JEE-Main-27-07-2022-Shift-2 (Memory Based)

Physics

Question: A proton is accelerated in cyclotron of radius of D as 60cm and magnetic field as 1 T.... Find kinetic energy of the proton

Options:

(a) 4.88 x 10^{-12} J = Mev

(b) 2.88 x 10^{-12} J = Mev

(c) 1.88 x 10^{-12} J = Mev

(d) 3.88 x 10^{-12} J = Mev

Answer: (b)

Solution:

$$k = \frac{B^2 q^2 R^2}{2m}$$

$$= \frac{1\left(1.6\times10^{-19}\right)^2\left(60\times10^{-2}\right)^2}{2\left(1.67\times10^{-27}\right)}$$

$$= 2.88\times10^{-12} J$$

Question: Base current changes from 20 μA to 25 μA and collector current changes from 4 mA to 6 mA current gain in common emitter transistor is

Options:

(a) 400

(b) 200

(c) 100

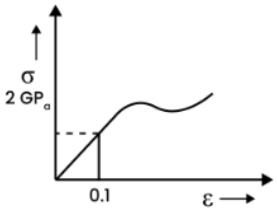
(d) 300

Answer: (a)

Solution:

$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{2mA}{5\mu A} = 400$$

Question: Following is the graph between longitudinal stress and strain for a wire, find value of young's modulus of elasticity for this wire



Options:

(a) 40 GP_a

(b) 30 GP_a
(c) 20 GP_a
(d) 10 GP_a
Answer: (c)
Solution:

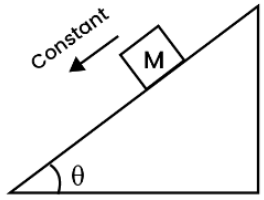
$$\gamma = \frac{\sigma}{6} = \frac{2\times10^9}{0.1} = 20GP_a$$

Question: The activity of radioactive material be 6.4 x 10^{-4} Curie. Its half life is 5 days, then the activity will be 5 x 10^{-6} Curie after.
Options:
(a) 7 days
(b) 15 days
(c) 25 days
(d) 21 days
Answer: (d)
Solution:

$$A = A_0\left(\frac{1}{2}\right)^{t/T_1/2} \Rightarrow 5\times10^{-6} = 64\times10^{-5}\left(\frac{1}{2}\right)^{t/3}$$

$$\frac{1}{128} = \left(\frac{1}{2}\right)^{t/3} \Rightarrow t = 21\,\text{days}$$

Question: A Block slides down cm a rough inclined plane with constant velocity. Find constant force.



Options:
(a) mg/2
(b) 2mg
(c) mg
(d) mg/3
Answer: (c)
Solution:

$\mu mg\cos\theta = mg\sin\theta$

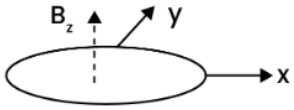
$\therefore$ Total force on plank

$= mg\sin\theta\left(\text{along the incline}\right)$

$mg\cos\theta^{+}\left(\perp\text{to incline}\right)$

= mg (resultant)

Question: $\overline{B} = 3t^3\hat{j} - 3t^2\hat{k}$ for a ring, in xy plane. Find EMF induced at t = 2 s?



Options:

(a) 10 A

(b) 12 A

(c) 14 A

(d) 16 A

Answer: (b)

Solution:

$\phi = B_z.A_x r$

$= -3t^2 A$

$\varepsilon = -\frac{d\phi}{dt}$

$= +3A.2t$

$= 6At$

$\varepsilon_2 = 6A(2)$

$\therefore \varepsilon_2 = 12 \rightarrow$ assumed

Question: In an LRC AC circuit, the frequency of voltage source is 60 % of resonance frequency. Find current in this circuit. L = 0.01 H, R = 10 Ω , C = 1 μF. Voltage rms value is 50

Options:

(a) 0.67 A

(b) 0.47 A

(c) 0.37 A

(d) 0.57 A

Answer: (b)

Solution:

$f_r = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01\times10^{-6}}}$

$f_r = \frac{10000}{2\pi} = 1592 H_g$

So, $(2\pi f.L) = 60, x_c = \frac{1}{2\pi fc} = 166$

$so\, i = \frac{v}{\sqrt{R^2 + (x_i - x_c)^2}} = \frac{50}{106} = 0.47\,\text{A}$

Question: how charge 4 μc should be split in two parts so that when kept at a fixed distance, force between them is maximum

Options:
(a) 3μC, 2μC
(b) 2μC, 2μC
(c) 1μC, 2μC
(d) 5μC, 2μC
Answer: (b)
Solution:

For maximum force, $\frac{dF}{dq}=0$

$\therefore \frac{d}{dq}\left(\frac{kq(Q-q)}{r^2}\right)=0$

Or, $\frac{d}{dq}\left(qQ-q^2\right)=0$

Or, $Q-2q=0$

$\Rightarrow q=\frac{Q}{2}$

Question: Which of the following statement is not true regarding photo electric effect
Options:
(a) Square of velocity depends on incident frequency linerly
(b) Existence of threshold frequency proves particle nature of light
(c) Increase in intensity changes the photocurrent
(d) Most of the energy in incident light is used in emission of photoelectron
Answer: (d)
Solution:
Most of the energy in incident light is used in emission of photoelectron

Question: If energy density is $\frac{\alpha}{\beta}\sin\left(\frac{\alpha x}{kT}\right)$ find the dimensions of β [k – Boltzmann constant, T – temperature, x is length]
Options:
(a) L^2
(b) L^3
(c) L^4
(d) L
Answer: (a)
Solution:
$\frac{\alpha}{\beta}=\frac{\text{Energy}}{\text{Volume}}$

Also, $\alpha x=KT$

$$\therefore \alpha = \frac{kT}{x} = \frac{\left[ML^2T^{-2}K^{-1}\right][K]}{L}$$

$$= \left[MLT^{-2}\right]$$

$$\therefore \beta = \alpha \times \frac{L^3}{ML^2T^{-2}} = \left[MLT^{-2}\right]\frac{\left[L^3\right]}{\left[ML^2T^{-2}\right]}$$

$$= \left[L^2\right]$$

Question: A Carnot engine is operating between T_1 and T_2 temperature. Find ratio of the efficiency of this engine in the 2 cases:

Case 1: $T_1 = 300k, T_2 = 200k$

Case 2: $T_1 = 200k, T_2 = 100k$

Options:

(a) $\frac{1}{2}$

(b) $\frac{2}{3}$

(c) $\frac{3}{2}$

(d) $\frac{5}{3}$

Answer: (b)

Solution:

$$\eta_1 = 1 - \frac{T_2}{T_1} = 1 - \frac{200}{300} = \frac{1}{3}$$

$$\eta_2 = 1 - \frac{T_2}{T_1} = 1 - \frac{100}{200} = \frac{1}{2}$$

$$\frac{\eta_1}{\eta_2} = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3}$$

Question: If $A_m = 1$ and $A_c = 5$. Find modulation index?

Options:

(a) 0.2

(b) 0.3

(c) 0.4

(d) 0.5

Answer: (a)

Solution:

$$m = \frac{A_m}{A_c} = \frac{1}{5} = 0.2$$

Question: The time taken by a particle at rest to reach down a smooth inclined plane inclined at $30°$ is 2 sec. Find time taken by the particle to travel same distance down the inclined plane at angle of $45°$.

Options:

(a) $\sqrt{2}$

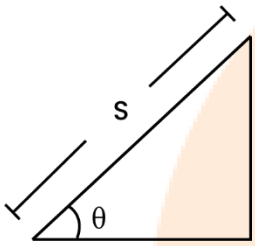
(b) $\sqrt{2\sqrt{2}}$

(c) $\sqrt{3}$

(d) $2\sqrt{2}$

Answer: (b)

Solution:



$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}g\sin\theta t^2$$

And $s_1 = s_2$

$$\frac{1}{2}g\sin 30(2)^2 = \frac{1}{2}g\sin 45°t^2$$

$$t^2 = \frac{(2)^2}{\sqrt{2}} = \sqrt{2\sqrt{2}}$$

Question: The speed of bullet becomes $\frac{1}{3}$ of initial speed after penetrating 2cm. If it starts at $4 + xcm$ find x?

Options:

(a) 0.1 cm

(b) 0.2 cm

(c) 0.3 cm

(d) 0.5 cm

Answer: (d)

Solution:

$$\frac{v^2}{9} = v^2 - 2a(4)$$

$$2a(9) = v^2\left(1-\frac{1}{9}\right) = v^2 \times \frac{8}{9}$$

$$2a = v^2\frac{2}{9}$$

Now, $0 = v^2 - 2a(s)$

$$v^2 = v^2 \times \frac{2}{9}s$$

$$s = \frac{9}{2} = 4.5cm$$

$$\therefore 4 + x = 4.5$$

$$\therefore x = 0.5cm$$

Question: Time taken for the particle to reach from point A to C is $l\left[1+\sqrt{2}\right]$ find value of t.

Assume all surfaces smooth. Velocity at A is just enough to reach B.

