + (8\sin^2 2x + 4\sin 2x)y = 0$

$\Rightarrow \displaystyle\int\frac{dy}{y} + \int(8 + 4\text{cosec } 2x)dx = 0$

$\Rightarrow \ln y + 8x + 2\ln|\text{cosec } 2x - \cot 2x| = c$

$y\left(\dfrac{\pi}{4}\right) = e^{-\pi}$ we get

$-\pi + 2\pi + 2\ln 1 = c \Rightarrow c = \pi$

$\Rightarrow \ln y + 8x + 2\ln|\text{cosec } 2x - \cot 2x| = \pi$

for $y\left(\dfrac{\pi}{6}\right)$ put $x = \dfrac{\pi}{6}$ we get

$\ln y + \dfrac{4\pi}{3} + 2\ln\left|\dfrac{1}{\sqrt{3}}\right| = \pi$

$\Rightarrow \ln\left(\dfrac{y}{3}\right) = -\dfrac{\pi}{3} \Rightarrow y = 3e^{-\frac{\pi}{3}}$

10. Let $f(x)=\frac{5x^2}{2}+\frac{\alpha}{x^5},(x>0)$ and minimum value of $f(x)$ is 14, then a is equal to

(1) 32 (2) 64

(3) 128 (4) 256

Answer (3)

Sol. Using weighted A.M.– G.M. Inequality

$$\frac{5\left(\frac{x^2}{2}\right)+2\left(\frac{\alpha}{2x^5}\right)}{5+2}\geq\left(\left(\frac{x^2}{2}\right)^5\cdot\left(\frac{\alpha}{2x^5}\right)^2\right)^{\frac{1}{7}}$$

$$\Rightarrow \frac{5x^2}{2}+\frac{\alpha}{x^5}\geq\frac{7}{2}(\alpha)^{\frac{2}{7}}$$

So least value of $\frac{5x^2}{2}+\frac{\alpha}{x^5}=\frac{7}{2}\alpha^{\frac{2}{7}}=14$

$$\Rightarrow a^{\frac{2}{7}}=4 \Rightarrow \alpha=128$$

11. The minimum value of function

$f(x)=\int_0^x e^{x-t}f'(t)dt+(x^2-x+1)e^x$, is then find $f(x)$

(1) $\frac{-2}{\sqrt{e}}$ (2) $-2\sqrt{e}$

(3) $\frac{2}{\sqrt{e}}$ (4) $\sqrt{e}$

Answer (3)

Sol. $f(x)=e^x\int_0^x e^{-t}f'(t)dt+e^x(x^2-x+1)$

$$f'(x)=e^x\int_0^x e^{-t}f'(t)dt+e^x.e^{-x}f'(x)$$
$$+e^x(x^2-x+1)+e^x(2x-1)$$

$$\Rightarrow 0=e^x\int_0^x e^{-t}(f'(t)dt+e^x(x^2+x)$$

$$\Rightarrow \int_0^x e^{-t}f'(t)dt=-(x^2+x)$$

$$\Rightarrow e^{-x}f'(x)=-(2x+1)$$

$$\Rightarrow f'(x)=-e^x(2x+1)$$

$$\Rightarrow f(x)=-e^x(2x-1)+c \quad (c=0 \text{ as } f(0)=1)$$

$$\therefore f(x)=(1-2x)e^x$$

$$\Rightarrow f(x)_{\min}=\frac{2}{\sqrt{e}}$$

12.

13.

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Let hyperbola $H:x^2-y^2=1$ and ellipse $E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and eccentricity of ellipse E is reciprocal of that of hyperbola H. If common tanget of H and E is $y=\sqrt{\frac{5}{2}}x+k$, then value of $4(a^2+b^2)$ is equal to

Answer (3)

Sol. $H:x^2-y^2=1,\ E:\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$$e_H=\sqrt{2},\ \text{and}\ e_E=\frac{1}{e_H}$$

$$\therefore 1-\frac{b^2}{a^2}=\frac{1}{2}$$

$$\Rightarrow \frac{b^2}{a^2}=\frac{1}{2} \quad \ldots\text{(i)}$$

$\because$ Common tangent of E and H is

$$y=\sqrt{\frac{5}{2}}x+k\left(i.e.\ m=\sqrt{\frac{5}{2}}\right)$$

Now, for common tangency

$$\frac{5}{2}a^2 + b^2 = \frac{5}{2} - 1$$

$$\frac{5}{2} + \frac{b^2}{a^2} = \frac{3}{2a^2}$$

$$\Rightarrow \quad a^2 = \frac{1}{2}$$

$$\therefore \quad a^2 + b^2 = \frac{3}{2}\cdot\frac{1}{2} = \frac{3}{4}$$

$\therefore$ $4(a^2 + b^2) = 3$

22. $A(\cos t, \sin t)$, $B(\sin t, -\cos t)$ and $C(a, b)$ form an equilateral triangle, $t \in [0, 2\pi]$ and orthocentre of this triangle lies on a circle with centre $\left(1, \frac{1}{3}\right)$ then $a^2 - b^2$ is equal to

Answer (08)

Sol. If ΔABC is an equilateral triangle, then orthocentre & centroid coincide.

So, Orthocentre of triangle is

$$(h, k) \equiv \left(\frac{\cos t + \sin t + a}{3}, \frac{\sin t - \cos t + b}{3}\right)$$

$\Rightarrow$ $3h - a = \cos t + \sin t$ ...(i)

$3k - b = \sin t - \cos t$ ...(ii)

Eliminate t from above two equations (i) & (ii)

$(3h - a)^2 + (3k - b)^2 = 2$

So, (h, k) lies on the circle whose centre is at $\left(\frac{a}{3}, \frac{b}{3}\right)$.

$a = 3$, $b = 1$; then $a^2 - b^2 = 8$

23. A password is to be made of 6 to 8 digits from either of the sets A, B, C, D, E or 1, 2, 3, 4, 5 having at least one numeric, then possible password is equal to $K \times 5^6$, then K is equal to

Answer ($7073 \cdot 5^6$ **)**

Sol. If password is six-digit then number of ways

$\therefore$ $10^6 - 5^6$

Similarly, if 7-ditis password

Number of ways = $10^7 - 5^7$

For 8-digit password number of ways

$= 10^8 - 5^8$

$\therefore$ Total ways

$$= \left(10^6 + 10^7 + 10^8\right) - \left(5^6 + 5^7 + 5^8\right)$$

$$= 5^6\left(2^6 + 5 \cdot 2^7 + 25 \cdot 2^8 - 1 - 5 - 5^2\right)$$

$$= 7073 \cdot 5^6$$

24.

25.

26.

27.

28.

29.

30.

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