Q1: Compound A contains 8.7% Hydrogen, 74% Carbon and 17.3% Nitrogen. The molecular formula of the compound is,

Given: Atomic masses of C, H and N are 12, 1 and 14 amu respectively.

The molar mass of the compound A is 162 g mol^{-1}.

(A) $C_4H_6N_2$

(B) C_2H_3N

(C) C_5H_7N

(D) $C_{10}H_{14}N_2$

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(A) $C_4H_6N_2$

(B) C_2H_3N

(C) C_5H_7N

(D) $C_{10}H_{14}N_2$

Solution:

Moles of hydrogen = $\frac{8.7}{1.008} \approx \frac{8.7}{1}$ = 8.7

Moles of carbon = $\frac{74}{12}$ = 6.17

Moles of nitrogen = $\frac{17.3}{14}$ = 1.24

Ratio of H, C and N is

8.7 : 6.17 : 1.24

7 : 5 : 1

Therefore, the empirical formula of compound is C_5H_7N.

Empirical formula mass of C_5H_7N = (5 × 12) + (7 × 1) + 14

= 60 + 7 + 14 = 81 g mol^{-1}

The molar mass of compound = 162 g mol^{-1}

Therefore, $\mathbf{n} = \frac{\text{Molar mass}}{\text{Empirical formula mass}} = \frac{162}{81} = \mathbf{2}$

Molecular formula of the compound can be obtained by multiplying 'n' with the empirical formula.

Therefore, the molecular formula of compound A is $C_{10}H_{14}N_2$.

Q2: Consider the following statements:

(A) The principal quantum number 'n' is a positive integer with values of 'n' = 1, 2, 3, ...

(B) The azimuthal quantum number 'l' for a given 'n' (principal quantum number) can have values as 'l' = 0, 1, 2, ... n

(C) Magnetic orbital quantum number 'm_l' for a particular 'l' (azimuthal quantum number) has $(2l + 1)$ values.

(D) ± 1/2 are the two possible orientations of electron spin.

(E) For l = 5, there will be a total of 9 orbital

Which of the above statements are correct?

(A) (A), (B) and (C)

(B) (A), (C), (D) and (E)

(C) (A), (C), and (D)

(D) (A), (B), (C) and (D)

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Which of the above statements are correct?

(A) (A), (B) and (C)

(B) (A), (C), (D) and (E)

(C) (A), (C), and (D)

(D) (A), (B), (C) and (D)

Solution:

Azimuthal quantum number, 'l' is also known as orbital angular momentum or subsidiary quantum number. It defines the three-dimensional shape of the orbital. For a given value of n, l can have n values ranging from 0 to n–1, that is, for a given value of n, the possible values of l are: l = 0, 1, 2, ….. (n–1)

For any sub-shell (defined by 'l' value) $2l$ + 1 values of m_l are possible.

So, for l = 5, there are (2 × 5 + 1) = 11 orbitals possible.

Q3: In the structure of SF_4, the lone pair of electrons on S is in

(A) equatorial position and there are two lone pair - bond pair repulsions at 90°.

(B) equatorial position and there are three lone pair - bond pair repulsions at 90°.

(C) axial position and there are three lone pair - bond pair repulsion at 90°.

(D) axial position and there are two lone pair - bond pair repulsion at 90°.

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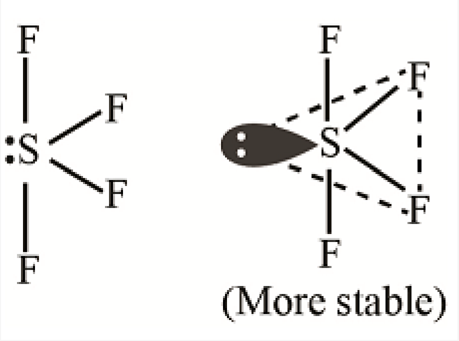
(B) equatorial position and there are three lone pair - bond pair repulsions at 90°.

(C) axial position and there are three lone pair - bond pair repulsion at 90°.

(D) axial position and there are two lone pair - bond pair repulsion at 90°.

Solution:

In SF_4 the lone pair is in an equatorial position, and there are two lone pair-bond pair repulsions. It has a see-saw shape.



Q4: A student needs to prepare a buffer solution of propanoic acid and its sodium salt with pH 4. The ratio of $\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]}$ required to make buffer is _____ .

Given: $K_a(CH_3CH_2COOH) = 1.3 \times 10^{-5}$

(A) 0.03

(B) 0.13

(C) 0.23

(D) 0.33

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Given: $K_a(CH_3CH_2COOH) = 1.3 \times 10^{-5}$

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(B) 0.13

(C) 0.23

(D) 0.33

Solution:

Using Handerson-Hasselbalch equation,

$$pH = pKa + \log\left(\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]}\right)$$

$$4 = -\log(1.3 \times 10^{-5}) + \log\left(\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]}\right)$$

$$4 = 4.88 + \log\left(\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]}\right)$$

$$-0.88 = \log\left(\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]}\right)$$

$$\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]} = \text{Antilog}\ (-0.88)$$

$= 0.13$

Q5: Match List–I with List–II:

List–I	**List–II**
(A) Negatively charged sol	(I) $Fe_2O_3 \cdot xH_2O$
(B) Macromolecular colloid	(II) CdS sol
(C) Positively charged sol	(III) Starch
(D) Cheese	(IV) a gel

Choose the correct answer from the options given below:

(A) A $\rightarrow$ II; B $\rightarrow$ III; C $\rightarrow$ IV; D $\rightarrow$ I

(B) A $\rightarrow$ II; B $\rightarrow$ I; C $\rightarrow$ III; D $\rightarrow$ IV

(C) A $\rightarrow$ II; B $\rightarrow$ III; C $\rightarrow$ I; D $\rightarrow$ IV

(D) A $\rightarrow$ I; B $\rightarrow$ III; C $\rightarrow$ II; D $\rightarrow$ IV

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(D) Cheese	(IV) a gel

Choose the correct answer from the options given below:

(A) A → II; B → III; C → IV; D → I

(B) A → II; B → I; C → III; D → IV

(C) A → II; B → III; C → I; D → IV

(D) A → I; B → III; C → II; D → IV

Solution:

(A) Negatively charged sol ⇒ CdS sol
(B) Macromolecular colloid ⇒ Starch
(C) Positively charged sol ⇒ $Fe_2O_3 \cdot xH_2O$
(D) Cheese ⇒ a gel

Q6: Match List–I with List–II:

List–I (Oxide)	**List–II (Nature)**
(A) Cl_2O_7	(I) Amphoteric
(B) Na_2O	(II) Basic
(C) Al_2O_3	(III) Neutral
(D) N_2O	(IV) Acidic

Choose the correct answer from the options given below

(A) A → IV; B → III; C → I; D → II

(B) A → IV; B → II; C → I; D → III

(C) A → II; B → IV; C → III; D → I

(D) A → I; B → II; C → III; D → IV

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(B) A → IV; B → II; C → I; D → III

(C) A → II; B → IV; C → III; D → I

(D) A → I; B → II; C → III; D → IV

Solution:

(A) $Cl_2O_7 \Rightarrow$ Acidic
(B) $Na_2O \Rightarrow$ Basic
(C) $Al_2O_3 \Rightarrow$ Amphoteric
(D) $N_2O \Rightarrow$ Neutral

Q7: In the metallurgical extraction of copper, following reaction is used :

$FeO + SiO_2 \rightarrow FeSiO_3$

FeO and $FeSiO_3$ respectively are:

(A) gangue and flux

(B) flux and slag

(C) slag and flux

(D) gangue and slag

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(C) slag and flux

(D) gangue and slag

Solution:

The sulphide ores of copper are heated in a reverberatory furnace. If the ore contains iron, it is mixed with silica before heating. Iron oxide 'slags of" as iron silicate and copper is produced in the form of copper matte which contains Cu_2S and FeS.

$$\underset{\text{Gangue}}{FeO} + SiO_2 \rightarrow \underset{\text{Slag}}{FeSiO_3}$$

Therefore, FeO and $FeSiO_3$ respectively are gangue and slag.

Q8: Hydrogen has three isotopes: protium (^{1}H), deuterium (^{2}H or D) and tritium (^{3}H or T). They have nearly the same chemical properties but different physical properties. They differ in

(A) number of protons

(B) atomic number

(C) electronic configuration

(D) atomic mass

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(A) number of protons

(B) atomic number

(C) electronic configuration

(D) atomic mass

Solution:

Hydrogen has three isotopes: protium, ($^{1}_{1}H$), deuterium, ($^{2}_{1}H$ or D) and tritium, ($^{3}_{1}H$ or T).
These isotopes differ from one another in respect of the presence of neutrons. Hence, they have the same atomic numbers but different mass numbers and atomic masses.

Q9: Among the following, basic oxide is :

(A) SO_3

(B) SiO_2

(C) CaO

(D) Al_2O_3

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Q9: Among the following, basic oxide is :

(A) SO_3

(B) SiO_2

(C) CaO

(D) Al_2O_3

Solution:

Since, oxides of metals are basic in nature, CaO is a basic oxide.
Acidic oxide- SO_3, SiO_2
Amphoteric oxide - Al_2O_3

Q10: Among the given oxides of nitrogen; N_2O, N_2O_3, N_2O_4 and N_2O_5, the number of compound/ (s) having N – N bond is:

(A) 1

(B) 2

(C) 3

(D) 4

Q10: Among the given oxides of nitrogen; N_2O, N_2O_3, N_2O_4 and N_2O_5, the number of compound/ (s) having N – N bond is:

(A) 1

(B) 2

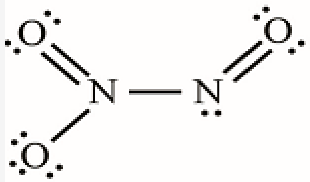
(C) 3

(D) 4

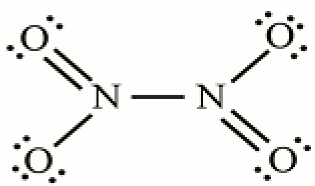
Solution:

N_2O

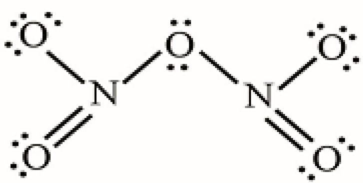
N_2O_3



N_2O_4



N_2O_5



Therefore, three compounds have an N-N bond.

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Q11: Which of the following oxoacids of sulphur contains “S” in two different oxidation states?

(A) $H_2S_2O_3$

(B) $H_2S_2O_6$

(C) $H_2S_2O_7$

(D) $H_2S_2O_8$

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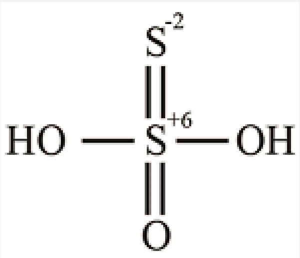
(B) $H_2S_2O_6$

(C) $H_2S_2O_7$

(D) $H_2S_2O_8$

Solution:

In $H_2S_2O_3$, sulphur exhibits two different oxidation states +6 and -2.



Q12: Correct statement about photo-chemical smog is:

(A) It occurs in humid climate

(B) It is a mixture of smoke, fog and SO_2

(C) It is a reducing smog

(D) It results from reaction of unsaturated hydrocarbons

Q12: Correct statement about photo-chemical smog is:

(A) It occurs in humid climate

(B) It is a mixture of smoke, fog and SO_2

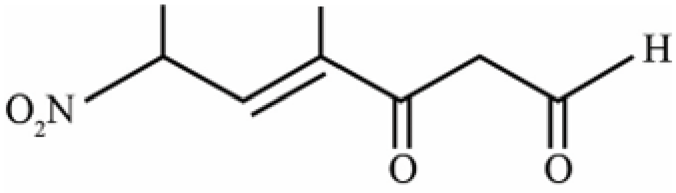
(C) It is a reducing smog

(D) It results from reaction of unsaturated hydrocarbons

Solution:

Photochemical smog occurs in warm, dry and sunny climate. The main components of the photochemical smog result from the action of sunlight on unsaturated hydrocarbons and nitrogen oxides produced by automobiles and factories. Photochemical smog has high concentration of oxidising agents and is, therefore, called as oxidising smog.

Q13: The correct IUPAC name of the following compound is



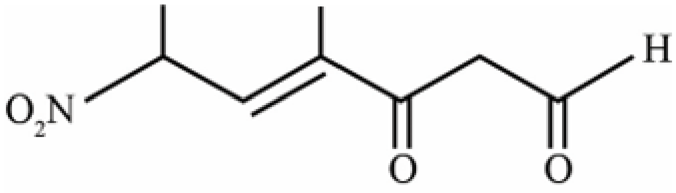
(A) 4-methyl-2-nitro-5-oxohept-3-enal

(B) 4-methyl-5-oxo-2-nitrohept-3-enal

(C) 4-methyl-6-nitro-3-oxohept-4-enal

(D) 6-formyl-4-methyl-2-nitrohex-3-enal

Q13: The correct IUPAC name of the following compound is



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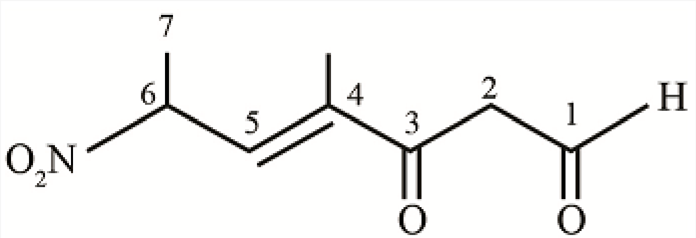
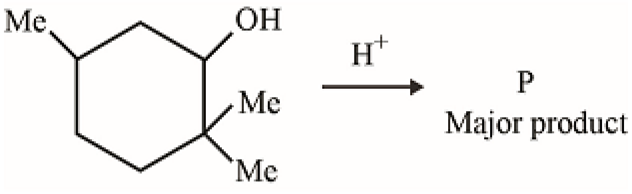
(D) 6-formyl-4-methyl-2-nitrohex-3-enal

Solution:

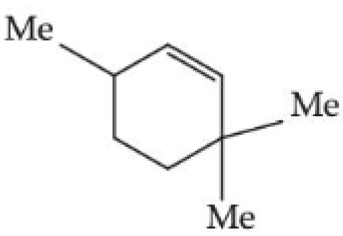


4-methyl-6-nitro-3-oxohept-4-enal

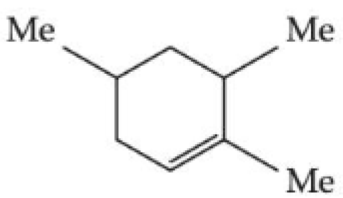
Q14: The major product (P) of the given reaction is
(where, Me is $-CH_3$)



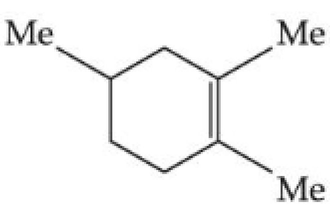
(A)



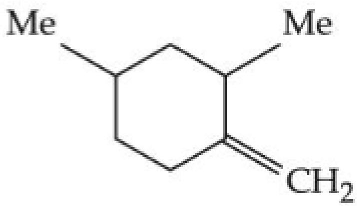
(B)



(C)



(D)



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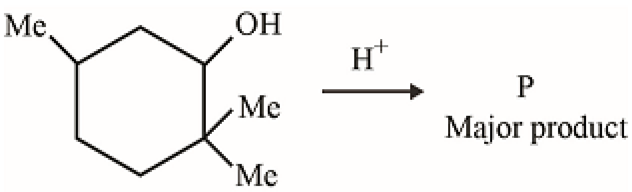
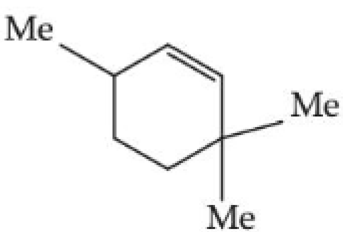
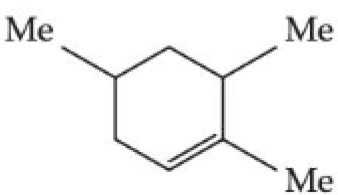


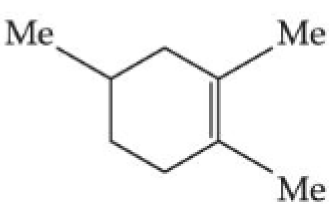
(A)



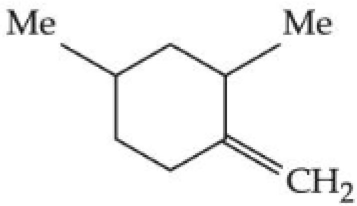
(B)