Options:

(a) $\sqrt{2}$

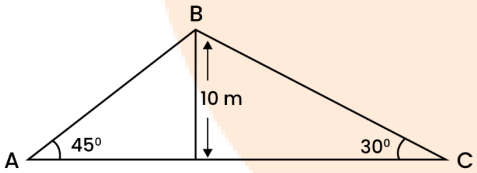
(b) $\left(1+\sqrt{2}\right)$

(c) $\sqrt{2}\left(1+\sqrt{2}\right)$

(d) $2\left(1+\sqrt{2}\right)$

Answer: (d)

Solution:



$A \to B$

$$\frac{1}{2}mu^2 = mgh$$

$$\Rightarrow u = \sqrt{2gh} = 10\sqrt{2}$$

So, $v = u + at \Rightarrow 0 = 10\sqrt{2} - \frac{10}{\sqrt{2}}.t$

$t = 2\sec$

$B \to C$

$$s = ut + \frac{1}{2}at^2 \Rightarrow \frac{10}{\sin 30°} = 0 + \frac{1}{2}g\sin 30°t^2$$

$$20 = \frac{5}{2}t^2 \Rightarrow t = \sqrt{8} = 2\sqrt{2}$$

So total time $= 2+\sqrt{2} = 2\left(1+\sqrt{2}\right)$

Question: For n^{th} frequency $\rightarrow 400Hz$

$(n+1)^{th}$ frequency $\rightarrow 450Hz$

On a string of 30 cm fixed at both ends. T = 2700 N. Find μ?

Options:

(a) 2 kg

(b) 3 kg

(c) 6 kg

(d) 8 kg

Answer: (b)

Solution:

$400 = nf_1$

$450 = (n+1) f_1$

$\therefore 450 = 400 + f_1$

$\therefore f_1 = 50Hz$

Now, $50 = \frac{1}{2 \times 0.3}\sqrt{\frac{2700}{\mu}}$

$(100 \times 0.3)^2 = \frac{2700}{\mu}$

$\therefore \mu = \frac{2700}{900}$

$\therefore \mu = 3kg/m$

Question: The drift speed of free electrons in a conductor does not depends upon:

Options:

(a) The material of the conductor

(b) The temperature of the conductor

(c) The potential difference applied across the ends of the conductor

(d) The area of cross section of the conductor

Answer: (d)

Solution:

The drift velocity is known to be primarily depends on the applied voltage and the molecular structure of the wire and hence the material of the conductor. Slight temperature depends is also observed.

Question: If in a meter bridge, area of cross section of wire is doubled, null point

Options:

(a) Increases

(b) Decreases

(c) Remains same

(d) None of these

Answer: (c)

Solution:

Null point will remain same as resistance of both sides will decrease in the same ratio.

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Chemistry

Question: The spin only magnetic moment of the metal in complex present in Fehling's reagent is ___ B.M.

Options:

(a) $\sqrt{3}$

(b) $2\sqrt{2}$

(c) 0

(d) $\sqrt{15}$

Answer: (a)

Solution: In Fehling solution, Cu is in +2 oxidation state

Electronic configuration of $Cu^{2+} \Rightarrow [Ar]\ 3d^9$

n = 1

$\mu = \sqrt{1(1+2)} = \sqrt{3}$ B M = 1.73 B.M.

Question: 5gm toluene reacts to form benzaldehyde with 92% yield. What is amount of benzaldehyde formed?

Options:

(a) 5.3 g

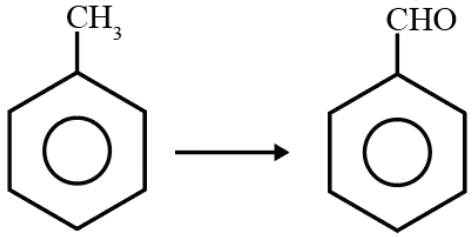
(b) 53 g

(c) 4.6 g

(d) 0.46 g

Answer: (a)

Solution:



(92% yield)

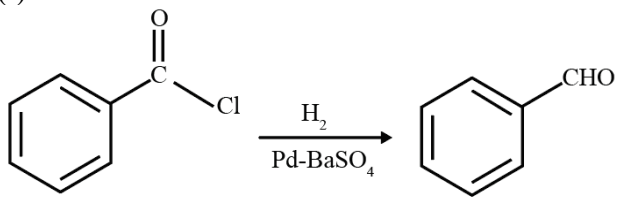
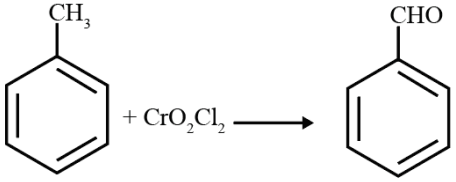
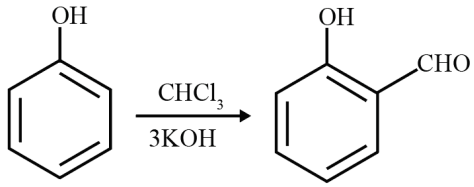
Molar mass of toluene = 92 g/mol

Number of moles of toluene = $\frac{5}{92}$

Moles of Benzaldehyde formed = $\frac{92}{100} \times \frac{5}{92}$ = 0.05 mole

Amount of Benzaldehyde = 0.05 × 106 = 5.3 g

Question: Match the following.

(Column I) Name reaction	(Column II) Reaction
(A) Stephen reaction	(i) 
(B) Etard reaction	(ii) $RCN + SnCl_2 + HCl \rightarrow RCH{=}NH \xrightarrow{H_3O^+} RCHO$
(C) Reimer Tiemann	(iii) 
(D) Rosenmund reaction	(iv) 

Options:

(a) A → (ii); B → (iii); C → (iv); D → (i)

(b) A → (iv); B → (iii); C → (i); D → (ii)

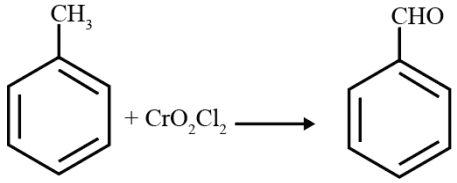
(c) A → (iii); B → (ii); C → (iv); D → (i)

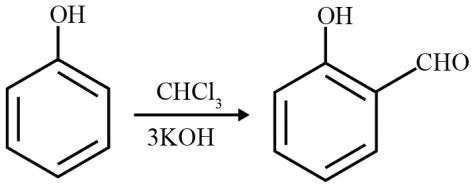
(d) A → (iii); B → (i); C → (ii); D → (iv)

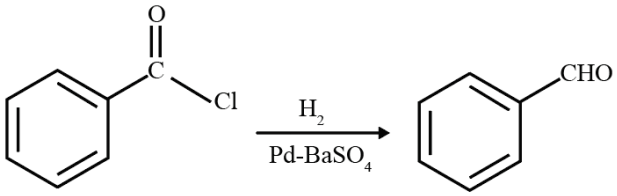
Answer: (a)

Solution:

Stephen reaction ⇒ $RCN + SnCl_2 + HCl \rightarrow RCH{=}NH \xrightarrow{H_3O^+} RCHO$

Etard reaction ⇒ 

Reimer Tiemann ⇒ 

Rosenmund reaction ⇒ 

Question: In neutral or alkaline solution, MnO_4^- oxidises thiosulphate to
Options:
(a) $S_2O_7^{2-}$
(b) $S_2O_8^{2-}$
(c) SO_3^{2-}
(d) SO_4^{2-}
Answer: (d)
Solution: $8MnO_4^- + S_2O_3^{2-} + 5OH^- \rightarrow 8MnO_4^{2-} + 5H^+ + 2SO_4^{2-}$

Question: Which of the following enhances the lathering property of soap?
Options:
(a) Sodium stearate
(b) Sodium carbonate
(c) Sodium rosinate
(d) Trisodium phosphate
Answer: (c)
Solution: Sodium rosinate is the filler which enhances the lathering property of soap.

Question: Low oxidation state of metals in their complexes are common when ligands
Options:
(a) Have good π - accepting character
(b) Have good σ - donor character
(c) Having good π - donating ability
(d) Having poor σ - donating ability
Answer: (a)
Solution: Low oxidation states are found when a complex compound has ligands capable of π-accepting character in addition to the σ-bonding.
For example, in $Ni(CO)_4$ and $Fe(CO)_3$, the oxidation state of nickel and iron is zero.

Question: An element A of group 1 shows similarity to an element B belonging to group 2. If A has maximum hydration enthalpy in group 1 then B is
Options:
(a) Mg
(b) Be
(c) Ca
(d) Sr
Answer: (a)
Solution: Li has maximum hydration enthalpy in group 1. So, A is Li and Li shows diagonal relationship with Mg.
Hence, B is Mg

Question: Statement 1: KI molar conductivity increases steeply with increasing dilution.
Statement 2: Carbonic molar conductivity increase/slowly with dilution.
Options:
(a) Both statement 1 and 2 are correct
(b) Statement 1 is correct but statement 2 is incorrect

(c) Statement 1 is incorrect but statement 2 is correct
(d) Both statement 1 and 2 are incorrect.
Answer: (d)
Solution: The molar conductivity of KI increases slowly with dilution.
Molar conductivity of carbonic acid increases steeply with dilution
Therefore, both the statement are false

Question: Correct decreasing order of energy of the orbitals having following set of quantum numbers
A) n = 3, l = 0, m = 0
B) n = 4, l = 0, m = 0
C) n = 3, l = 1, m = 0
D) n = 3, l = 2, m = 1
Options:
(a) A > C > B > D
(b) D > B > C > A
(c) A > B > C > D
(d) D > C > B > A
Answer: (b)
Solution: Following the principle of $(n+l)$ rule, the correct order of energy is D > B > C > A
The subshells with the lowest $(n+l)$ value has the lowest energy. When two or more subshells have the same $(n+l)$ value, then the subshell with the lowest value of n have lowest energy

Question: Outermost electronic configuration of 4 elements A, B, C, D
A-$3s^2$, B-$3s^2 3p^1$, C-$3s^2 3p^3$, D-$3s^2 3p^4$
Correct order of 1st ionisation enthalpy
Options:
(a) B > A > C > D
(b) D > C > B > A
(c) C > D > B > A
(d) D > C > A > B
Answer: (c)
Solution: Ionization energy increases along a period
But ionisation energy of C-$3s^2 3p^3$ is more than D-$3s^2 3p^4$ as C has half-filled p subshells giving extra stability
∴ Correct order of first ionisation energy is C > D > B > A

Question: Match the reaction of glucose with their products.