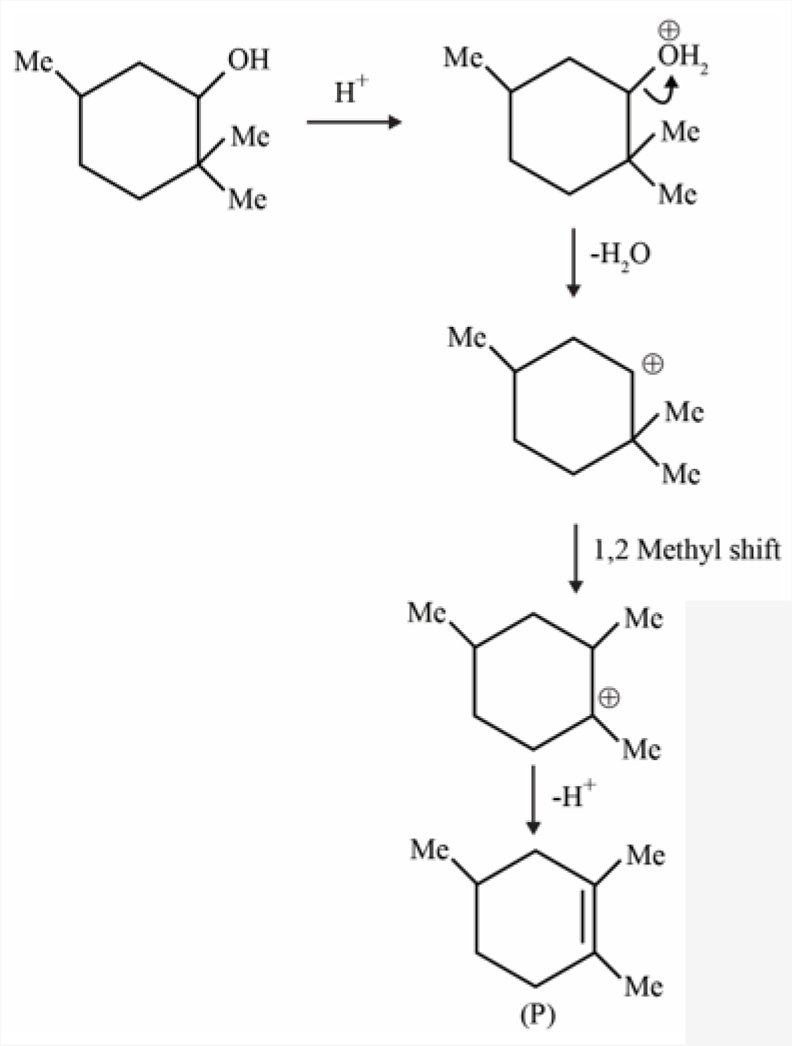
(C)



(D)



Solution:

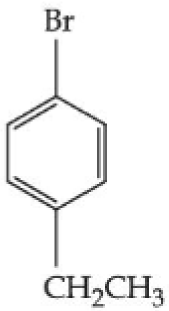


Me
OH
Me
Me
H^+
Me
$\overset{\oplus}{O}H_2$
Me
Me
$-H_2O$
Me
Me
Me
1,2 Methyl shift
Me
Me
Me
$-H^+$
Me
Me
Me
(P)

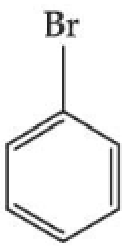
Q15: $A \xrightarrow[\text{(ii)}CN^-,\ \text{(iii)}H_2O/H^+]{\text{(i)}Cl_2,\Delta} \text{4 - Bromophenyl acetic acid}$

In the above reaction' A' is

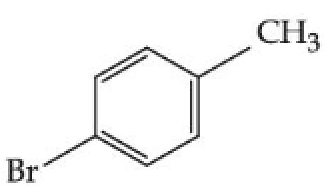
(A)



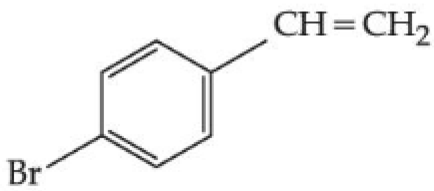
(B)



(C)



(D)



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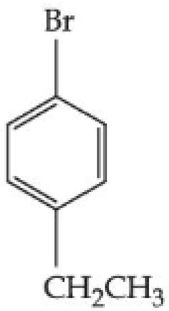
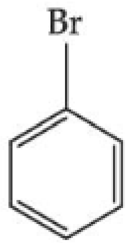
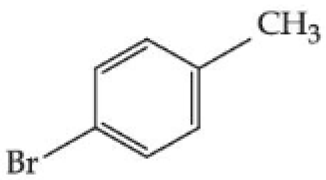
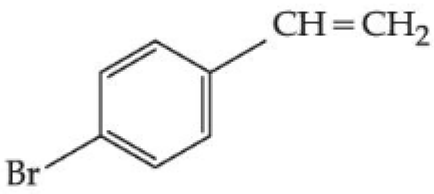


(B)

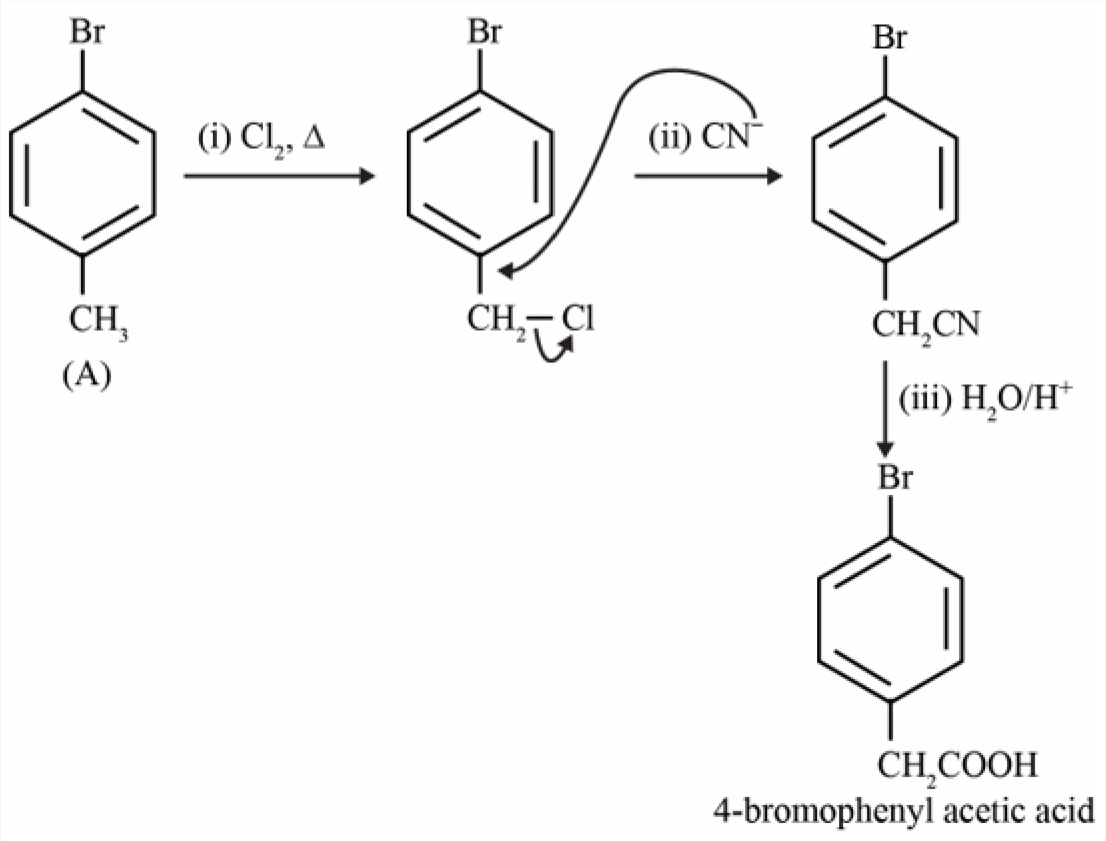


(C)



(D) 

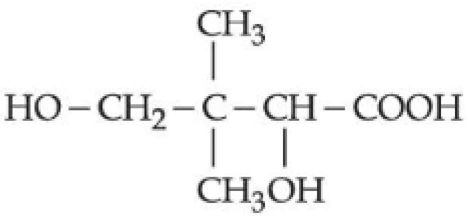
Solution:



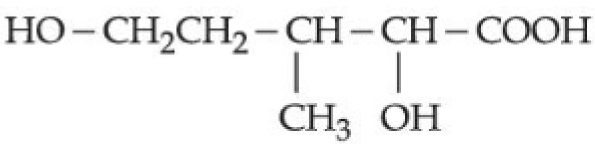
Br
CH3
(A)
(i) Cl2, Δ
Br
CH2—Cl
(ii) CN⁻
Br
CH2CN
(iii) H2O/H⁺
Br
CH2COOH
4-bromophenyl acetic acid

Q16: Isobutyraldehyde on reaction with formaldehyde and K_2CO_3 gives compound ‘A’. Compound ‘A’ reacts with KCN and yields compound ‘B’, which on hydrolysis gives a stable compound ‘C’. The compound ‘C’ is

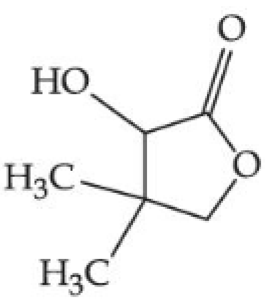
(A)



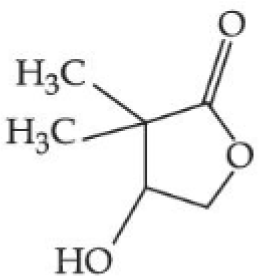
(B)



(C)

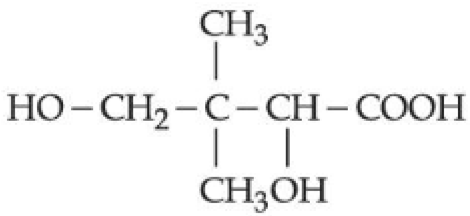


(D)

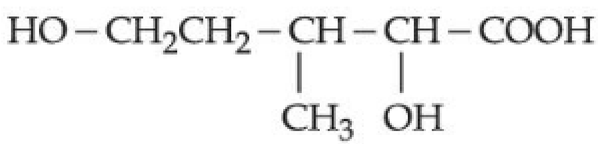


Q16: Isobutyraldehyde on reaction with formaldehyde and K_2CO_3 gives compound ‘A’. Compound ‘A’ reacts with KCN and yields compound ‘B’, which on hydrolysis gives a stable compound ‘C’. The compound ‘C’ is

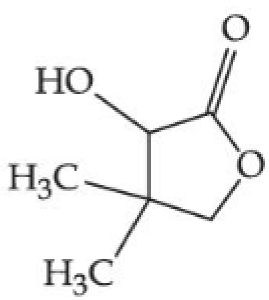
(A)



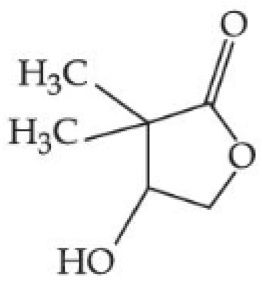
(B)



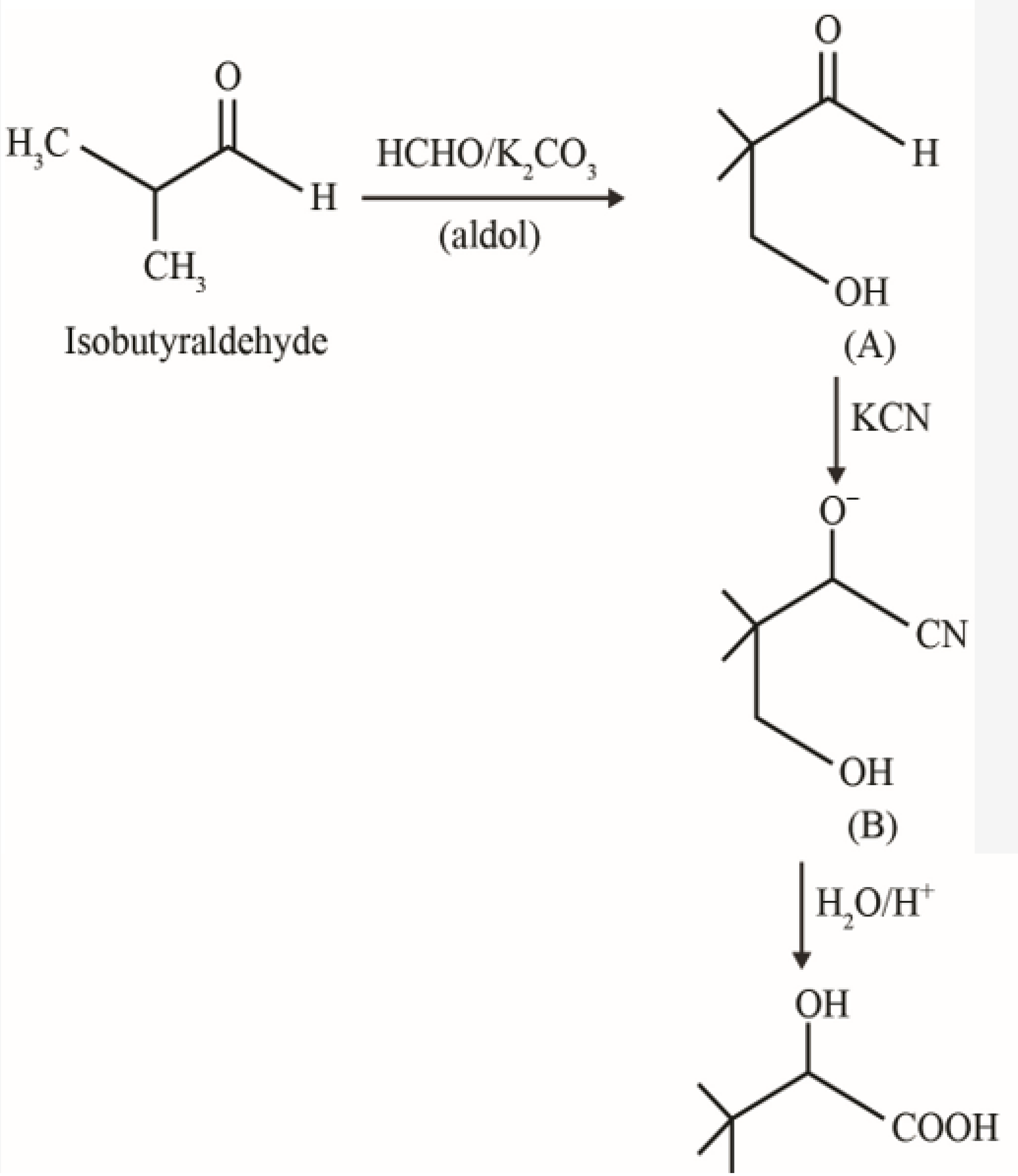
(C)



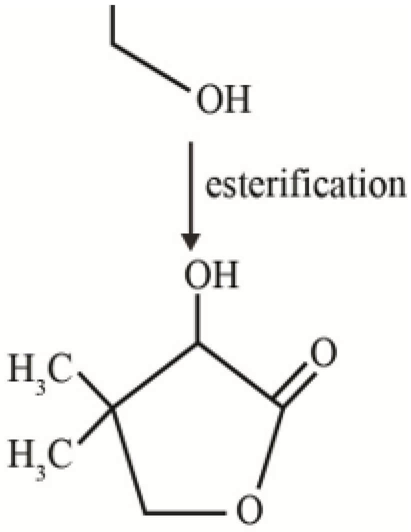
(D)



Solution:

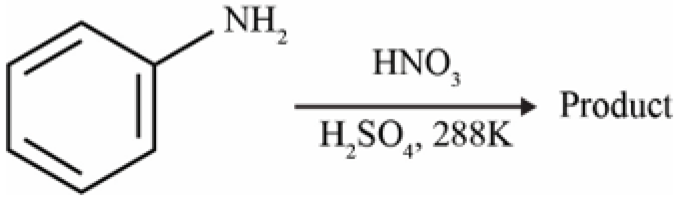


O
H_3C
H
CH_3
Isobutyraldehyde
$HCHO/K_2CO_3$
(aldol)
O
H
OH
(A)
KCN
O^-
CN
OH
(B)
H_2O/H^+
OH
COOH



(C)

Q17: With respect to the following reaction, consider the given statements:



(A) o-Nitroaniline and p-nitroaniline are the predominant products

(B) p-Nitroaniline and m-nitroaniline are the predominant products

(C) HNO_3 acts as an acid.

(D) H_2SO_4 acts as an acid.

Choose the correct option.

(A) (A) and (C) are correct statements

(B) (A) and (D) are correct statements

(C) (B) and (D) are correct statements

(D) (B) and (C) are correct statements

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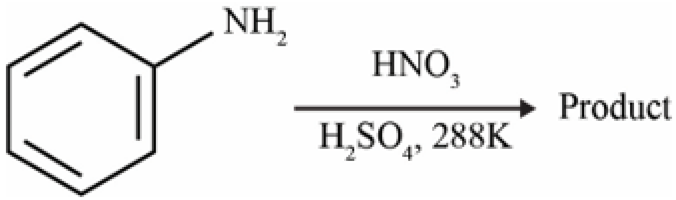
Q17: With respect to the following reaction, consider the given statements:



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Choose the correct option.

(A) (A) and (C) are correct statements

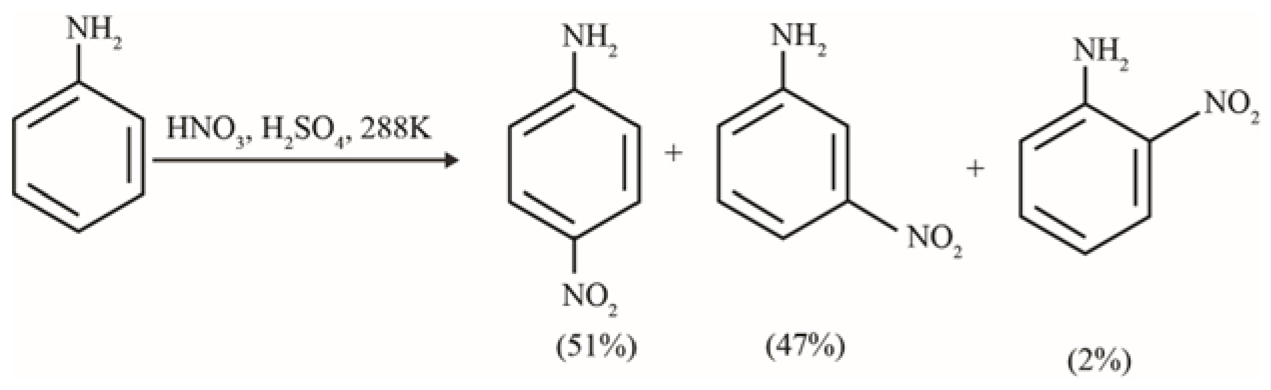
(B) (A) and (D) are correct statements

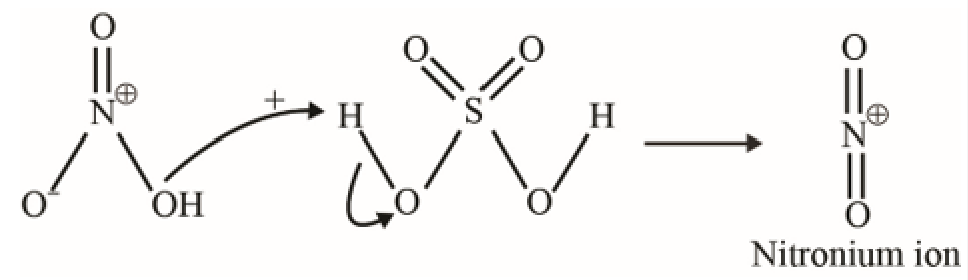
(C) (B) and (D) are correct statements

(D) (B) and (C) are correct statements

Solution:

Direct nitration of aniline yields three oxidation products in addition to the nitro derivatives. Moreover, in the strongly acidic medium, aniline is protonated to form the anilinium ion which is meta directing. That is why besides the ortho and para derivatives, a significant amount of meta derivative is also formed.