(Column I) Reaction of Glucose	(Column II) Products
(A) With HI	(i) Gluconic acid
(B) With HNO_3	(ii) Glucose pentaacetate
(C) With Br_2	(iii) n - hexane

(D) With Ac_2O	(iv) Saccharic acid

Options:
(a) A → (i); B → (iii); C → (ii); D → (iv)
(b) A → (iv); B → (iii); C → (i); D → (ii)
(c) A → (iii); B → (ii); C → (iv); D → (i)
(d) A → (iii); B → (iv); C → (i); D → (ii)
Answer: (d)
Solution:
With HI ⇒ n - hexane
With HNO_3 ⇒ Saccharic acid
With Br_2 ⇒ Gluconic acid
With Ac_2O ⇒ Glucose pentaacetate

Question: Statement 1: True solution and colloidal solution can be separated using parchment paper.
Statement 2: Particles of true solution does not pass through parchment paper whereas colloidal solution passes.
Options:
(a) Both statement 1 and 2 are correct
(b) Statement 1 is correct but statement 2 is incorrect
(c) Statement 1 is incorrect but statement 2 is correct
(d) Both statement 1 and 2 are incorrect.
Answer: (d)
Solution: True solution cannot be separated using parchment paper
Particles of colloidal solution cannot pass through parchment paper
Therefore, both the statements are false

Question: Assertion: BF_6^{3-} does not exist.
Reason: It is because of small size of boron
Options:
(a) Both assertion and reason are true, reason is correct explanation of assertion.
(b) Both assertion and reason are true, but reason is not a correct explanation of the assertion.
(c) Assertion is true, but reason is false
(d) Assertion is false, but reason is true
Answer: (b)
Solution: BF_6^{3-} does not exist because of non availability of vacant d orbitals. Reason statement is also correct B has small size but it does not explain assertion. Hence, B is correct.

Question: Compound that is of Prussian Blue Colour?
Options:
(a) $Na_4[Fe(CN)_6S]$
(b) $Na_4[Fe(CN)_6NCS]$
(c) $Na_4[Fe(CN)_5NOS]$
(d) $Na_2[Fe(CN)_5NOS]$
Answer: (c)

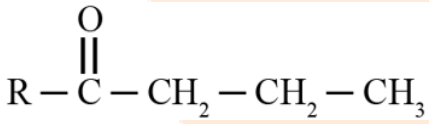
Solution: $Na_2S + Na_2[Fe(CN)_5NO] \rightarrow Na_4[Fe(CN)_5NOS]$
Sodium sulphide reacts with sodium nitroprusside to form a violet colour compound, which confirms the presence of sulfur.

Question: Match the following.

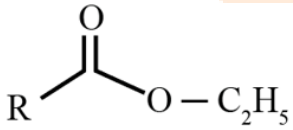
(Column I)	(Column II)
(A) Neoprene	(i) Prop-2-enal
(B) Isoprene	(ii) Chloroprene
(C) Teflon	(iii) Natural rubber
(D) Acrolein	(iv) Chlorofluoroethene

Options:
(a) A → (i); B → (iii); C → (ii); D → (iv)
(b) A → (iv); B → (iii); C → (i); D → (ii)
(c) A → (iii); B → (ii); C → (iv); D → (i)
(d) A → (ii); B → (iii); C → (iv); D → (i)
Answer: (d)
Solution:
Neoprene ⇒ Chloroprene
Isoprene ⇒ Natural rubber
Teflon ⇒ Chlorofluoroethene
Acrolein ⇒ Prop-2-enal

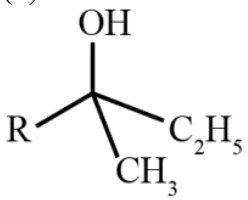
Question: $RCOCH_3 + NaOH \rightarrow A + C_2H_5Br \rightarrow$ Major product
Options:
(a)



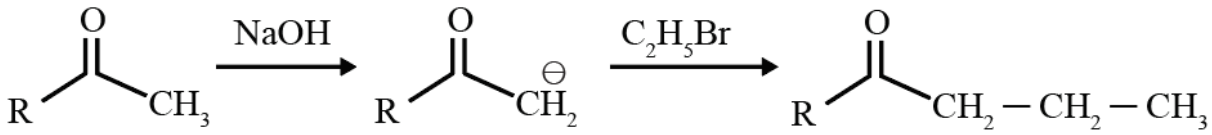
(b) R–COOH
(c)



(d)



Answer: (a)
Solution:



Question: H_2O_2 + potassium permanganate in basic medium. What's the magnetic moment of Mn? (Round off to nearest integer)
Answer: 4.00
Solution: $2K\overset{+7}{M}nO_4 + 3H_2O_2 \rightarrow 2\overset{+4}{M}nO_2 + 3O_2 + 2KOH + 4H_2O$
$Mn^{+4} \rightarrow [Ar]\ 3d^3$
$\mu = \sqrt{n(n+2)}$
where n = 3
$\mu = \sqrt{3(3+2)} = 3.87 \approx 4$ B M

Question: A solid 'A' (Molecular weight 280) is added to 100 g water to make solution dilute. The vapour pressure of solution becomes half of vapour pressure of pure water. If vapour pressure of pure water is 23.4 mm Hg. Find moles of solid 'A' added. (Round off to nearest integer)
Answer: 3.00
Solution:
$$\frac{P^o - P}{P^o} = \frac{n_2}{n_1}$$
$$\frac{23.4 - 11.7}{11.7} = \frac{n_2 \times 18}{100}$$
$n_2 = 2.7 \approx 3$

Question: The normality of H_2SO_4 in the solution obtained on mixing 100 ml of 0.1 M H_2SO_4 with 50 ml of 0.1 M NaOH is ___ $\times 10^{-1}$ N
Answer: 1.00
Solution:
$$N_{H_2SO_4} = \frac{N_{H_2SO_4}V_{H_2SO_4} - N_{NaOH}V_{NaOH}}{V_{H_2SO_4} + V_{NaOH}}$$
$$= \frac{(2 \times 0.1 \times 100) - (0.1 \times 50)}{150} = \frac{20 - 5}{150} = \frac{15}{150} = 0.1N$$
$= 1 \times 10^{-1}$ N

Question: Number of non-planar structures
NO_3^-, H_2O_2, BF_3, PCl_3, XeF_4, SF_4, XeO_3, PH_4^+, SO_3, $[Al(OH)_4]^-$.
Answer: 6.00
Solution: PCl_3, H_2O_2, SF_4, XeO_3, PH_4^+ and $[Al(OH)_4]^-$ are non-planar structures.

Question: Among the following, number of iron ores is/are:
Malachite, Siderite, Hematite, Magnetite, Bauxite, cryolite
Answer: 3.00
Solution: Magnetite(Fe_3O_4), Siderite($FeCO_3$), and Hematite(Fe_2O_3) are ores of iron

JEE-Main-27-07-2022-Shift-2 (Memory Based)

MATHEMATICS

Question: Let $A=\begin{bmatrix}4 & -2\\ \alpha & \beta\end{bmatrix}$. If $A^2+\gamma A+18I=0,$ then $\det(A)$ equals:

Options:

(a) -18

(b) 18

(c) -50

(d) 50

Answer: (b)

Solution:

Characteristic equation of matrix:

$$\begin{bmatrix}4-\lambda & -2\\ \alpha & \beta-\lambda\end{bmatrix}=0$$

$$\Rightarrow 4\beta+\lambda^2-(\beta+4)\lambda+2\alpha=0$$

$$\therefore A^2-(\beta+4)A+2\alpha I=0$$

$$\Rightarrow \gamma=0-\beta+4 \ \& \ 2\alpha+4\beta=18$$

$$\det(A)=4\beta+2\alpha=18$$

Question: The area of region enclosed by $y\le 4x^2, x^2\le 9y, y\le 4$ is equal to:

Options:

(a) $\frac{40}{3}$

(b) $\frac{56}{3}$

(c) $\frac{112}{3}$

(d) $\frac{80}{3}$

Answer: (d)

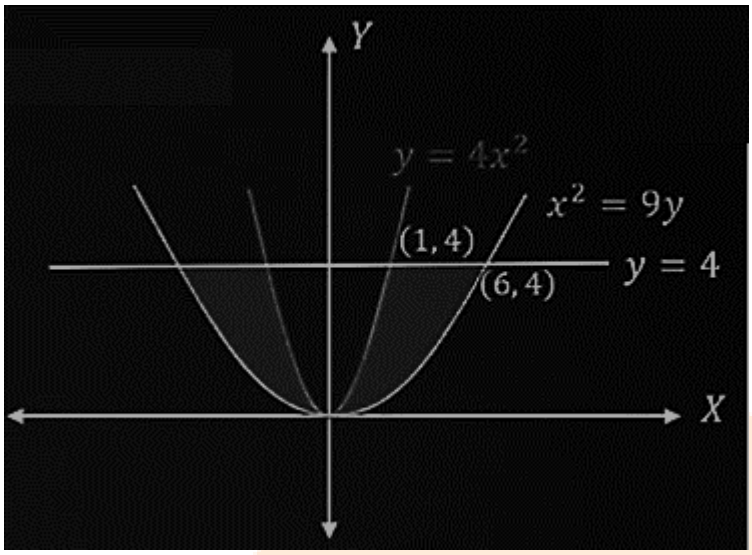
Solution:

$$\text{Required Area } =2\int_0^4\left(3\sqrt{y}-\frac{\sqrt{y}}{2}\right)dy$$

$$=2\cdot\frac{5}{2}\int_0^4\sqrt{y}\,dy$$

$$=5\left[\frac{2}{3}y^{\frac{3}{2}}\right]_0^4$$

$= \frac{10}{3}(4)^{\frac{3}{2}} = \frac{80}{3}$



Question: If the length of the latus rectum of a parabola whose focus is (a, a) and tangent at its vertex is $x + y = a,$ is 16. Then $|a|$ is equal to:

Options:

(a) $2\sqrt{3}$

(b) $2\sqrt{2}$

(c) $4\sqrt{2}$

(d) 4

Answer: (c)

Solution:

Length of perpendicular from focus to tangent at vertex:

$l = \left|\frac{a}{\sqrt{2}}\right|$

So length of latus rectum will be, $4l = 16$

$\Rightarrow 2\sqrt{2}|a| = 16$

$\Rightarrow |a| = 4\sqrt{2}$

Question: Let $f(x) = \frac{\left(729p(1+x)^{\frac{1}{7}}\right) - 3}{\left(729(1+qx)^{\frac{1}{3}}\right) - 9}$, and $f(x)$ is continuous at $x = 0,$ then:

Options:

(a) $21qf(0) - p = 0$

(b) $21q^2 f(0) - p^3 = 0$

(c) $21p^2 f(0) - q^3 = 0$

(d) $p^2 f(0) - 7q^2 = 0$

Answer: (a)

Solution:

$\lim_{x\to 0} f(x)$ exists if numerator of $f(x)$ is zero at $x = 0$.

Clearly, $p = 3$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{3\left[(x+1)^{\frac{1}{7}} - 1\right]}{9\left[(1+qx)^{\frac{1}{3}} - 1\right]}$$

$$= \frac{1}{3}\left(\frac{\frac{1}{7}}{\frac{q}{3}}\right) = \frac{1}{7q} = f(0)$$

So, $21qf(0) = 3 = p$

$\Rightarrow 21qf(0) - p = 0$

Question: Let $f(x) = \min\{[x],[x-1],[x-2],...,[x-10]\}$ where [ ] denotes greatest integer function. Then $\int_0^{10} \left(f(x) + |f(x)| + f^2(x)\right) dx$ is equal to:

Options:

(a) 55

(b) 385

(c) 5050

(d) 270

Answer: (b)

Solution:

Clearly $f(x) = [x-10]$

Here $f(x) \le 0 \ \forall \ x \in (0,10)$

So, $\int_0^{10} \left(f(x) + |f(x)|\right) dx = 0$

Now, $\int_0^{10} f^2(x)\,dx = \int_0^{10} \left([x]-10\right)^2 dx$

$$= \int_0^1 100\,dx + \int_1^2 81\,dx + \int_2^3 64\,dx + ... + \int_9^{10} 1\,dx$$

$=\left(1^2+2^2+3^2+....+10^2\right)$

$=\frac{10\times11\times21}{6}$

$=385$

Question: The value of $\int_0^2\left(\left|2x^3-3x\right|+\left[x-\frac{1}{2}\right]\right)dx,$ where $[.]$ is greatest integer function is:

Options:

(a) $\frac{7}{6}$

(b) $\frac{19}{12}$

(c) $\frac{17}{4}$

(d) $\frac{3}{2}$

Answer: (c)

Solution:

Given, $\int_0^2\left(\left|2x^3-3x\right|+\left[x-\frac{1}{2}\right]\right)dx$

$\int_0^2\left|2x^3-3x\right|dx+\int_0^2\left[x-\frac{1}{2}\right]dx$

$=\int_0^{\sqrt{\frac{3}{2}}}\left(3x-2x^3\right)dx+\int_{\sqrt{\frac{3}{2}}}^2\left(2x^3-3x\right)dx+\int_0^{\frac{1}{2}}\left[x-\frac{1}{2}\right]dx+\int_{\frac{1}{2}}^{\frac{3}{2}}\left[x-\frac{1}{2}\right]dx+\int_{\frac{3}{2}}^2\left[x-\frac{1}{2}\right]dx$

$=\left[\frac{3x^2-x^4}{2}\right]_0^{\sqrt{\frac{3}{2}}}+\left[\frac{x^4-3x^2}{2}\right]_{\sqrt{\frac{3}{2}}}^2+\left(-\frac{1}{2}\right)+0+\left(\frac{1}{2}\right)$