Hence, H_2SO_4 acts as an acid.

Q18: Given below are two statements, one is Assertion (A) and other is Reason (R).

Assertion (A): Natural rubber is a linear polymer of isoprene called cis-polyisoprene with elastic properties.

Reason (R): The cis-polyisoprene molecules consist of various chains held together by strong polar interactions with coiled structure.

In the light of the above statements, choose the correct one from the options given below:

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true

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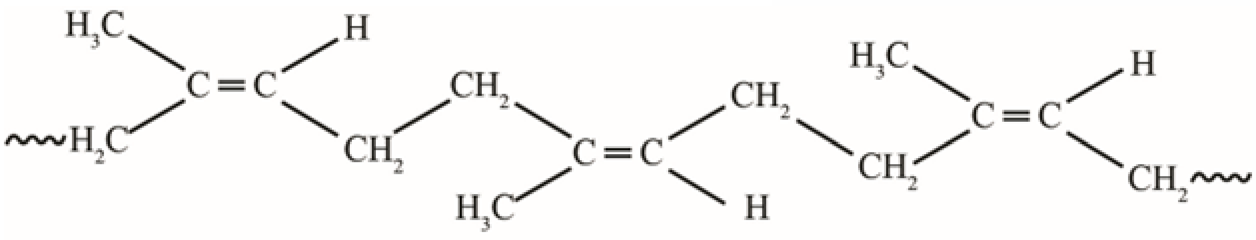
(C) (A) is true but (R) is false

(D) (A) is false but (R) is true

Solution:



Natural rubber

The cis-polyisoprene molecule consists of various chains held together by weak van der Waals interactions and has a coiled structure. Thus, it can be stretched like a spring and exhibits elastic properties.

Hence, the assertion is true, but the reason is false.

Q19: When sugar 'X' is boiled with dilute H_2SO_4 in alcoholic solution, two isomers 'A' and 'B' are formed. 'A' on oxidation with HNO_3 yields saccharic acid whereas 'B' is laevorotatory. The compound 'X' is:

(A) Maltose

(B) Sucrose

(C) Lactose

(D) Strach

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(A) Maltose

(B) Sucrose

(C) Lactose

(D) Strach

Solution:

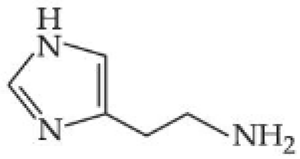
$$\underset{\substack{\text{Sucrose} \\ \text{(X)}}}{C_{12}H_{22}O_{11}} + H_2O \xrightarrow{H^+} \underset{\substack{\text{D-(+)-Glucose} \\ \text{(A)}}}{C_6H_{12}O_6} + \underset{\substack{\text{D-(-)-Fructose} \\ \text{(B)}}}{C_6H_{12}O_6}$$

$$\underset{\text{Glucose}}{\begin{array}{c} CHO \\ | \\ (CHOH)_4 \\ | \\ CH_2OH \end{array}} \xrightarrow{\text{Nitric acid}} \underset{\text{Saccharic acid}}{\begin{array}{c} COOH \\ | \\ (CHOH)_4 \\ | \\ COOH \end{array}}$$

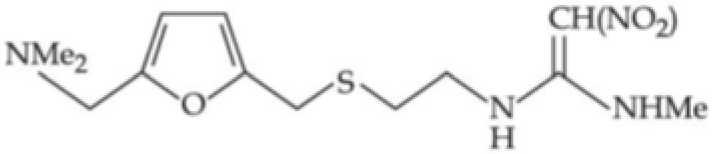
D-(-)- Fructose is a laevorotatory compound.

Q20: The drug tegamet is:

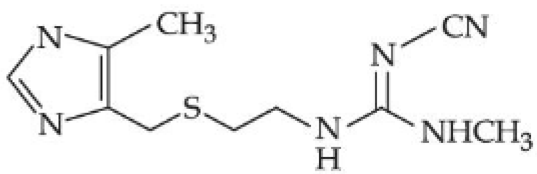
(A)



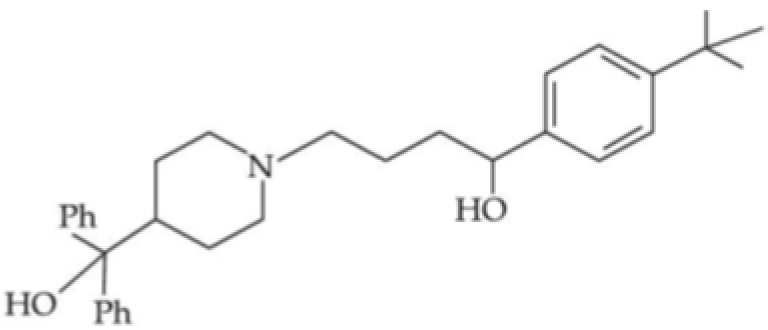
(B)



(C)



(D)



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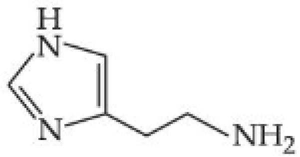
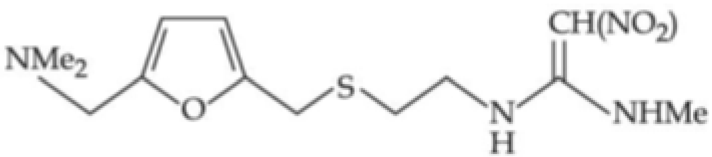
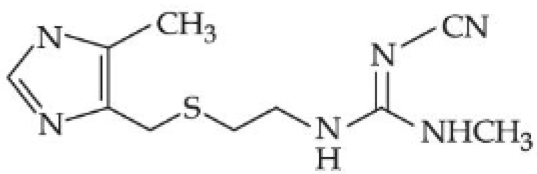
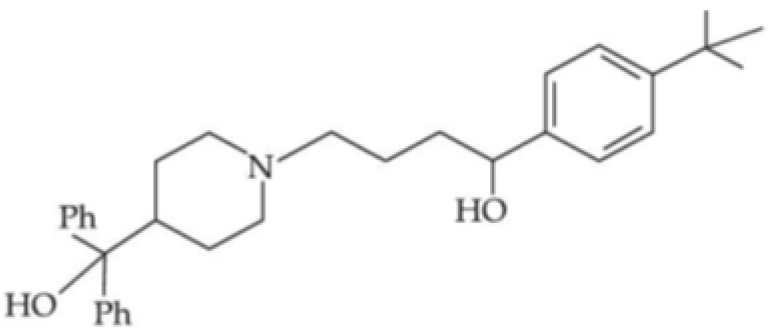
Q20: The drug tegamet is:

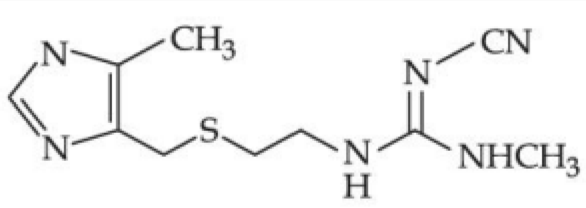
(A)


(B)


(C)


(D)


Solution:



Q21: 100 g of an ideal gas is kept in a cylinder of 416 L volume at 27 °C under 1.5 bar pressure. The molar mass of the gas is _____ g mol^{-1}. (Nearest integer)
(Given: R = 0.083 L bar K^{-1} mol^{-1})

Q21: 100 g of an ideal gas is kept in a cylinder of 416 L volume at 27 °C under 1.5 bar pressure. The molar mass of the gas is _____ g mol^{-1}. (Nearest integer)
(Given: R = 0.083 L bar K^{-1} mol^{-1})

4

Solution:

Using ideal gas equation

PV = nRT

PV = $\frac{w}{M}$RT

$1.5 \times 416 = \frac{100}{M} \times 0.083 \times 300$

$$M = \frac{2490}{624} \approx 4\,g\,mol^{-1}$$

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Q22: For combustion of one mole of magnesium in an open container at 300 K and 1 bar pressure, $\Delta_C H^{\ominus} = -601.70$ kJ mol^{-1}, the magnitude of change in internal energy for the reaction is _____ kJ. (Nearest integer)
(Given: R = 8.3 J K^{-1} mol^{-1})

Q22: For combustion of one mole of magnesium in an open container at 300 K and 1 bar pressure, $\Delta_C H^\ominus = -601.70$ kJ mol^{-1}, the magnitude of change in internal energy for the reaction is _____ kJ. (Nearest integer)
(Given: R = 8.3 J K^{-1} mol^{-1})

600

Solution:

$Mg(s) + \frac{1}{2}O_2(g) \rightarrow MgO(s)$
$\Delta H = \Delta U + \Delta n_g RT$
$\Delta n_g = 0 - (\frac{1}{2}) = -\frac{1}{2}$
$-601.70 = \Delta U - \frac{1}{2}(8.3) \times 300 \times 10^{-3}$
$\Delta U = -601.70 + 1.245$
$\Delta U = -600.45$ kJ
Nearest integer of the magnitude of change in internal energy for the reaction is 600.

Q23: 2.5 g of protein containing only glycine ($C_2H_5NO_2$) is dissolved in water to make 500 mL of solution. The osmotic pressure of this solution at 300 K is found to be 5.03×10^{-3} bar. The total number of glycine units present in the protein is _____.
(Given: R= 0.083 L bar K^{-1} mol^{-1})

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Q23: 2.5 g of protein containing only glycine ($C_2H_5NO_2$) is dissolved in water to make 500 mL of solution. The osmotic pressure of this solution at 300 K is found to be 5.03×10^{-3} bar. The total number of glycine units present in the protein is ______.
(Given: R= 0.083 L bar K^{-1} mol^{-1})

330

Solution:

π = CRT

$$5.03 \times 10^{-3} = \frac{2.5 \times 0.083 \times 300 \times 1000}{M \times 500}$$

$$M = \frac{62{,}250}{5.03 \times 10^{-3} \times 500} = 24751.5 \text{ g mol}^{-1}$$

Number of glycine units in the protein = $\frac{24751.5}{75}$ = 330

Q24: For the given reactions

$Sn^{2+} + 2e^- \rightarrow Sn$

$Sn^{4+} + 4e^- \rightarrow Sn$

the electrode potentials are; $E^o_{Sn^{2+}/Sn}$ = –0.140 V and $E^o_{Sn^{4+}/Sn}$ = 0.010 V. The magnitude of standard electrode potential for Sn^{4+}/Sn^{2+} i.e. $E^o_{Sn^{4+}/Sn^{2+}}$ is ______ $\times 10^{-2}$ V.

(Nearest integer)

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(Nearest integer)

16

Solution:

$Sn^{2+} + 2e^- \rightarrow Sn$ .... E^o_1 = -0.140 V

$Sn^{4+} + 4e^- \rightarrow Sn$ .... E^o_2 = 0.010 V

Overall equation

$Sn^{4+} + 2e^- \rightarrow Sn^{2+}$ E^o_{cell}

$$E^o_{cell} = \frac{n_2E^o_2 - n_1E^o_1}{n} = \frac{4(0.010) - 2(-0.14)}{2}$$

$$E^o_{cell} = \frac{0.04 + 0.28}{2} = \frac{0.32}{2}$$

= 0.16 V

= 16 × 10^{-2} V

Q25: A radioactive element has a half life of 200 days. The percentage of original activity remaining after 83 days is _____. (Nearest integer)
(Given: antilog 0.125 = 1.333, antilog 0.693 = 4.93)

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(Given: antilog 0.125 = 1.333, antilog 0.693 = 4.93)

75

Solution:

$$t_{1/2} = \frac{0.693}{k_c}$$
$$k_c = \left(\frac{0.693}{200}\right)$$
$$k_c = \frac{2.303}{t} \log \frac{[R]_0}{[R]}$$
$$\frac{0.693}{200} = \frac{2.303}{83} \log \frac{[R]_0}{[R]}$$
$$\log \frac{[R]_0}{[R]} = \frac{0.693 \times 83}{200 \times 2.303}$$
$\frac{[R]_0}{[R]}$ = 0.75

Hence, the percentage of original activity remaining after 83 days is 75%.

Q26: Among the given complexes, the number of paramagnetic complexes is ______.

$[Fe(CN)_6]^{4-}$

$[Fe(CN)_6]^{3-}$

$[Ti(CN)_6]^{3-}$

$[Ni(CN)_4]^{2-}$

$[Co(CN)_6]^{3-}$

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Q26: Among the given complexes, the number of paramagnetic complexes is _____.

$[Fe(CN)_6]^{4-}$

$[Fe(CN)_6]^{3-}$

$[Ti(CN)_6]^{3-}$

$[Ni(CN)_4]^{2-}$

$[Co(CN)_6]^{3-}$

2

Solution:

	Valence shell configuration	Magnetic nature
$[Fe(CN)_6]^{4-}$	$3d^6$ (pairing)	Diamagnetic
$[Fe(CN)_6]^{3-}$	$3d^5$ (pairing)	Paramagnetic
$[Ti(CN)_6]^{3-}$	$3d^1$	Paramagnetic
$[Ni(CN)_4]^{2-}$	$3d^8$ (pairing)	Diamagnetic
$[Co(CN)_6]^{3-}$	$3d^6$ (pairing)	Diamagnetic

Number of paramagnetic complexes, is 2

Q27: Number of complex(es) which will exist in cis-trans form is/are ___.

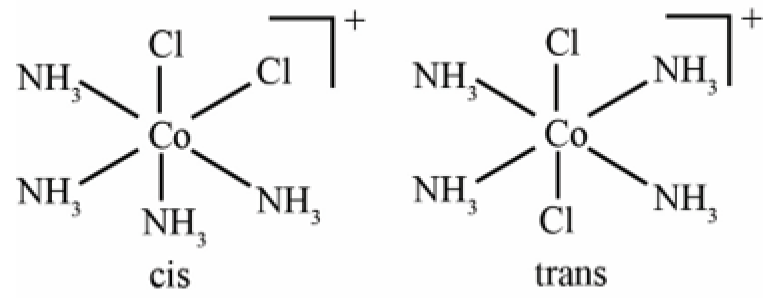
A) $CoCl_3 \cdot 4\ NH_3$, B) $CoCl_3 \cdot 5NH_3$, C) $CoCl_3 \cdot 6NH_3$ and D) $CoCl(NO_3)_2 \cdot 5NH_3$

Q27: Number of complex(es) which will exist in cis-trans form is/are ___.
A) $CoCl_3 \cdot 4\ NH_3$, B) $CoCl_3 \cdot 5NH_3$, C) $CoCl_3 \cdot 6NH_3$ and D) $CoCl(NO_3)_2 \cdot 5NH_3$

1

Solution:

$CoCl_3 \cdot 4NH_3 \Rightarrow [Co(NH_3)_4Cl_2]Cl$
$CoCl_3 \cdot 5NH_3 \Rightarrow [Co(NH_3)_5Cl]Cl_2$
$CoCl_3 \cdot 6NH_3 \Rightarrow [Co(NH_3)_6]Cl_3$
Only $[Co(NH_3)_4Cl_2]^+$ can show geometrical isomerism.
Hence, can exist in cis-trans form.



Geometrical isomers (cis and trans) of $[Co(NH_3)_4Cl_2]^+$

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Q28: The complete combustion of 0.492 g of an organic compound containing 'C', 'H' and 'O' gives 0.793g of CO_2 and 0.442 g of H_2O. The percentage of oxygen composition in the organic compound is ______. (nearest integer)

Q28: The complete combustion of 0.492 g of an organic compound containing 'C', 'H' and 'O' gives 0.793g of CO_2 and 0.442 g of H_2O. The percentage of oxygen composition in the organic compound is _______. (nearest integer)

46

Solution:

$C_xH_yO_z + O_2 \rightarrow CO_2 + H_2O$

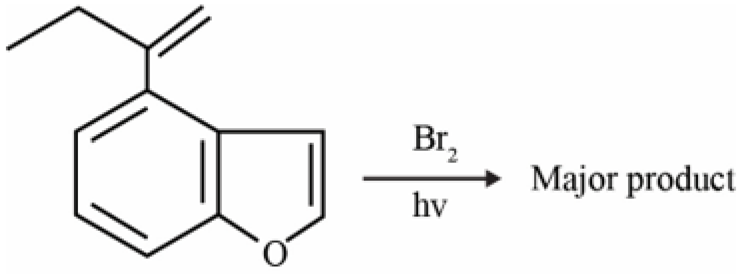
Weight of carbon = $\frac{0.793}{44} \times 12 = 0.216$ g

Weight of hydrogen = $\frac{0.442}{18} \times 2 = 0.05$ g

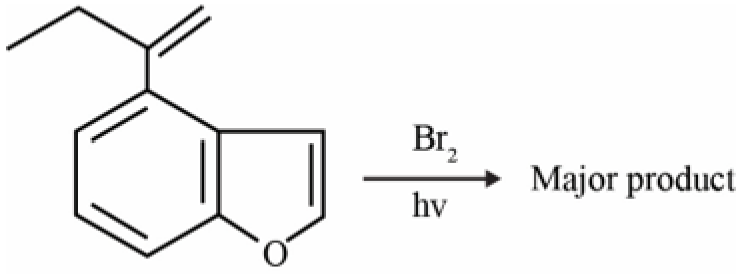
Weight of oxygen = 0.492 – (0.216 + 0.05) = 0.226 g

% by mass of oxygen in compound = $\frac{0.226}{0.492} \times 100 = 46\%$

Q29: The major product of the following reaction contains ____ bromine atom(s).

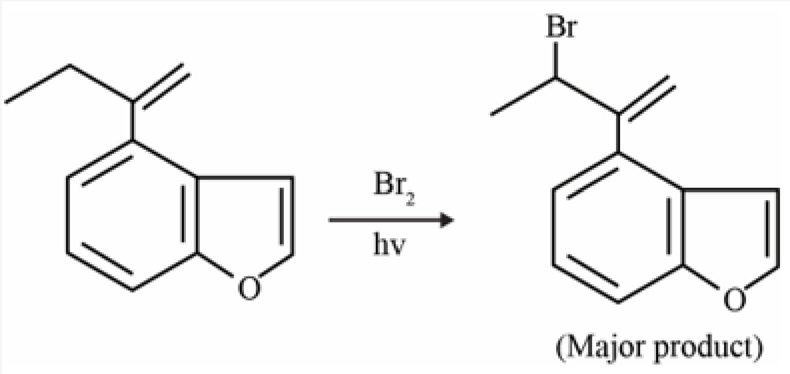


Q29: The major product of the following reaction contains _____ bromine atom(s).



1

Solution:



The major product contains only 1 bromine atom.

Q30: 0.01 M $KMnO_4$ solution was added to 20.0 mL of 0.05 M Mohr's salt solution through a burette. The initial reading of 50 mL burette is zero. The volume of $KMnO_4$ solution left in the burette after the end point is _____ ml. (nearest integer)

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Q30: 0.01 M $KMnO_4$ solution was added to 20.0 mL of 0.05 M Mohr's salt solution through a burette. The initial reading of 50 mL burette is zero. The volume of $KMnO_4$ solution left in the burette after the end point is _____ ml. (nearest integer)

30

Solution:

$$(\mathrm{M} \times \mathrm{V} \times \mathrm{n_F})_{\mathrm{KMnO_4}} = (\mathrm{M} \times \mathrm{V} \times \mathrm{n_F})_{\text{Mohr's salt}}$$

0.01 × 20 × 5 = 0.05 × V × 1

Volume required = 20 ml

The initial volume of $KMnO_4$ in a burette is 50 ml. Hence, the volume of $KMnO_4$ left in the burette after the end point is 30 ml.

Q31: Let $R_1 = \{(a, b) \in N \times N : |a - b| \leqslant 13\}$ and $R_2 = \{(a, b) \in N \times N : |a - b| \neq 13\}$. Then on N :

(A) Both R_1 and R_2 are equivalence relations

(B) Neither R_1 nor R_2 is an equivalence relation

(C) R_1 is an equivalence relation but R_2 is not

(D) R_2 is an equivalence relation but R_1 is not

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(A) Both R_1 and R_2 are equivalence relations

(B) Neither R_1 nor R_2 is an equivalence relation

(C) R_1 is an equivalence relation but R_2 is not

(D) R_2 is an equivalence relation but R_1 is not

Solution:

$R_1 = \{(a, b) \in N \times N : |a - b| \leqslant 13\}$
$R_2 = \{(a, b) \in N \times N : |a - b| \neq 13\}$.
For R_1 :
(i) Reflexive relation
$(a, a) \in N \times N : |a - a| \leqslant 13$
(ii) Symmetric relation
$(a, b) \in R_1, (b, a) \in R_1 : |b - a| \leqslant 13$
(iii) Transitive relation
$(a, b) \in R_1, (b, c) \in R_1, (a, c) \in R_1 :$
$(1, 3) \in R_1, (3, 16) \in R_1$, but $(1, 16) \notin R_1$
For R_2 :
(i) Reflexive relation
$(a, a) \in N \times N : |a - a| \neq 13$
(ii) Symmetric relation
$(b, a) \in N \times N : |b - a| \neq 13$
(iii) Transitive relation
$(a, b) \in R_2, (b, c) \in R_2, (a, c) \in R_2$
$(1, 3) \in R_2, (3, 14) \in R_2$, but $(1, 14) \notin R_2$

Q32: Let $f(x)$ be a quadratic polynomial such that $f(-2)+f(3)=0$. If one of the roots of $f(x)=0$ is -1, then the sum of the roots of $f(x)=0$ is equal to :

(A) $\frac{11}{3}$

(B) $\frac{7}{3}$

(C) $\frac{13}{3}$

(D) $\frac{14}{3}$

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(A) $\frac{11}{3}$

(B) $\frac{7}{3}$

(C) $\frac{13}{3}$

(D) $\frac{14}{3}$

Solution:

$f(-2) + f(3) = 0$
$f(x) = (x+1)(ax+b)$
$f(-2) + f(3) = -1(-2a+b) + 4(3a+b) = 0$
$2a - b + 12a + 4b = 0$
$14a + 3b = 0$
$\frac{-b}{a} = \frac{14}{3}$
Sum of roots $= \left(-1 + \frac{-b}{a}\right) = -1 + \frac{14}{3} = \frac{11}{3}$

Q33: The number of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4 so that C_2 receives atleast 4 and atmost 7 candies, C_3 receives atleast 2 and atmost 6 candies, is equal to

(A) 205

(B) 615

(C) 510

(D) 430

Q33: The number of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4 so that C_2 receives atleast 4 and atmost 7 candies, C_3 receives atleast 2 and atmost 6 candies, is equal to

(A) 205

(B) 615

(C) 510

(D) 430

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Solution:

$t_1 + t_2 + t_3 + t_4 = 30$

Coefficient of x^{30} in $(1 + x + x^2 + \ldots\ldots + x^{30})^2$

$(x^4 + x^5 + x^6 + x^7)(x^2 + x^3 + x^4 + x^5 + x^6)$

$x^6 \left(\frac{1-x^{31}}{1-x}\right)(1 + x + x^2 + x^3)(1 + x + x^2 + x^3 + x^4)$

$x^6 (1 - x^{31})^2 (1 - x^4)(1 - x^5)(1 - x)^{-4}$

$x^6 (1 - x^4 - x^5 + x^9)(1 + x^{62} - 2x^{31} (1 - x)^{-4})$

$x^6 (1 - x^4 - x^5 + x^9)(1 - x)^{-4}$

Coefficient of x^n in $(1 - x)^{-r}$ is ${}^{n+r-1}C_{r-1}$

$\Rightarrow {}^{27}C_3 - {}^{23}C_3 - {}^{22}C_3 + {}^{18}C_3$

$2925 - 1771 - 1540 + 816$

$= 430$

OR

$x_2 \in [4. 7],\ x_3 \in [2, 6]$

$\Rightarrow t_1 + t_2 + t_3 + t_4 = 24$

total ways =

${}^{24+4-1}C_{4-1} - {}^{20+4-1}C_{4-1} - {}^{19+4-1}C_{4-1} + {}^{15+4-1}C_{4-1}$

$= {}^{27}C_3 - {}^{23}C_3 - {}^{22}C_3 + {}^{18}C_3 = 430$

Q34: The term independent of x in the expression of $(1-x^2+3x^3)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$, $x \neq 0$ is

(A) $\frac{7}{40}$

(B) $\frac{33}{200}$

(C) $\frac{39}{200}$

(D) $\frac{11}{50}$

Q34: The term independent of x in the expression of $(1-x^2+3x^3)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$, $x \neq 0$ is

(A) $\frac{7}{40}$

(B) $\frac{33}{200}$

(C) $\frac{39}{200}$

(D) $\frac{11}{50}$

Solution:

$(1 - x^2 + 3x^3)\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$

General term of $\left(\frac{5}{2}\,x^3 - \frac{1}{5x^2}\right)^{11}$ is

${}^{11}C_r\ \left(\frac{5}{2}x^3\right)^{11-r}\ \left(-\frac{1}{5x^2}\right)^r$

General term is ${}^{11}C_r\ \left(\frac{5}{2}\right)^{11-r}\ \left(-\frac{1}{5}\right)^r\ x^{33\,-\,5r}$

Now, term independent of x

1 × coefficient of x^0 in $\left(\frac{5}{2}\,x^3 - \frac{1}{5x^2}\right)^{11}$

$-1\ \times$ coefficient of x^{-2} in $\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11} +$

$3\ \times$ coefficient of x^{-3} in $\left(\frac{5}{2}x^3 - \frac{1}{5x^2}\right)^{11}$

for coefficient of x^0 $33 - 5r = 0$ not possible

for coefficient of x^{-2} $33 - 5r = -2$

$35 = 5r \Rightarrow r = 7$

for coefficient of x^{-3} $33 - 5r = -3$

36 = 5r not possible

So term independent of x is

$(-1)\ {}^{11}C_7\ \left(\frac{5}{2}\right)^4\ \left(-\frac{1}{5}\right)^7 = \frac{33}{200}$

Q35: If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is $1:7$ and $a+n=33$, then the value of n is

(A) 21

(B) 22

(C) 23

(D) 24

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Q35: If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is $1 : 7$ and $a + n = 33$, then the value of n is

(A) 21

(B) 22

(C) 23

(D) 24

Solution:

$d = \frac{100-a}{n+1}$

$A_1 = a + d$

$A_n = 100 - d$

$\Rightarrow \frac{A_1}{A_n} = \frac{1}{7} \Rightarrow \frac{a+d}{100-d} = \frac{1}{7}$

$\Rightarrow 7a + 8d = 100$

$\Rightarrow 7a + 8\left(\frac{100-a}{n+1}\right) = 100 \quad \ldots.(1)$

$\because \ a + n = 33 \quad \ldots.(2)$

Now, by Eq. (1) and (2)

$7n^2 - 132n - 667 = 0$

$\boxed{n = 23}$ and $n = \frac{-29}{7}$ reject.

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Q36: Let $f,\ g\ :\ R\ \to\ R$ be function defined by

$$g(x) = \begin{cases} [x] & , \quad x < 0 \\ |1-x| & , \quad x \geqslant 0 \end{cases}$$ and

$$g(x) = \begin{cases} e^x - x & , \quad x < 0 \\ (x-1)^2 - 1 & , \quad x \geqslant 0 \end{cases}$$

where $[x]$ denote the greatest integer less than or equal to x. Then, the function fog is discontinuous at exactly :

(A) one point

(B) two points

(C) three points

(D) four points

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where $[x]$ denote the greatest integer less than or equal to x. Then, the function fog is discontinuous at exactly :

(A) one point

(B) two points

(C) three points

(D) four points

Solution:

Check continuity at $x = 0$ and also check continuity at those x where $g(x) = 0$

$g(x) = 0$ at $x = 0, 2$

$fog(0^+) = -1$

$fog(0) = 0$

Hence, discontinuous at $x = 0$

$fog(2^+) = 1$

$fog(2^-) = -1$

Hence, discontinuous at $x = 2$

Q37: Let $f : R \rightarrow R$ be a differentiable function such that $f\left(\frac{\pi}{4}\right)=\sqrt{2},\ f\left(\frac{\pi}{2}\right)=0$ and $f'\left(\frac{\pi}{2}\right)=1$ and let $g(x)=\int_{x}^{\pi/4}\left(f'(t)\sec t+\tan t\sec t\, f(t)\right)dt$ for $x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then $\lim_{x \rightarrow \left(\frac{\pi}{2}\right)^{-}} g(x)$ is equal to

(A) 2

(B) 3

(C) 4

(D) -3

Q37: Let $f : R \rightarrow R$ be a differentiable function such that $f\left(\frac{\pi}{4}\right)=\sqrt{2},\ f\left(\frac{\pi}{2}\right)=0$ and $f'\left(\frac{\pi}{2}\right)=1$ and let $g(x)=\int_{x}^{\pi/4}\left(f'(t)\sec t+\tan t\sec t\, f(t)\right) dt$ for $x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then $\lim_{x \rightarrow \left(\frac{\pi}{2}\right)^{-}} g(x)$ is equal to

(A) 2

(B) 3

(C) 4

(D) -3

Solution:

$$g(x) = \int_x^{\pi/4} (f'(t) \sec t + \tan t \sec t\, f(t)\, dt$$

$$g(x) = \int_x^{\pi/4} d(f(t) \cdot \sec t) = f(t) \sec t \Big|_x^{\pi/4}$$

$$g(x) = f\left(\frac{\pi}{4}\right) \sec \frac{\pi}{4} - f(x) . \sec x$$

$$g(x) = 2 - f(x) \sec x = 2 - \left(\frac{f(x)}{\cos x}\right)$$

$$\lim_{x \to \left(\frac{\pi}{2}\right)^-} g(x) = 2 - \lim_{x \to \left(\frac{\pi}{2}\right)^-} \left(\frac{f(x)}{\cos x}\right)$$

using L'Hopital Rule

$$= 2 - \lim_{x \to \left(\frac{\pi}{2}\right)^-} \frac{f'(x)}{(-\sin x)}$$

$$= 2 + \frac{f'\left(\frac{\pi}{2}\right)}{\sin \frac{\pi}{2}} = 2 + \frac{1}{1} = 3$$

Q38: Let $f : R \rightarrow R$ be continuous function satisfying $f(x) + f(x+k) = n$, for all $x \in R$ where $k > 0$ and n is a positive integer. If $I_1 = \int_0^{4nk} f(x)\,dx$ and $I_2 = \int_{-k}^{3k} f(x)\,dx$, then

(A) $I_1 + 2I_2 = 4nk$

(B) $I_1 + 2I_2 = 2nk$

(C) $I_1 + nI_2 = 4n^2k$

(D) $I_1 + nI_2 = 6n^2k$

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(A) $I_1 + 2I_2 = 4nk$

(B) $I_1 + 2I_2 = 2nk$

(C) $I_1 + nI_2 = 4n^2k$

(D) $I_1 + nI_2 = 6n^2k$

Solution:

$f(x)+f(x+k)=n$

$\Rightarrow f(x)=f(x+2k)$

$f(x)$ is periodic with period $2k$

$I_1=\int_0^{4nk} f(x)\,dx=2n\int_0^{2k} f(x)\,dx$

Now,

$f(x)+f(x+k)=n$

$\Rightarrow \int_0^{k} f(x)\,dx+\int_0^{k} f(x+k)\,dx=nk$

$\Rightarrow \int_0^{k} f(x)\,dx+\int_0^{2k} f(x)\,dx=nk$

$\Rightarrow \int_0^{2k} f(x)\,dx=nk$

$\Rightarrow I_1=2n^2k,\ I_2=2nk$

$\Rightarrow I_1+nI_2=4n^2k$

Q39: The area of the bounded region enclosed by the curve $y = 3 - \left|x - \frac{1}{2}\right| - |x + 1|$ and the x-axis is

(A) $\frac{9}{4}$

(B) $\frac{45}{16}$

(C) $\frac{27}{8}$

(D) $\frac{63}{16}$

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Q39: The area of the bounded region enclosed by the curve $y = 3 - \left|x - \frac{1}{2}\right| - |x + 1|$ and the x-axis is

(A) $\frac{9}{4}$

(B) $\frac{45}{16}$

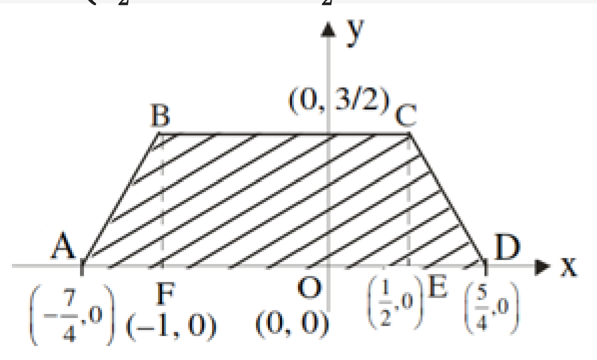
(C) $\frac{27}{8}$

(D) $\frac{63}{16}$

Solution:

$$y=\begin{cases}3+(x+1)+\left(x-\frac{1}{2}\right), & x\leqslant-1\\ 3-(x+1)+\left(x-\frac{1}{2}\right), & -1\leqslant x<\frac{1}{2}\\ 3-(x+1)-\left(x-\frac{1}{2}\right), & \frac{1}{2}\leqslant x\end{cases}$$

$$y=\begin{cases}\frac{7}{2}+2x, & x<-1\\ \frac{3}{2}, & -1\leqslant x<\frac{1}{2}\\ \frac{5}{2}-2x, & \frac{1}{2}\leqslant x\end{cases}$$



Area bounded = $ar\ ABF+ar\ BCEF+ar\ CDE$

$=\frac{1}{2}\left(\frac{3}{4}\right)\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)+\frac{1}{2}\left(\frac{3}{4}\right)\left(\frac{3}{2}\right)$

$=\frac{27}{8}$ sq. units.