$=\frac{9}{8}+2+\frac{9}{8}$

$=\frac{17}{4}$

Question: If the line of intersection of the planes $ax+by=3$ and $ax+by+cz=0$ makes an angle $30°$ with the plane $y-z+2=0$, then the direction cosines of line are:

Options:

(a) $\frac{1}{\sqrt{2}},0,\frac{1}{2}$

(b) $\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},0$

(c) $\frac{1}{\sqrt{5}},-\frac{2}{\sqrt{5}},0$

(d) $\frac{1}{2},-\frac{\sqrt{3}}{2},0$

Answer: (b)

Solution:

Direction ratios of line of intersection $(b,-a,0)$

As angle between this line and $y-z+2=0$ is $30°$

$\therefore \sin\theta=\left|\frac{a}{\sqrt{a^2+b^2}\cdot\sqrt{2}}\right|$

$\Rightarrow a^2=b^2$

$\therefore$ Possible combination is $\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}},0$

Question: If $A=\begin{bmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ \beta+\gamma & \alpha+\gamma & \alpha+\beta\end{bmatrix}$ and $\frac{\left|adj\left(adj\left(adj\left(adj\left(A\right)\right)\right)\right)\right|}{(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}}=2^{32}\cdot3^{16}$, where α,β,γ are distinct natural number, then number of triplets of (α,β,γ) is _____.

Answer: 55.00

Solution:

$A=\begin{bmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ \beta+\gamma & \alpha+\gamma & \alpha+\beta\end{bmatrix}$

$R_3\to R_3+R_1$

$\Rightarrow |A|=(\alpha+\beta+\gamma)\begin{vmatrix}\alpha & \beta & \gamma\\ \alpha^2 & \beta^2 & \gamma^2\\ 1 & 1 & 1\end{vmatrix}$

$=(\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$

$\because \left|adj\left(adj\left(adj\left(adj\left(A\right)\right)\right)\right)\right|=|A|^{(2)^4}=|A|^{16}$

$|A|=(\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$

Clearly $(\alpha+\beta+\gamma)^{16}=2^{32}\cdot3^{16}$

$\Rightarrow(\alpha+\beta+\gamma)=12$

Number of positive integral solutions $={}^{11}C_2=55$

Question: $\frac{\left(2^3-1^3\right)}{(1\times 7)}+\frac{\left\{\left(4^3-3^3\right)+\left(2^3-1^3\right)\right\}}{(2\times 11)}+\frac{\left\{\left(6^3-5^3\right)+\left(4^3-3^3\right)+\left(2^3-1^3\right)\right\}}{(3\times 15)}+\ldots$ upto 15 terms

Answer: 120.00

Solution:

$\frac{2^3-1^3}{1\times 7}+\frac{4^3-3^3+2^3-1^3}{2\times 11}+\ldots.$

$=1+2+3+\ldots.$

$=\left(\frac{15\times 16}{2}\right)$

$=120$

Question: Domain of $f(x)=\sin^{-1}\left[2x^2-3\right]+\log_2\left(\log_{\frac{1}{2}}\left(x^2-5x+5\right)\right)$

Answer: $1,\frac{5-\sqrt{5}}{2}$

Solution:

$-1\le\left[2x^2-3\right]\le 1$

$-1\le\left(2x^2-3\right)<2$

$2\le 2x^2<5$

$1\le x^2<\frac{5}{2}\quad\ldots.(1)$

$\log_{\frac{1}{2}}\left(x^2-5x+5\right)>0$

$0<x^2-5x+5<1$

$\Rightarrow x^2-5x+5=0$ and $x^2-5x+4<0$

$x\in\left(-\infty,\frac{5-\sqrt{5}}{2}\right)\cup\left(\frac{5+\sqrt{5}}{2},\infty\right)\quad\ldots.(2)$

and $x\in(1,4)$

Taking intersection of (1) and (2)

$x\in\left(1,\frac{5-\sqrt{5}}{2}\right)$

Question: Let n^{th} term of any sequence is given by $T_n=\frac{-1^3+2^3-3^3+4^3+\ldots+(2n)^3}{n(4n+3)}$, then $\sum_{n=1}^{15}T_n$ is equal to ____.

Answer: 120.00

Solution:

$$T_n = \frac{2\left[2^3 + 4^3 + \ldots + \left(2n^3\right)\right] - \left[1^3 + 2^3 + 3^3 + \ldots + \left(2n\right)^3\right]}{n\left(4n+3\right)}$$

$$T_n = \frac{16\left(\frac{n\left(n+1\right)}{2}\right)^2 - \left(\frac{2n\left(2n+1\right)}{2}\right)^2}{n\left(4n+2\right)}$$

$$= \frac{n^2\left(4n+3\right)}{n\left(4n+3\right)} = n$$

$$\therefore \sum_{n=1}^{15} T_n = \frac{15\times 16}{2} = 120$$