Q40: Let $x = x(y)$ be the solution of the differential equation $2ye^{x/y^2}\,dx + \left(y^2 - 4xe^{x/y^2}\right)dy = 0$ such that $x(1) = 0$. Then, $x(e)$ is equal to

(A) $e\log_e(2)$

(B) $-e\log_e(2)$

(C) $e^2\log_e(2)$

(D) $-e^2\log_e(2)$

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$2ye^{x/y^2}\,dx + \left(y^2 - 4xe^{x/y^2}\right)dy = 0$ such that $x(1) = 0$. Then, $x(e)$ is equal to

(A) $e\log_e(2)$

(B) $-e\log_e(2)$

(C) $e^2\log_e(2)$

(D) $-e^2\log_e(2)$

Solution:

$$2y\,e^{x/y^2}\ dx + \ (y^2 - 4x\ e^{x/y^2})\ dy\ =\ 0$$

$$2e^{x/y^2}\,[ydx - \ 2xdy + \ y]^2\ dy = \ 0$$

$$2e^{x/y^2}\ \left[\frac{y^2 dx - x\cdot\ (2y)\ dy}{y}\right] + \ y^2 dy = \ 0$$

Divide by y^3

$$2e^{x/y^2}\ \left[\frac{y^2 dx - x\cdot\ (2y)\ dy}{y^4}\right] + \ \frac{1}{y}\ dy = \ 0$$

$$2e^{x/y^2}\ d\ \left(\frac{x}{y^2}\right) + \ \frac{1}{y}\ dy = \ 0$$

integrating

$$\int 2e^{x/y^2} d\ \left(\frac{x}{y^2}\right) + \ f\ \frac{1}{y}\ dy = \ 0$$

$$2e^{x/y^2}\ + \ \ell ny\ + \ c = \ 0$$

(0, 1) lies on it.

$$2e^0\ + \ \ell n1\ + \ c\ =\ 0\ \ \Rightarrow\ c = -\ 2$$

Required curve : $\boxed{2e^{x/y^2}\ + \ \ell ny - 2\ =\ 0}$

For $x\ (e)$

$$2e^{x/e^2}\ + \ \ell ne - \ 2 = \ 0\ \ \Rightarrow\ \ x = -\ e^2\log_e\ 2$$

Q41: Let the slope of the tangent to a curve $y = f(x)$ at (x, y) be given by $2\ \tan x\ (\cos x - y)$. If the curve passes through the point $\left(\frac{\pi}{4},\ 0\right)$, then the value of $\int_0^{\pi/2} y dx$ is equal to

(A) $(2 - \sqrt{2}) + \frac{\pi}{\sqrt{2}}$

(B) $2 - \frac{\pi}{\sqrt{2}}$

(C) $(2 + \sqrt{2}) + \frac{\pi}{\sqrt{2}}$

(D) $2 + \frac{\pi}{\sqrt{2}}$

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(B) $2 - \frac{\pi}{\sqrt{2}}$

(C) $(2 + \sqrt{2}) + \frac{\pi}{\sqrt{2}}$

(D) $2 + \frac{\pi}{\sqrt{2}}$

Solution:

$\frac{dy}{dx} = 2\ \tan\ x\ \cos\ x - 2\ \tan\ x \cdot\ y$

$\frac{dy}{dx} +\ (2\ \tan)\ x \cdot y\ =\ 2 \sin\ x$

Integrating factor $= e^{\int 2\ \tan\ x\ dx}\ =\ \frac{1}{\cos^2\ x}$

$y\ \left(\frac{1}{\cos^2 x}\right) =\ \int \frac{2 \sin\ x}{\cos^2 x}\ dx$

$y \sec^2 x =\ \frac{2}{\cos\ x}\ +\ C$

$y =\ 2\ \cos\ x +\ C \cos^2\ x$

Passes through $\left(\frac{\pi}{4},\ 0\right)$

$0 =\ \sqrt{2} +\ \frac{C}{2}\ \ \Rightarrow\ C = -\ 2\sqrt{2}$

$f\,(x) =\ 2\ \cos\ x -\ 2\sqrt{2} \cos^2\ x$: Required curve

$\int_0^{\pi/2} y dx =\ 2\ \int_0^{\pi/2} \cos\ x\, dx -\ 2\sqrt{2}\ \int_0^{\pi/2} \cos^2 x\, dx$

$=\ \left[2 \sin\ x\right]_0^{\pi/2}\ - 2\sqrt{2}\ \left[\frac{x}{2} +\ \frac{\sin\ 2x}{4}\right]_0^{\pi/2}$

$=\ 2 -\ \frac{\pi}{\sqrt{2}}$

Q42: Let a triangle be bounded by the lines $L_1 : 2x + 5y = 10$; $L_2 : -4x + 3y = 12$ and the line L_3, which passes through the point P (2, 3), intersect L_2 at A and L_1 at B. If the point P divides the line-segment AB, internally in the ratio 1 : 3, then the area of the triangle is equal to

(A) $\frac{110}{13}$

(B) $\frac{132}{13}$

(C) $\frac{142}{13}$

(D) $\frac{151}{13}$

Q42: Let a triangle be bounded by the lines $L_1 : 2x + 5y = 10$; $L_2 : -4x + 3y = 12$ and the line L_3, which passes through the point P (2, 3), intersect L_2 at A and L_1 at B. If the point P divides the line-segment AB, internally in the ratio 1 : 3, then the area of the triangle is equal to

(A) $\frac{110}{13}$

(B) $\frac{132}{13}$

(C) $\frac{142}{13}$

(D) $\frac{151}{13}$

Solution:

Points A lies on L_2

$A\ \left(\alpha,\ 4+\ \frac{4}{3}\ \alpha\right)$

Points B lies on L_1

$B\ \left(\beta,\ 2-\ \frac{2}{5}\ \beta\right)$

Points P divides AB internally in the ratio 1 : 3

$$\Rightarrow\ P\,(2,\ 3)\ =\ P\ \left(\frac{3\alpha+\beta}{4},\ \frac{3\ \left(4+\ \frac{4}{3}\ \alpha\right)+\ 1\ \left(2-\ \frac{2}{5}\ \beta\right)}{4}\right)$$

$$\Rightarrow\ \ \alpha = \frac{3}{13},\ \ \beta\ =\ \frac{95}{13}$$

Point $A\ \left(\frac{3}{13},\ \frac{56}{13}\right),\ B\ \left(\frac{95}{13}, -\ \frac{12}{13}\right)$

Vertex C of triangle is the point of intersection

$L_1\ \&\ L_2$

$$\Rightarrow\ \ C\ \left(-\frac{15}{13},\ \frac{32}{13}\right)$$

$$area\ \Delta ABC = \frac{1}{2}\left\|\begin{matrix}\frac{3}{13} & \frac{56}{13} & 1\\ \frac{95}{13} & -\frac{12}{13} & 1\\ -\frac{15}{13} & \frac{32}{13} & 1\end{matrix}\right\|$$

$$= \frac{1}{2\ \times\ 13^3}\left\|\begin{matrix}3 & 56 & 13\\ 95 & -12 & 13\\ -15 & 32 & 13\end{matrix}\right\|$$

area $\Delta ABC = \frac{132}{13}$ sq. units.

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Q43: Let $a > 0, b > 0$. Let e and ℓ respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Let e' and ℓ' respectively the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2 = \frac{11}{14}\ell$ and $(e')^2 = \frac{11}{8}\ell'$, then the value of $77a + 44b$ is equal to

(A) 100

(B) 110

(C) 120

(D) 130

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Q43: Let $a > 0, b > 0$. Let e and ℓ respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Let e' and ℓ' respectively the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2 = \frac{11}{14}\ell$ and $(e')^2 = \frac{11}{8}\ell'$, then the value of $77a + 44b$ is equal to

(A) 100

(B) 110

(C) 120

(D) 130

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Solution:

$e = \sqrt{1 + \frac{b^2}{a^2}}, \quad \ell = \frac{2b^2}{a}$

Given $e^2 = \frac{11}{14}\ell$

$1 + \frac{b^2}{a^2} = \frac{11}{14} \cdot \frac{2b^2}{a}$

$\frac{a^2 + b^2}{a^2} = \frac{11}{7} \cdot \frac{b^2}{a} \qquad \ldots.(1)$

Also $e' = \sqrt{1 + \frac{a^2}{b^2}}, \; \ell' = \frac{2a^2}{b}$

Given $(e')^2 = \frac{11}{8}\ell'$

$1 + \frac{a^2}{b^2} = \frac{11}{8} \cdot \frac{2a^2}{b}$

$\frac{a^2 + b^2}{b^2} = \frac{11}{4} \cdot \frac{a^2}{b} \qquad \ldots.(2)$

New $(1) \div (2)$

$\frac{b^2}{a^2} = \frac{4}{7} \cdot \frac{b^3}{a^3}$

$\therefore \; 7a = 4b \qquad \ldots.(3)$

From (2)

$\frac{\frac{16b^2}{49} + b^2}{b^2} = \frac{11}{4} \cdot \frac{16b^2}{49b}$

$\frac{65}{49} = \frac{11}{4} \cdot \frac{16}{49} \cdot b$

$\therefore \; b = \frac{4 \times 65}{11 \times 16} \qquad \ldots.(4)$

We have to find value of $77a + 44b$

$11\,(7a + 4b) = 11\,(4b + 4b) = 11 \times 8b$

$\therefore$ Value of $11 \times 8b = 11 \times 8 \times \frac{4 \times 65}{16 \times 11} = 130$

Q44: Let $\vec{a} = \alpha\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + \alpha\hat{j} + \hat{k}$, where $\alpha \in R$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $\sqrt{15(\alpha^2 + 4)}$, then the value of $2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2$ is equal to

(A) 10

(B) 7

(C) 9

(D) 14

Q44: Let $\vec{a} = \alpha\hat{i} + 2\hat{j} - \hat{k}$ and $\vec{b} = -2\hat{i} + \alpha\hat{j} + \hat{k}$, where $\alpha \in R$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $\sqrt{15(\alpha^2 + 4)}$, then the value of $2|\vec{a}|^2 + (\vec{a} \cdot \vec{b})|\vec{b}|^2$ is equal to

(A) 10

(B) 7

(C) 9

(D) 14

Solution:

$\vec{a} = \alpha\hat{i} + 2\hat{j} - \hat{k},\ \vec{b} = -2\hat{i} + \alpha\hat{j} + \hat{k},$

area of parallelogram $= |\hat{a} \times \hat{b}|$

$|\hat{a} \times \hat{b}| = \sqrt{(\alpha + 2)^2 + (\alpha - 2)^2 + (\alpha^2 + 4)^2}$

Given $|\hat{a} \times \hat{b}| = \sqrt{15(\alpha^2 + 4)}$

$2(\alpha^2 + 4) + (\alpha^2 + 4)^2 = 15(\alpha^2 + 4)$

$(\alpha^2 + 4)^2 = 13(\alpha^2 + 4)$

$\Rightarrow \alpha^2 + 4 = 13 \quad \therefore \alpha^2 = 9$

$2|\vec{a}|^2 + (\vec{a}.\vec{b})|\vec{b}|^2$

$|\vec{a}|^2 = \alpha^2 + 4 + 1 = \alpha^2 + 5$

$|\vec{b}|^2 = 4 + \alpha^2 + 1 = \alpha^2 + 5$

$\vec{a}.\vec{b} = -2\alpha + 2\alpha - 1 = -1$

$\therefore \quad 2|\vec{a}|^2 + (\vec{a}.\vec{b})|\vec{b}|^2$

$2(\alpha^2 + 5) - 1(\alpha^2 + 5) = \alpha^2 + 5 = 14$

Q45: If vertex of a parabola is $(2,\ -1)$ and the equation of its directrix is $4x-\ 3y=\ 21$, then the length of its latus rectum is

(A) 2

(B) 8

(C) 12

(D) 16

Q45: If vertex of a parabola is $(2, -1)$ and the equation of its directrix is $4x - 3y = 21$, then the length of its latus rectum is

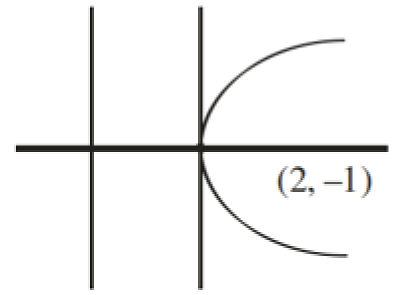
(A) 2

(B) 8

(C) 12

(D) 16

Solution:



$4x - 3y = 21$

$a = \frac{18+3-21|}{5} = \frac{10}{5} = 2$

$\therefore$ latus rectum $= 4a = 8$

Q46: Let the plane $ax + by + cz = d$ pass through (2, 3, –5) and is perpendicular to the planes $2x + y - 5z = 10$ and $3x + 5y - 7z = 12$.
If a, b, c, d are integers $d > 0$ and gcd $(|a|,\ |b|,\ |c|,\ d)$ = 1, then the value of $a +\ 7b +\ c +\ 20d$ is equal to

(A) 18

(B) 20

(C) 24

(D) 22

Q46: Let the plane $ax + by + cz = d$ pass through (2, 3, –5) and is perpendicular to the planes $2x + y - 5z = 10$ and $3x + 5y - 7z = 12$.
If a, b, c, d are integers $d > 0$ and gcd $(|a|,\ |b|,\ |c|,\ d)$ = 1, then the value of $a + \ 7b + \ c + \ 20d$ is equal to

(A) 18

(B) 20

(C) 24

(D) 22

Solution:

DR's normal of plane

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -5 \\ 3 & 5 & -7 \end{vmatrix} = 18\hat{i} - \hat{j} + 7\hat{k}$$

$\therefore \quad eq^n$ of plane

$18x - y + 7z = d$

It passes through $(2, 3, -5)$

$36 - 3 - 35 = d \therefore \; d = -2$

$\therefore \quad Eq^n$ of plane

$18x + y - 7z = 2$

$-18x + y - 7z = 2$

$\therefore \; a = -18, \; b = 1, \; c = -7, \; d = 2$

$a + 7b + c + 20d = -18 + 7 - 7 + 40 = 22$

Q47: The probability that a randomly chosen one-one function from the set $\{a, b, c, d\}$ to the set {1, 2, 3, 4, 5} satisfies $f(a) + 2f(b) - f(c) = f(d)$ is :

(A) $\frac{1}{24}$

(B) $\frac{1}{40}$

(C) $\frac{1}{30}$

(D) $\frac{1}{20}$

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Q47: The probability that a randomly chosen one-one function from the set $\{a, b, c, d\}$ to the set {1, 2, 3, 4, 5} satisfies $f(a) + 2f(b) - f(c) = f(d)$ is :

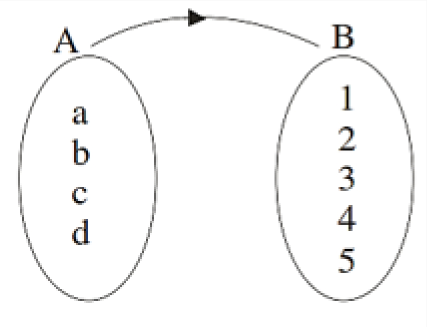
(A) $\frac{1}{24}$

(B) $\frac{1}{40}$

(C) $\frac{1}{30}$

(D) $\frac{1}{20}$

Solution:



$n(s) = 5_{C_4} \times 4! = 120$

f (a) + 2f (b) = f (c) + f (d)			
5	2 × 1	3	4
4	2 × 2	3	5
1	2 × 3	2	5

$n(A) = 2! \times 3 = 6$

$\therefore \; P(A) = \frac{n(A)}{n(s)} = \frac{6}{120} = \frac{1}{20}$

Q48: The value of $\lim_{n \to \infty} 6 \tan \left\{ \sum_{r=1}^{n} \tan^{-1} \left(\frac{1}{r^2 + 3x + 3} \right) \right\}$ is equal to

(A) 1

(B) 2

(C) 3

(D) 6

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Q48: The value of $\lim_{n \to \infty} 6 \tan \left\{ \sum_{r=1}^{n} \tan^{-1} \left(\frac{1}{r^2 + 3x + 3} \right) \right\}$ is equal to

(A) 1

(B) 2

(C) 3

(D) 6

Solution:

$$T_r = \tan^{-1} \left[\frac{(r+2) - (r+1)}{1 + (r+2)(r+1)} \right]$$
$$= \tan^{-1}(r+2) - \tan^{-1}(r+1)$$
$$T_1 = \tan^{-1} 3 - \tan^{-1} 2$$
$$T_2 = \tan^{-1} 4 - \tan^{-1} 3$$
$$T_n = \tan^{-1}(n+2) - \tan^{-1}(n+1)$$
$$S_n = \tan^{-1}(n+2) - \tan^{-1} 2 = \tan^{-1} \left(\frac{n+2-2}{1+2(n+2)} \right) = \tan^{-1} \left(\frac{n}{2n+5} \right)$$
$$\lim_{n \to \infty} 6 \tan \left(\tan^{-1} \left(\frac{n}{2n+5} \right) \right)$$
$$\lim_{n \to \infty} \frac{6n}{2n+5} = \frac{6}{2} = 3$$

Q49: Let $\vec{a}$ be a vector which is perpendicular to the vector $3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}$. If $\vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$, then the projection of the vector $\vec{a}$ on the vector $2\hat{i} + 2\hat{j} + \hat{k}$ is

(A) $\frac{1}{3}$

(B) 1

(C) $\frac{5}{3}$

(D) $\frac{7}{3}$

Q49: Let $\vec{a}$ be a vector which is perpendicular to the vector $3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}$. If $\vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$, then the projection of the vector $\vec{a}$ on the vector $2\hat{i} + 2\hat{j} + \hat{k}$ is

(A) $\frac{1}{3}$

(B) 1

(C) $\frac{5}{3}$

(D) $\frac{7}{3}$

Solution:

$$(\vec{a} \times (2\hat{i} + \hat{k})) \times \left(3\hat{i} + \tfrac{1}{2}\hat{j} + 2\hat{k}\right)$$
$$= (2\hat{i} - 13\hat{j} - 4\hat{k}) \times \left(3\hat{i} + \tfrac{1}{2}\hat{j} + 2\hat{k}\right)$$
$$-(6 + 2)\,\vec{a} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -13 & -4 \\ 3 & \frac{1}{2} & 2 \end{vmatrix}$$
$$\vec{a} = 3\hat{i} + 2\hat{j} - 5\hat{k}$$

Projection of $\vec{a}$ on vector $2\hat{i} + 2\hat{j} + \hat{k}$ is

$$\vec{a}\,.\,\frac{(2\hat{i} + 2\hat{j} + \hat{k})}{3} = \frac{5}{3}$$

Q50: If $\cot\ \alpha = \ 1$ and $\sec\ \beta = -\ \frac{5}{3}$, where $\pi\ <\ \alpha\ <\ \frac{3\pi}{2}$ and $\frac{\pi}{2} <\ \beta\ <\ \pi$, then the value of $\tan\ (\alpha + \ \beta)$ and the quadrant in which $\alpha + \ \beta$ lies, respectively are

(A) $-\ \frac{1}{7}$ and IVth quadrant

(B) 7 and Ist quadrant

(C) – 7 and IVth quadrant

(D) $\frac{1}{7}$ and Ist quadrant

Q50: If $\cot \alpha = 1$ and $\sec \beta = -\frac{5}{3}$, where $\pi < \alpha < \frac{3\pi}{2}$ and $\frac{\pi}{2} < \beta < \pi$, then the value of $\tan(\alpha + \beta)$ and the quadrant in which $\alpha + \beta$ lies, respectively are

(A) $-\frac{1}{7}$ and IVth quadrant

(B) 7 and Ist quadrant

(C) – 7 and IVth quadrant

(D) $\frac{1}{7}$ and Ist quadrant

Solution:

$$\cot \alpha = 1, \sec \beta = \frac{-5}{3}, \cos \beta = \frac{-3}{5}, \tan \beta = \frac{-4}{3}$$

$$\tan(\alpha + \beta) = \frac{1 - \frac{4}{3}}{1 + \frac{4}{3} \times 1} = \frac{-1}{7}$$

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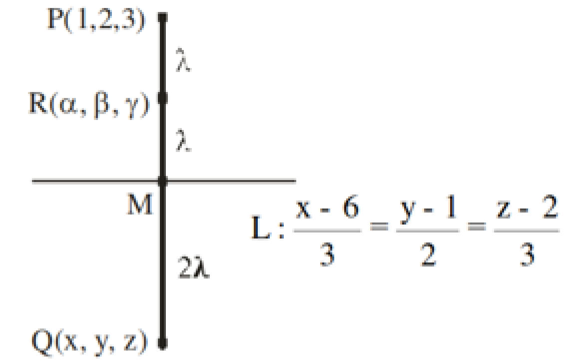
Q51: Let the image of the point P (1, 2, 3) in the line $L\ :\ \frac{x-6}{3}=\frac{y-1}{2}=\frac{z-2}{3}$ be Q. let $R\ (\alpha,\ \beta,\ \gamma)$be a point that divides internally the line segment PQ in the ratio 1 : 3. Then the value of $22\ (\alpha+\ \beta\ +\ \gamma)$is equal to

Q51: Let the image of the point P (1, 2, 3) in the line $L\ :\ \frac{x-6}{3} = \frac{y-1}{2} = \frac{z-2}{3}$ be Q. let $R\ (\alpha,\ \beta,\ \gamma)$be a point that divides internally the line segment PQ in the ratio 1 : 3. Then the value of $22\ (\alpha +\ \beta\ +\ \gamma)$is equal to

125.00

Solution:



Let M be the mid-point of PQ

$\therefore\ \ M = (3\lambda +\ 6,\ 2\lambda + 1,\ 3\lambda +\ 2)$

Now, $\overrightarrow{PM}\ =\ (3\lambda +\ 5)\ \hat{i} +\ (2\lambda - 1)\ \hat{j} +\ (3\lambda - 1)\ \hat{k}$

$\because\ \ \overrightarrow{PM} \perp (3\hat{i} +\ 2\hat{j} +\ 3\hat{k})$

$\therefore\ \ 3\ (3\lambda +\ 5)\ +\ 2\ (2\lambda - 1)\ +\ 3\ (3\lambda - 1) =\ 0$

$\lambda = \frac{-5}{11}$

$\therefore\ \ M\ \left(\frac{51}{11},\ \frac{1}{11},\ \frac{7}{11}\right)$

Since R is mid-point of PM

$22\ (\alpha +\ \beta +\ \gamma)\ \ =\ \ 125$

Q52: Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62, and their variance is 20. A student fails in the examination if he/she gets less than 50 marks, then in worst case, the number of students can fail is

Q52: Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62, and their variance is 20. A student fails in the examination if he/she gets less than 50 marks, then in worst case, the number of students can fail is

0.00

Solution:

$$20 = \frac{\sum_{i=1}^{7} |x_i - 62|^2}{7}$$

$$\Rightarrow |x_1 - 62|^2 + |x_2 - 62|^2 + \ldots\ldots + |x_7 - 62|^2 = 140$$

If $x_1 = 49$

$$|49 - 62|^2 = 169$$

then,

$|x_2 - 62|^2 + \ldots\ldots + |x_7 - 62|^2 =$ Negative Number which is not possible, therefore, no student can fail.

Q53: If one of the diameters of the circle $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$ is chord of the circle $(x - 2\sqrt{2})^2 + (y - 2\sqrt{2})^2 = r^2$, then the value of r^2 is equal to

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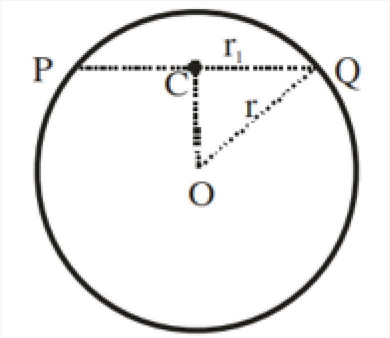
Q53: If one of the diameters of the circle $x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$ is chord of the circle $(x - 2\sqrt{2})^2 + (y - 2\sqrt{2})^2 = r^2$, then the value of r^2 is equal to

10.00

Solution:



PQ is diameter of circle

$S : x^2 + y^2 - 2\sqrt{2}x - 6\sqrt{2}y + 14 = 0$

$C(\sqrt{2}, 3\sqrt{2}), O(2\sqrt{2}, 2\sqrt{2})$

$r_1 = \sqrt{6}$

$S_1 : (x - 2\sqrt{2})^2 + (y - 2\sqrt{2})^2 = r^2$

Now in ΔOCQ

$|OC|^2 + |CQ|^2 = |OQ|^2$

$4 + 6 = r^2$

$r^2 = 10$

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Q54: If $\lim_{x \to 1} \frac{\sin(3x^2-4x+1)-x^2+1}{2x^3-7x^2+ax+b} = -2$, then the value of $(a - b)$ is equal to

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Q54: If $\lim_{x \to 1} \frac{\sin(3x^2-4x+1)-x^2+1}{2x^3-7x^2+ax+b} = -2$, then the value of $(a-b)$ is equal to

11.00

Solution:

$$\lim_{x \to 1} \frac{\sin(3x^2-4x+1)-x^2+1}{2x^3-7x^2+ax+b} = -2$$

For finite limit

$$a+b-5=0 \qquad ....(1)$$

Apply L'H rule

$$\lim_{x \to 1} \frac{\cos(3x^2-4x+1)(6x-4)-2x}{(6x^2-14x+a)} = -2$$

For finite limit

$$6-14+a=0$$

$$\boxed{a=8}$$

From (1), $\boxed{b=-3}$

Now $(a-b) = 11$

Q55: Let for n = 1, 2, ..., 50, S_n be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\frac{1}{(n+1)^2}$. Then the value of

$$\frac{1}{26} + \sum_{n=1}^{50}\left(S_n + \frac{2}{n+1} - n - 1\right)$$ is equal to

Q55: Let for n = 1, 2, ..., 50, S_n be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\frac{1}{(n+1)^2}$. Then the value of

$$\frac{1}{26} + \sum_{n=1}^{50}\left(S_n + \frac{2}{n+1} - n - 1\right)$$ is equal to

41651.00

Solution:

$$S_n = \frac{n^2}{1-\frac{1}{(n+1)^2}} = \frac{n\,(n+1)^2}{(n+2)}$$

$$S_n = \frac{n\,(n^2+2n+1)}{(n+2)}$$

$$S_n = \frac{n\,[n\,(n+2)+1]}{(n+2)}$$

$$S_n = n\left[n + \frac{1}{n+2}\right]$$

$$S_n = n^2 + \frac{n+2-2}{(n+2)}$$

$$S_n = n^2 + 1 - \frac{2}{(n+2)}$$

Now $\frac{1}{26} + \sum_{n=1}^{50}\left[(n^2 - n) - 2\left(\frac{1}{n+2} - \frac{1}{n+1}\right)\right]$

$$= \frac{1}{26} + \left[\frac{50 \times 51 \times 101}{6} - \frac{50\times51}{2} - 2\left(\frac{1}{52} - \frac{1}{2}\right)\right]$$

$$= 41651$$

Q56: If the system of linear equations
$2x - 3y = \gamma + 5, \alpha x + 5y = \beta + 1$, where $\alpha, \beta, \gamma \in R$ has infinitely many solutions, then the value of $|9\alpha + 3\beta + 5\gamma|$ is equal to

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Q56: If the system of linear equations $2x - 3y = \gamma + 5, \alpha x + 5y = \beta + 1$, where $\alpha, \beta, \gamma \in R$ has infinitely many solutions, then the value of $|9\alpha + 3\beta + 5\gamma|$ is equal to

58.00

Solution:

$2x - 3y = \gamma + 5$
$\alpha x + 5y = \beta + 1$
Infinite many solution
$\frac{\alpha}{2} = \frac{5}{-3} = \frac{\beta+1}{\gamma+5}$
$\alpha = \frac{-10}{2}, 5\gamma + 25 = -3\beta - 3$
$9\alpha = -30, 3\beta + 5\gamma = -28$
Now, $9\alpha + 3\beta + 5\gamma = -58$
$|9\alpha + 3\beta + 5\gamma| = 58$

Q57: Let $A = \begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix}$ where $i = \sqrt{-1}$. Then, the number of elements in the set $\{n \in \{1, 2, \ldots\ldots\ldots, 100\} : A^n = A\}$ is

Q57: Let $A = \begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix}$ where $i = \sqrt{-1}$. Then, the number of elements in the set $\{n \in \{1, 2, \ldots\ldots\ldots, 100\} : A^n = A\}$ is

25.00

Solution:

$$A = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix}$$
$$A^2 = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix} \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix}$$
$$A^2 = \begin{bmatrix} i & 1+i \\ -i+1 & -i \end{bmatrix}$$
$$A^4 = \begin{bmatrix} i & 1+i \\ -i+1 & -i \end{bmatrix} \begin{bmatrix} i & 1+i \\ -i+1 & -i \end{bmatrix}$$
$$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$
$$A^{4n+1} = A$$
$$n = 1, 5, 9, \ldots\ldots\ldots\ldots, 97$$
$\Rightarrow$ total elements in the set is 25.

Q58: Sum of squares of modulus of all the complex numbers z satisfying $\bar{z} = iz^2 + z^2 - z$ is equal to

Q58: Sum of squares of modulus of all the complex numbers z satisfying $\bar{z} = iz^2 + z^2 - z$ is equal to

2.00

Solution:

$z + \bar{z} = iz^2 + z^2$

Consider $z = x + iy$

$2x = (i + 1)(x^2 - y^2 + 2xyi)$

$\Rightarrow 2x = x^2 - y^2 - 2xy$ and $x^2 - y^2 + 2xy = 0$

$\Rightarrow 2x = -4xy$

$\Rightarrow x = 0 \ or \ y = \frac{-1}{2}$

Case 1 : $x = 0 \Rightarrow y = 0$ here $z = 0$

Case 2 : $y = \frac{-1}{2}$

$\Rightarrow 4x^2 - 4x - 1 = 0$

$(2x - 1)^2 = 2$

$2x - 1 = \pm\sqrt{2}$

$x = \frac{1 \pm \sqrt{2}}{2}$

Here $z = \frac{1+\sqrt{2}}{2} - \frac{i}{2} \ or \ z = \frac{1-\sqrt{2}}{2} - \frac{i}{2}$

Sum of squares of modulus of z

$= 0 + \frac{(1+\sqrt{2})^2 + 1}{4} + \frac{(1-\sqrt{2})^2 + 1}{4} = \frac{8}{4} = 2$

Q59: Let $S = \{1, 2, 3, 4\}$. Then the number of elements in the set $\{f : S \times S \rightarrow S : f$ is onto and $f(a, b) = f(b, a) \geqslant a \forall (a, b) \in S \times S\}$ is

Q59: Let $S = \{1, 2, 3, 4\}$. Then the number of elements in the set $\{f : S \times S \to S : f$ is onto and $f(a, b) = f(b, a) \geqslant a \forall (a, b) \in S \times S\}$ is

37.00

Solution:

(1, 1), (1, 4), (4, 1), (2, 4), (4, 2), (3, 4), (4, 3), (4, 4) – all have one choice for image.
(2, 1), (1, 2), (2, 2) – all have three choices for image.
(3, 2), (2, 3), (3, 1), (1, 3), (1, 3), (3, 3) – all have two choices for image.
So the total functions $= 3 \times 3 \times 2 \times 2 \times 2 = 72$
Case 1 : None of the pre-images have 3 as image
Total function $= 2 \times 2 \times 1 \times 1 \times 1 = 4$
Case 2 : None of the pre-images have 2 as image
Total function $= 2 \times 2 \times 2 \times 2 \times 2 = 32$
Case 3 : None of the pre-images have either 3 or 2 as image
Total functions $= 1 \times 1 \times 1 \times 1 \times 1 = 1$
$\therefore$ Total onto functions $= 72 - 4 - 32 + 1 = 37$

Q60: The maximum number of compound propositions, out of
$p \vee r \vee s$, $p \vee r \vee \sim s$, $p \vee \sim q \vee s$,
$\sim p \vee \sim r \vee s$, $\sim p \vee \sim r \vee \sim s$, $\sim p \vee q \vee \sim s$,
$q \vee r \vee \sim s$, $q \vee \sim r \vee \sim s$, $\sim p \vee \sim q \vee \sim s$
that can be made simultaneously true by an assignment of the truth values to p, q, r and s, is equal to

Q60: The maximum number of compound propositions, out of
$p \vee r \vee s, p \vee r \vee \sim s, p \vee \sim q \vee s,$
$\sim p \vee \sim r \vee s, \sim p \vee \sim r \vee \sim s, \sim p \vee q \vee \sim s,$
$q \vee r \vee \sim s, q \vee \sim r \vee \sim s, \sim p \vee \sim q \vee \sim s$
that can be made simultaneously true by an assignment of the truth values to p, q, r and s, is equal to

9.00

Solution:

If we take

p	q	r	s
F	F	T	F

The truth value of all the propositions will be true.

Q61: Velocity (v) and acceleration (a) in two system of units 1 and 2 are related as $v_2 = \frac{n}{m^2} v_1$ and $a_2 = \frac{a_1}{mn}$ respectively. Here m and n are constant. The relations for distance and time in two systems respectively are:

(A) $\frac{n^3}{m^2} L_1 = L_2$ and $\frac{n^2}{m} T_1 = T_2$

(B) $L_1 = \frac{n^4}{m^2} L_2$ and $T_1 = \frac{n^2}{m} T_2$

(C) $L_1 = \frac{n^2}{m} L_2$ and $T_1 = \frac{n^4}{m^2} T_2$

(D) $\frac{n^2}{m} L_1 = L_2$ and $\frac{n^4}{m^2} T_1 = T_2$

Q61: Velocity (v) and acceleration (a) in two system of units 1 and 2 are related as $v_2 = \frac{n}{m^2} v_1$ and $a_2 = \frac{a_1}{mn}$ respectively. Here m and n are constant. The relations for distance and time in two systems respectively are:

(A) $\frac{n^3}{m^2} L_1 = L_2$ and $\frac{n^2}{m} T_1 = T_2$

(B) $L_1 = \frac{n^4}{m^2} L_2$ and $T_1 = \frac{n^2}{m} T_2$

(C) $L_1 = \frac{n^2}{m} L_2$ and $T_1 = \frac{n^4}{m^2} T_2$

(D) $\frac{n^2}{m} L_1 = L_2$ and $\frac{n^4}{m^2} T_1 = T_2$

Solution:

$$\frac{v_1}{v_2} = \frac{a_1 t_1}{a_2 t_2}$$
$$\frac{v_1}{v_2} = \frac{m^2}{n}$$
$$\frac{a_1}{a_2} = mn$$
$$\frac{m^2}{\eta} = mn \frac{t_1}{t_2}$$
$$T_2 = \frac{\eta^2}{m} T_1$$

Q62: A ball is spun with angular acceleration $\alpha = 6t^2 - 2t$ where t is in second and α is in $rads^{-2}$.at $t = 0$,the ball has angular velocity of $10 rads^{-1}$ and angular position of 4 rad. The most appropriate expression for the angular position of the ball is:

(A) $\frac{3}{2}t^4 - t^2 + 10t$

(B) $\frac{t^4}{2} - \frac{t^3}{3} + 10t + 4$

(C) $\frac{2t^4}{3} - \frac{t^3}{6} + 10t + 12$

(D) $2t^4 - \frac{t^3}{2} + 5t + 4$

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(A) $\frac{3}{2}t^4 - t^2 + 10t$

(B) $\frac{t^4}{2} - \frac{t^3}{3} + 10t + 4$

(C) $\frac{2t^4}{3} - \frac{t^3}{6} + 10t + 12$

(D) $2t^4 - \frac{t^3}{2} + 5t + 4$

Solution:

$$\alpha = 6t^2 - 2t$$
$$\frac{d\omega}{dt} = 6t^2 - 2t$$
$$\int_{10}^{\omega} d\omega = \int_{0}^{t} \left(6t^2 - 2t\right) dt$$
$$\omega - 10 = 2t^3 - t^2$$
$$\frac{d\theta}{dt} = 10 + 2t^3 - t^2$$
$$\int_{4}^{\theta} d\theta = \int_{0}^{t} \left(10 + 2t^3 - t^2\right) dt$$
$$\theta - 4 = 10t + \frac{t^4}{2} - \frac{t^3}{3}$$
$$\theta = \frac{t^4}{2} - \frac{t^3}{3} + 10t + 4$$

Q63: A block of mass 2 kg moving on a horizontal surface with speed of $4ms^{-1}$ enters a rough surface ranging from x = 0.5 m to x = 1.5 in. The retarding force in this range of rough surface is related to distance by $F = -kx$ where$k = 12Nm^{-1}$. The speed of the block as it just crosses the rough surface will be:

(A) Zero

(B) $1.5ms^{-1}$

(C) $2.0ms^{-1}$

(D) $2.5ms^{-1}$

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(A) Zero

(B) $1.5ms^{-1}$

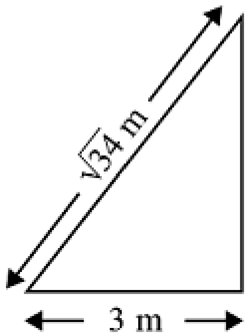
(C) $2.0ms^{-1}$

(D) $2.5ms^{-1}$

Solution:

$F=-kx$
$K=12N^{-1}$
$a=6x$
$v\frac{dv}{dx}=-6x$
$\int_4^v vdv=\int_{0.5}^{1.5}-3xdx$
$\frac{v^2-16}{2}=\frac{6}{2}[2.25-0.25]$
$V^2=-12+16$
$V=\sqrt{4}$
$V=2m/s$

Q64: A $\sqrt{34}m$long ladder weighting 10 kg leans on a frictionless wall. Its feet rest on the floor 3m away from the wall as shown in the figure. If $F_f$and F_w are the reaction forces of the floor and the wall< then ratio of $\frac{F_w}{F_f}$will be:



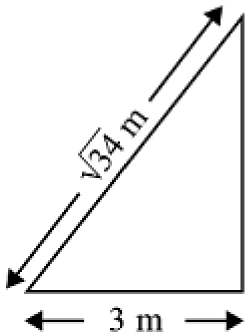
(A) $\frac{6}{\sqrt{110}}$

(B) $\frac{3}{\sqrt{113}}$

(C) $\frac{3}{\sqrt{109}}$

(D) $\frac{2}{\sqrt{109}}$

Q64: A $\sqrt{34}m$long ladder weighting 10 kg leans on a frictionless wall. Its feet rest on the floor 3m away from the wall as shown in the figure. If $F_f$and F_w are the reaction forces of the floor and the wall< then ratio of $\frac{F_w}{F_f}$will be:



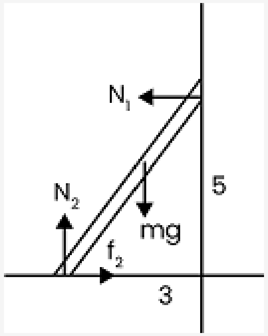
(A) $\frac{6}{\sqrt{110}}$

(B) $\frac{3}{\sqrt{113}}$

(C) $\frac{3}{\sqrt{109}}$

(D) $\frac{2}{\sqrt{109}}$

Solution:



$N_1 = f_2, N_2 = mg$

$N_1 \times 5 = mg \times \frac{3}{2} \Rightarrow N_1 = \frac{3}{10} mg$

$R_1 = N_1 = \frac{3}{10} mg, R_2 = \sqrt{N_2^2 + t_2^2} = \frac{\sqrt{109}}{10} mg$

$\frac{R_1}{R_2} = \frac{3}{\sqrt{109}} = \frac{F_w}{F_1} = \frac{3}{\sqrt{109}}$

Q65: Water falls from a 40 in high dam at the rate of 9 x 10^4 kg per hour. Fifty percentage of gravitational potential energy can be converted into electrical energy. Using this hydro electric energy number of 100 W lamps, that can be lit, is : (Take $g = 10ms^{-2}$)

(A) 25

(B) 50

(C) 100

(D) 18

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(A) 25

(B) 50

(C) 100

(D) 18

Solution:

$$\frac{40\times9\times10^4}{1\text{hr}}\text{g}\times\frac{50}{100}=\frac{40\times9\times10^4}{3600}\times10\times\frac{50}{100}=100\text{ N}$$

$$N=50$$

Q66: Two objects of equal masses placed at certain distance from each other attracts each other with a force of F. If one-third mass of one object is transferred to the other object, then the new force will be

(A) $\frac{2}{9}F$

(B) $\frac{16}{9}F$

(C) $\frac{8}{9}F$

(D) F

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(B) $\frac{16}{9}F$

(C) $\frac{8}{9}F$

(D) F

Solution:

$$F=\frac{Gmm}{d^2}$$

$$F=\frac{G\frac{2m}{3}\times\frac{4}{3}m}{d^2}=\frac{8}{9}\frac{Gmm}{d^2}$$

$$\frac{F}{F}=\frac{8}{9}$$

$$F'=\frac{8}{9}F$$

Q67: A water drop of radius $1\mu m$ falls in a situation where the effect of buoyant force is negligible. Co-efficient of viscosity of air is $1.8 \times 10^{-5} Nsm^2$ and its density is negligible as compared to that of water $10^6 gm^{-3}$. Terminal velocity of the water drop is: (Take acceleration due to gravity $= 10ms^{-2}$)

(A) $145.4 \times 10^{-6} ms^{-1}$

(B) $118.0 \times 10^{-6} ms^{-1}$

(C) $132.6 \times 10^{-6} ms^{-1}$

(D) $123.4 \times 10^{-6} ms^{-1}$

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Solution:

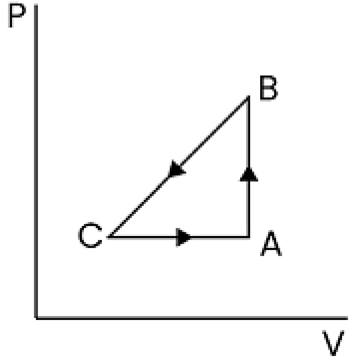
$\frac{4}{3}\pi r^3 \rho g = 6\pi n r V$

$\frac{4}{3\times6} r^2 \frac{\rho g}{n} = v$

$\frac{4}{3} \times \frac{10^{-12}\times10^3\times10}{1.8\times10^{-5}\times6}$

$v = 123.4 \times 10^{-6} m/s$

Q68: A sample of an ideal gas is taken through the cyclic process ABCA as shown in figure. It absorbs, 40 J of heat during the part AB, no heat during BC and rejects 60 J of heat during CA. A work of 50 J is done on the gas during the part BC. The internal energy of the gas at A is 1560 J. The work done by the gas during the part CA is:



(A) 20 J

(B) 30 J

(C) -30 J

(D) -60 J

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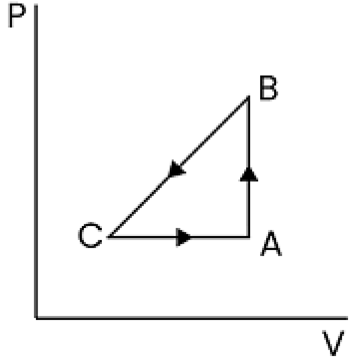

(A) 20 J

(B) 30 J

(C) -30 J

(D) -60 J

Solution:

Internal energy at B point $= 1600 + 50 = 1650$

ΔUin CA = 1560 – 1650 = -90

ΔQin CA = -60 J

Work done $\Delta W = \Delta Q - \Delta U = -60 - (-90) = 30$

Q69: What will be the effect on the root mean square velocity of oxygen molecules if the temperature is doubled and oxygen molecule dissociates into atomic oxygen?

(A) The velocity of atomic oxygen remains same

(B) The velocity of atomic oxygen doubles

(C) The velocity of atomic oxygen becomes half

(D) The velocity of atomic oxygen becomes four times

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Solution:

The velocity of atomic oxygen doubles

Q70: Two point charges A and B of magnitude $+8 \times 10^{-6} C$and $-8 \times 10^{-6} C$ respectively are placed at a distance d apart. The electric field at the middle point 0 between the charges is$6.4 \times 10^{4} NC^{-1}$. The distance ad' between the point charges A and B is:

(A) 2.0 m

(B) 3.0 m

(C) 1.0 m

(D) 4.0 m

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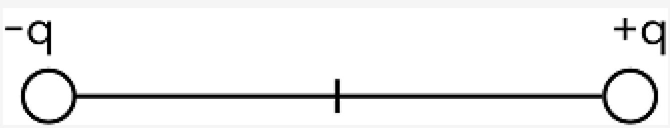

E at mid point

$$E = \frac{2kq}{\frac{d^2}{4}}$$

$$6.4 \times 10^4 = \frac{8kq}{d^2}$$

$$d^2 = \frac{8 \times k \times 8 \times 10^{-6}}{6.4 \times 10^4} = \frac{8 \times 9 \times 10^9 \times 8 \times 10^{-6}}{6.4 \times 10^4} = 3m$$

Q71: Resistance of the wire is measured as 2Ω and 3Ω at 10°C and 30°C respectively. Temperature co-efficient of resistance of the material of the wire is :

(A) $0.033^{\circ}C^{-1}$

(B) $-0.033^{\circ}C^{-1}$

(C) $0.011^{\circ}C^{-1}$

(D) $0.055^{\circ}C^{-1}$

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Solution:

$$R = R_0 (1 + \alpha \Delta T)$$
$$2 = R_0 (1 + 10\alpha)$$
$$3 = R_0 (1 + 30\alpha)$$
$$1 = 30\alpha$$
$$\alpha = \frac{1}{30} = 0.033$$

Q72: The space inside a straight current carrying solenoid is filled with a magnetic material having magnetic susceptibility equal to 1.2×10^{-5}. What is fractional increase in the magnetic field inside solenoid with respect to air as medium inside the solenoid ?

(A) 1.2×10^{-5}

(B) 1.2×10^{-3}

(C) 1.8×10^{-3}

(D) 2.4×10^{-5}

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(A) 1.2×10^{-5}

(B) 1.2×10^{-3}

(C) 1.8×10^{-3}

(D) 2.4×10^{-5}

Solution:

$$\chi = 1.2 \times 10^{-5}$$
$$\mu_r = \chi - 1$$
$$B = \mu n i$$
$$= \mu_r \mu_0 n i$$

Q73: Two parallel, long wires are kept 0.20 m apart in vacuum, each carrying current of x A in the same direction. If the force of attraction per meter of each wire is $2 \times 10^{-6} N$, then the value of x is approximately:

(A) 1

(B) 2.4

(C) 1.4

(D) 2

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(A) 1

(B) 2.4

(C) 1.4

(D) 2

Solution:

$\frac{F}{\ell} = \frac{\mu_0 ii}{2\pi d}$

$2 \times 10^{-6} = \frac{4\pi^2 \times 10^{-7} i^2}{2\pi \times 0.2}$

$i^2 = \sqrt{2} = 1.4$

Q74: A coil is placed in a time varying magnetic field. if the number of turns in the coil were to be halved and the radius of wire doubled, the electrical power dissipated due to the current induced in the coil would be : (Assume the coil to be short circuited.)

(A) Halved

(B) Quadrupled

(C) The same

(D) Doubled

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(A) Halved

(B) Quadrupled

(C) The same

(D) Doubled

Solution:

Resistance of coil remains same if number of turn becomes half and radius is doubled.

$E = \frac{Nd\phi}{dt} = -\frac{NAdB}{dt}$

$P = \frac{e^2}{R}$

$P \propto e^2 \propto N^2 A^2 \propto N^2 r^4$

$\left(\frac{1}{2}\right)^2 (2)^4 = 2^2 = 4$

Q75: An EM wave propagating in x-direction has a wavelength of 8 mm. The electric field vibrating y-direction has maximum magnitude of $60 Vm^{-1}$. Choose the correct equations for electric and magnetic fields if the EM wave is propagating in vacuum:

(A) $\mathrm{E}_y = 60 \sin\left[\frac{\pi}{4} \times 10^3 \left(x - 3 \times 10^8 t\right)\right] \hat{j}\ \mathrm{Vm}^{-1} \mathrm{B}_z = 2 \sin\left[\frac{\pi}{4} \times 10^3 \left(x - 3 \times 10^8 \mathrm{t}\right)\right] \hat{k}\ \mathrm{T}$

(B) $\mathrm{E}_y = 60 \sin\left[\frac{\pi}{4} \times 10^3 \left(x - 3 \times 10^8 t\right)\right] \hat{j} \mathrm{Vm}^{-1} \mathrm{B}_z = 2 \times 10^{-7} \sin\left[\frac{\pi}{4} \times 10^3 \left(x - 3 \times 10^8 \mathrm{t}\right)\right] \hat{k}\ \mathrm{T}$

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(D) $E_y = 2 \times 10^{-7} \sin\left[\frac{\pi}{4} \times 10^3 \left(x - 4 \times 10^8 \mathrm{t}\right)\right] \hat{j} V m^{-1} B_z = 60 \sin\left[\frac{\pi}{4} \times 10^4 \left(x - 4 \times 10^8 t\right)\right] \hat{k} T$

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Solution:

$\mathrm{E}_y = 60 \sin\left[\frac{\pi}{4} \times 10^3 \left(x - 3 \times 10^8 t\right)\right] \hat{j} \mathrm{Vm}^{-1}$

$\mathrm{B}_z = 2 \times 10^{-7} \sin\left[\frac{\pi}{4} \times 10^3 \left(x - 3 \times 10^8 \mathrm{t}\right)\right] \hat{k}\ \mathrm{T}$

Q76: In young's double slit experiment performed using a monochromatic light of wavelength λ. when a glass plate $(\mu = 1.5)$ of thickness $x\lambda$ is introduced in the path of the one or the interfering beams, the intensity at the position where the central maximum occurred previously remains unchanged. The value of x will be:

(A) 3

(B) 2

(C) 1.5

(D) 0.5

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(A) 3

(B) 2

(C) 1.5

(D) 0.5

Solution:

$$\Delta x = (\mu - 1)\, t$$
$$= (1.5 - 1)\, x\lambda = n\lambda$$
$$n = 1$$
$$x\lambda = \frac{\lambda}{0.5}; x = 2$$

Q77: Let K_1 and K_2 be the maximum kinetic energies of photo-electrons emitted when two monochromatic beams of wavelength λ_1 and λ_2, respectively are incident on a metallic surface. If $\lambda_1 = 3\lambda_2$ then :

(A) $K_1 > \frac{K_2}{3}$

(B) $K_1 < \frac{K_2}{3}$

(C) $K_1 = \frac{K_2}{3}$

(D) $K_2 = \frac{K_1}{3}$

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(B) $K_1 < \frac{K_2}{3}$

(C) $K_1 = \frac{K_2}{3}$

(D) $K_2 = \frac{K_1}{3}$

Solution:

$$K_1 = \frac{hc}{\lambda_1} - \phi$$
$$K_2 = \frac{hc}{\lambda_2} - \phi$$
$$\frac{K_1}{K_2} = \frac{\frac{hc}{3\lambda_2} - \phi}{\frac{hc}{\lambda_2} - \phi}$$
$$K_1 < \frac{K_2}{3}$$

Q78: Following statements related to radioactivity are given below

A) Radioactivity is a random and spontaneous process and is dependent on physical and chemical conditions.

(B) The number of un-decayed nuclei in the radioactive sample decays exponentially with time. (C) Slope of the graph of $\log_e$ (no. of undecayed nuclei) Vs. time represents the reciprocal of mean life time (τ).

(D) Product of decay constant (λ) and half-life time $\left(T_{\frac{1}{2}}\right)$ is not constant

Choose the most appropriate answer from the options given below :

(A) (A) and (B) only

(B) (B) and (D) only

(C) (B) and (C) only

(D) (C) and (D) only

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Choose the most appropriate answer from the options given below :

(A) (A) and (B) only

(B) (B) and (D) only

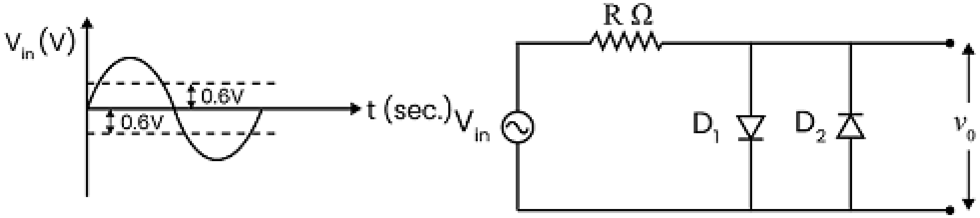
(C) (B) and (C) only

(D) (C) and (D) only

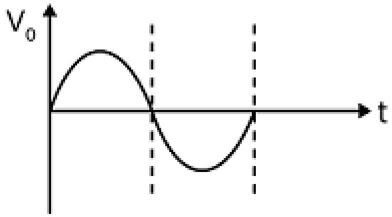
Solution:

(B) and (D) only

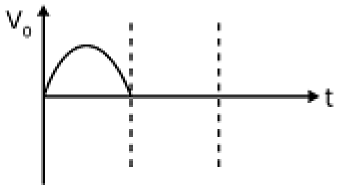
Q79: In the given circuit the input voltage V_{in} is shown in figure. The cut-in voltage of p-n junction diode (D_1 or D_2) is 0.6 V. Which of the following output voltage (V_0) waveform across the diode is correct?



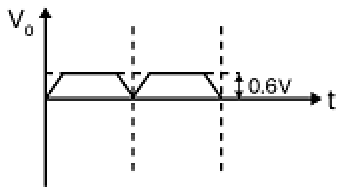
(A)



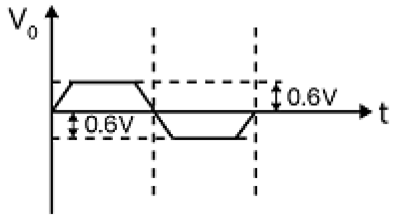
(B)



(C)



(D)



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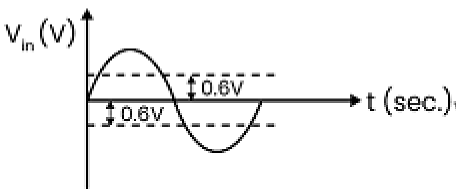
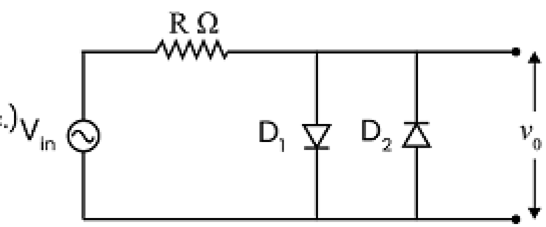
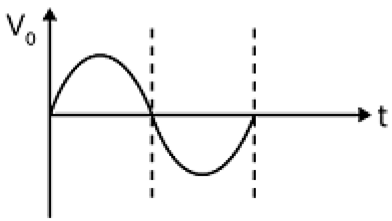
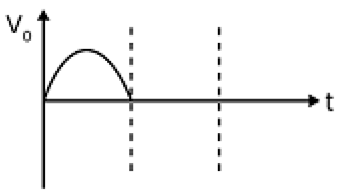




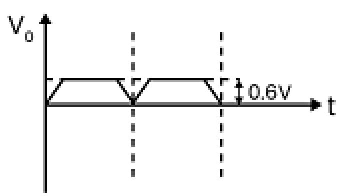
(A)



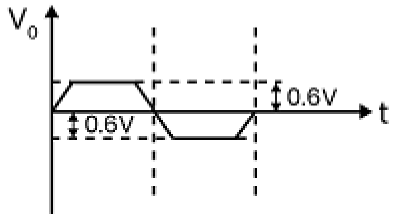
(B)



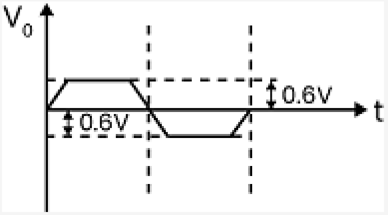
(C)



(D)



Solution:



Q80: Amplitude modulated wave is represented by $V_{AM} = 10\left[1 + 0.4\cos\left(2\pi \times 10^4 t\right)\right]\cos\left(2\pi \times 10^7\right)t$. The total bandwidth of the amplitude modulated wave is

(A) 10 kHz

(B) 20 MHz

(C) 20 kHz

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(A) 10 kHz

(B) 20 MHz

(C) 20 kHz

(D) 10 MHz

Solution:

$f = \frac{\omega}{2\pi}$

Band width = 2f

Q81: A student in the laboratory measures thickness of a wire using screw gauge. The readings are 1.22 mm, 1.23 mm, 1.19 mm and 1.20 mm. The percentage error is $\frac{x}{121}\%$. The value of x is ______

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150

Solution:

$$X_{\text{avg}} = \frac{1.19+1.20+1.22+1.23}{4}$$

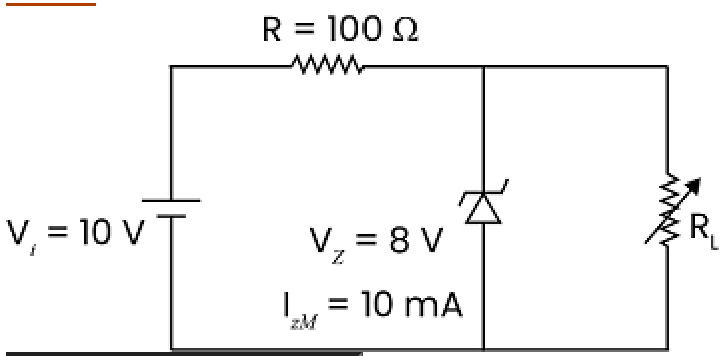
$$\Delta \mathrm{x} = \frac{0.02+0.01+0.01+0.02}{4} = \frac{0.06}{4}$$

$$\Delta x = \frac{\frac{0.03}{2}}{1.21} \times 100$$

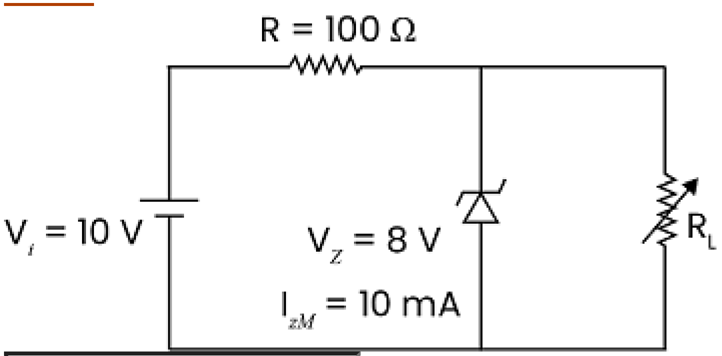
$$\Delta x = \frac{150}{121}$$

$$\mathrm{X} = 150$$

Q82: A zener of breakdown voltage $V_Z = 8V$ and maximum zener current, $I_{ZM} = 10mA$is subjected to an input voltage V_i =10 V with series resistance $R = 100\Omega$. In the given circuit RL represents the variable load resistance. The ratio of maximum and minimum value of R_L is ____.



Q82: A zener of breakdown voltage $V_Z = 8V$ and maximum zener current, $I_{ZM} = 10mA$is subjected to an input voltage V_i =10 V with series resistance $R = 100\Omega$. In the given circuit RL represents the variable load resistance. The ratio of maximum and minimum value of R_L is ____.



2

Solution:

$R_L = \frac{8}{10} = 0.8$
$R_{\max} = \frac{8}{20}$
$\frac{8}{10} \times \frac{20}{8} = 2$

Q83: In a Young's double slit experiment, an angular width of the fringe is 0.35° on a screen placed at 2 m away for particular wavelength of 450 nm. The angular width of the fringe, when whole system is immersed in a medium of refractive index $\frac{7}{5}$, is $\frac{1}{\alpha}$. The value of α is ______.

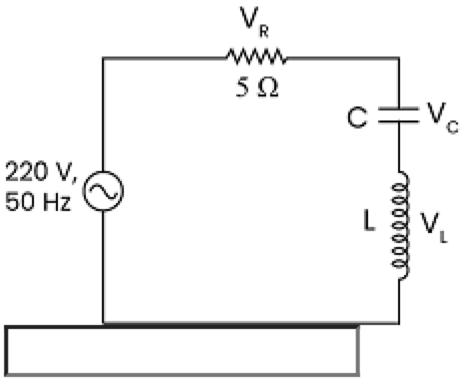
Q83: In a Young's double slit experiment, an angular width of the fringe is 0.35° on a screen placed at 2 m away for particular wavelength of 450 nm. The angular width of the fringe, when whole system is immersed in a medium of refractive index $\frac{7}{5}$, is $\frac{1}{\alpha}$. The value of $\alpha$is ______.

4

Solution:

4

Q84: In the given circuit, the magnitude of V_L and V_C are twice that of V_R. Given that f = 50Hz, the inductance of the coil is $\frac{1}{K\pi} mH$. The value of K is ______.



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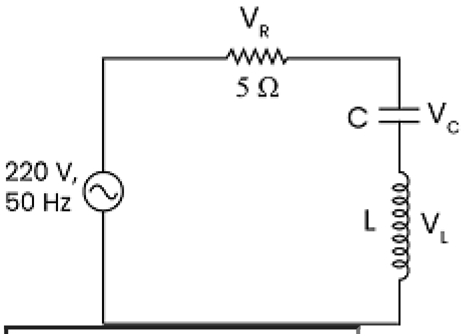


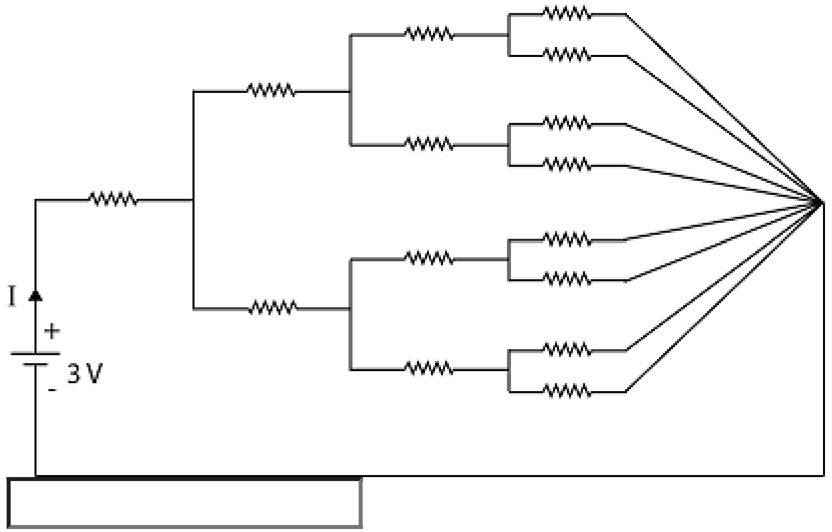
0

Solution:

$$v=\sqrt{v_2^2+\left(v_L-v_C\right)^2}$$

$V_L=V_C=2 V_R$
$\mathrm{V_S}=V_A=220\ \mathrm{V}$
$I_{\mathrm{rms}}=\frac{220}{5}=44\ \mathrm{A}$
$\mathrm{X_L}=\frac{440}{44}=10 \Omega$
$\mathrm{L}=\frac{10}{100 \pi}=\frac{1}{10 \pi} \mathrm{Hz}$
$\frac{1}{\mathrm{K} \pi} \times 10^3=\frac{1}{10 \pi}$
$K=\frac{1}{100}$
Alternate solutions:
$V_L=V_C=2 V_R$
$V_S=\sqrt{V_R^2+\left(V_L-V_C\right)^2}$
$\mathrm{V}_S=V_R=220$
$\mathrm{I}_{rms}(5)=220$
$I_{\mathrm{rms}}=444$
$\mathrm{V}_L=2 V_R$
$I_{rms} X_L=440$
$X_L=\frac{440}{44}=10$
$L=\frac{10}{100 \pi}=\frac{1}{10 \pi} H$
$\frac{1}{\mathrm{K} \pi}=\frac{100}{\pi}$
$K=0.01$
Nearest integer zero.

Q85: All resistance in figure are 1Ω each. The value of current 'T' is $\frac{a}{5}A$.The value of a is _______.



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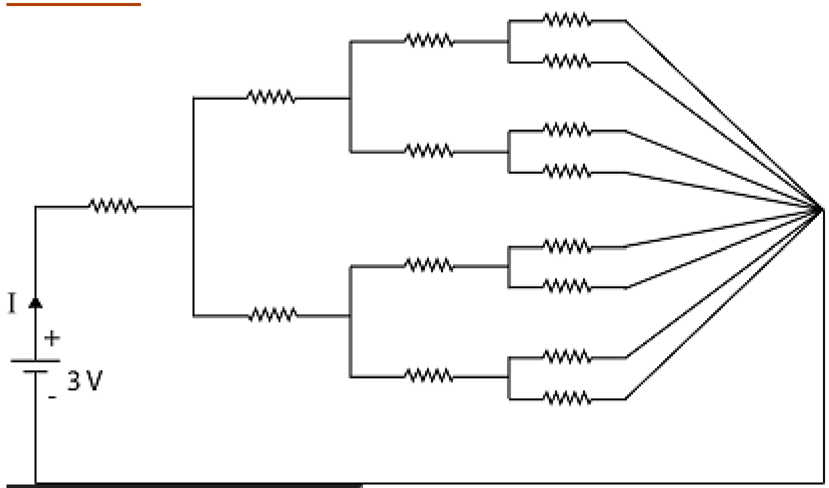

8

Solution:

$$R_{eq} = \frac{15}{8}$$
$$i = \frac{3}{\frac{15}{8}} = \frac{8}{5}A$$

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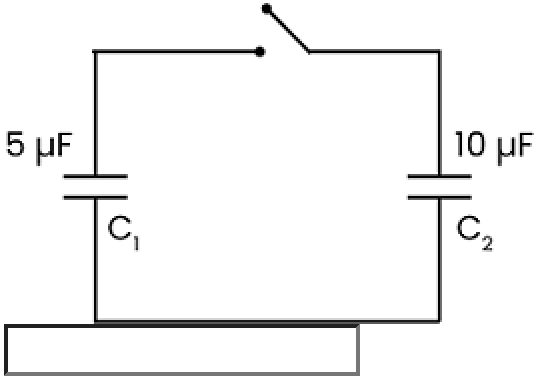
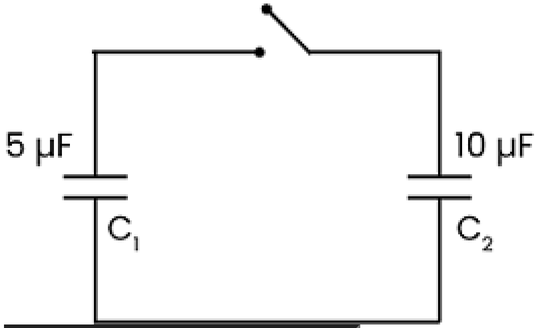
Q86: A capacitor C_1 of capacitance $5\mu F$ is charged to a potential of 30 V using a battery. The battery is then removed and the charged capacitor is connected to an uncharged capacitor C_2 of capacitance $10\mu F$ as shown in figure. When the switch is closed charge flows between the capacitors. At equilibrium, the charge on the capacitor C_2 is _______μC



Q86: A capacitor C_1 of capacitance $5\mu F$ is charged to a potential of 30 V using a battery. The battery is then removed and the charged capacitor is connected to an uncharged capacitor C_2 of capacitance $10\mu F$ as shown in figure. When the switch is closed charge flows between the capacitors. At equilibrium, the charge on the capacitor C_2 is ______μC



100

Solution:

$$V = \frac{C_1 V_1 + C_2 V_2}{C_1 + C_2}$$
$$= \frac{5 \times 30 + 0}{5 + 10} = 10$$
$$Q_2 = C_2 V = 10 \times 10 = 100\mu C$$

Q87: A tuning fork of frequency 340 Hz resonates in the fundamental mode with an air column of length 125 cm in a cylindrical tube closed at one end. When water is slowly poured in it, the minimum height of water required for observing resonance once again is cm. (Velocity of sound in air is $340ms^{-1}$)

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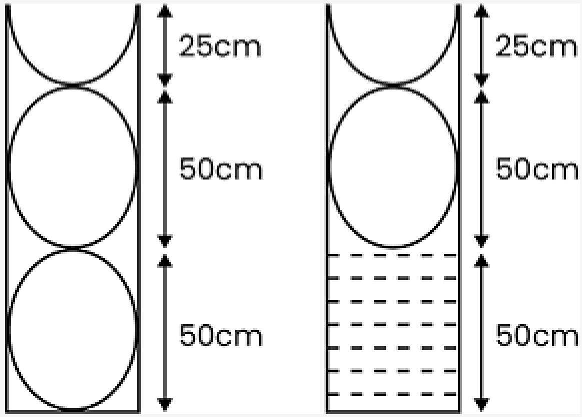
50

Solution:



Q88: A liquid of density $750 kgm^{-3}$ flows smoothly through a horizontal pipe that tapers in cross-sectional area from $A_1 = 1.2 \times 10^{-2} m^2$ to $A_2 = \frac{A_1}{2}$. The pressure difference between the wide and narrow sections of the pipe is 4500Pa. The rate of flow of liquid is ______ $\times 10^{-3} m^3 s^{-1}$.

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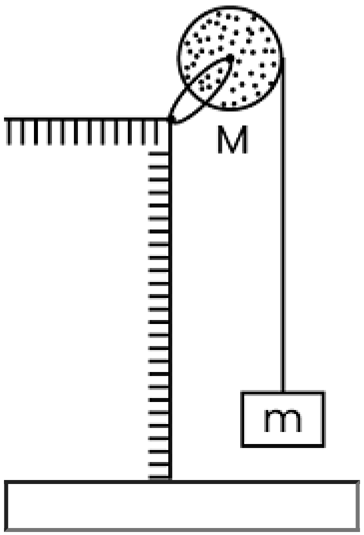
24

Solution:

$$\mathrm{P}_1 + \frac{\rho \mathrm{v}_1^2}{2} = \mathrm{P}_2 + \frac{\rho \mathrm{v}_2^2}{2}$$
$$P_1 - P_2 = \rho \left(\frac{v_2^2 - v_1^2}{2} \right)$$
$$4500 = 750 \left(\frac{3v^2}{2} \right)$$
$$v = 24$$

Q89: A uniform disc with mass M=4 kg and radius R=10 cm is mounted on a fixed horizontal axle as shown in figure. A block with mass m=2 kg hangs from a massless cord that is wrapped around the rim of the disc. During the fall of the Nock, the cord does not slip and there is no friction at the aide. The tension in the cord is N. (Take $g = 10ms^{-2}$)



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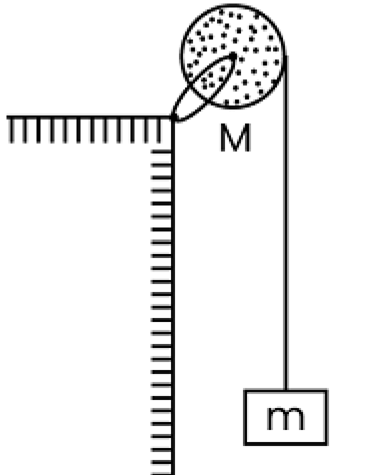
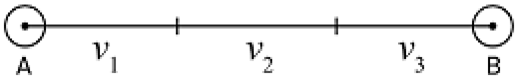


10

Solution:

$$\tau = Ia$$
$$= \frac{4t^2}{2}\alpha$$
$$\alpha = \frac{T}{2r} = \frac{T}{2\times0.1} = 5T$$
$$2g - T = 2a = 2 \times 0.1 \times \alpha$$
$$20 - T = 0.2 \times 5T$$
$$20 = 2T$$
$$T = 10N$$

Q90: A car covers AB distance with first one-third at velocity $v_1 ms^{-1}$,second one-third at $v_2 ms^{-1}$and last one-thrid at $v_3 ms^{-1}$.If $v_3 = 3v_1, v_2 = 2v_1$ and $v_1 = 11ms^{-1}$then the average velocity of the car is _______ ms^{-1}.



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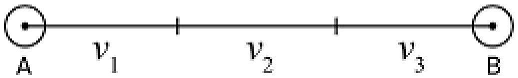

18

Solution:

$$V_{avg} = \frac{3d}{\frac{d}{11}+\frac{d}{22}+\frac{d}{33}} = \frac{3}{\frac{6+3+2}{66}} = 18m/s$$