29/07/2022
Morning

Corporate Office : Aakash Tower, 8, Pusa Road, New Delhi-110005 | Ph.: 011-47623456

Memory Based
Answers & Solutions

Time : 3 hrs. *for* **M.M. : 300**

JEE (Main)-2022 (Online) Phase-2

(Physics, Chemistry and Mathematics)

IMPORTANT INSTRUCTIONS:

(1) The test is of **3 hours** duration.

(2) The Test Booklet consists of 90 questions. The maximum marks are 300.

(3) There are **three** parts in the question paper consisting of **Physics, Chemistry** and **Mathematics** having 30 questions in each part of equal weightage. Each part (subject) has two sections.

(i) **Section-A:** This section contains 20 multiple choice questions which have only one correct answer. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer.

(ii) **Section-B:** This section contains 10 questions. In Section-B, attempt any **five questions out of 10.** The answer to each of the questions is a numerical value. Each question carries **4 marks** for correct answer and **–1 mark** for wrong answer. For Section-B, the answer should be rounded off to the nearest integer.

PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Position of a particle, x and time t are related as $t = \sqrt{2x+4}$. Then velocity of the particle at t = 4 seconds is equal to (Unit in SI)

(1) 4 (2) 2

(3) 1 (4) 5

Answer (1)

Sol. $t = \sqrt{2x+4}$

$\Rightarrow \quad x = \frac{1}{2}(t^2 - 4)$

So $\frac{dx}{dt} = t$

at t = 4 sec, v = 4 m/s

2. A projectile with kinetic energy ε at point of projection is projected at angle 45°. Its kinetic energy at top most point is equal to

(1) $\frac{\varepsilon}{2}$ (2) $\frac{3\varepsilon}{2}$

(3) $\frac{\varepsilon}{4}$ (4) $\frac{\varepsilon}{3}$

Answer (1)

Sol. $KE_i = \frac{1}{2}mv^2 = \varepsilon$

Speed of highest point $v' = v\cos 45°$

$KE_f = \frac{1}{2}mv'^2 = \frac{1}{4}mv^2$

$KE_f = \frac{\varepsilon}{2}$

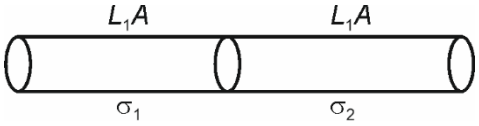
3. Two rods with identical length any area of cross-section are connected in series. If σ_1 and σ_2 is the thermal conductance of material of two rods then equivalent conductance of combination is equal to

(1) $\frac{2\sigma_1\sigma_2}{\sigma_1+\sigma_2}$ (2) $\frac{\sigma_1\sigma_2}{\sigma_1+\sigma_2}$

(3) $\frac{\sigma_1\sigma_2}{\sigma_1-\sigma_2}$ (4) $\frac{2\sigma_1\sigma_2}{\sigma_1-\sigma_2}$

Answer (1)

Sol.



$R_1 = \frac{L}{\sigma_1 A}$ and $R_2 = \frac{L}{\sigma_2 A}$

$R_{net} = R_1 + R_2 = \frac{L}{A}\left(\frac{1}{\sigma_1} + \frac{1}{\sigma_2}\right)$

Must be equivalent to $R_{net} = \frac{2L}{\sigma A}$

So, $\frac{2L}{\sigma A} = \frac{L}{A}\left(\frac{1}{\sigma_1} + \frac{1}{\sigma_2}\right)$

$\sigma = \frac{2\sigma_1\sigma_2}{\sigma_1+\sigma_2}$

4. A particle thrown at angle 45° with horizontal with speed u has its range equal to R. At what angle should it be thrown with same speed for its range to be half of its initial value.

(1) 60° (2) 30°

(3) 15° (4) 70°

Answer (3)

Sol. $R = \frac{u^2 \sin 2\times 45°}{g} = \frac{u^2}{g}$

For range $\frac{R}{2}$

$\frac{u^2}{2g} = \frac{u^2 \sin 2\theta}{g}$

$\sin 2\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = 15°$

5. A travelling microscope has vernier scales with 9 MSD = 10 VSD. If one main scale division (MSD) is equal to 1 mm then least count of travelling microscope is

(1) 0.005 m (2) 0.002 m

(3) 0.0001 m (4) 0.0005 m

Answer (3)

Sol. LC = 1 MSD – 1 VSD

$= 1\,\text{MSD} - \frac{9}{10}\,\text{VSD}$

$= \frac{1}{10}$ mm

= .0001 m

6. A cart is moving down the smooth incline of inclination α. What is the time period of a bob hanging from the roof of the cart with a light string.

(1) $2\pi\sqrt{\frac{l}{g\cos\alpha}}$

(2) $2\pi\sqrt{\frac{l}{g}}$

(3) $2\pi\sqrt{\frac{l}{g\sin\alpha}}$

(4) $2\pi\sqrt{\frac{l}{g\cot\alpha}}$

Answer (1)

Sol. $\vec{g}_{\text{eff}} = \vec{g} - \vec{a}$

$|\vec{g}_{\text{eff}}| = g\cos\alpha$

So, $T = 2\pi\sqrt{\frac{l}{g\cos\alpha}}$

7. If the length of wire is doubled and the radius is halved, then the Young's modulus (Y) becomes

(1) Same

(2) 8 times the original value

(3) 4 times the original value

(4) None of these

Answer (1)

Sol. Young's modulus is independent of length and area.

8. Find the ratio of energy of electron when it transition from second to first energy state in comparison to highest state to first energy state of hydrogen atom

(1) $\frac{1}{4}$ (2) $\frac{5}{36}$

(3) $\frac{8}{9}$ (4) $\frac{3}{4}$

Answer (4)

Sol. $E_1(2\to1) = 13.6\left(1-\frac{1}{4}\right) = 13.6\times\frac{3}{4}$ eV

$E_1(\infty\to1) = 13.6\left(1-\frac{1}{\infty}\right) = 13.6$ eV

So, $\frac{E_1}{E_3} = \frac{3}{4}$

9. Which gas has specific heats ratio of $\frac{7}{5}$? (Choose most appropriate option)

(1) Monoatomic

(2) Diatomic

(3) Polyatomic linear

(4) Both (2) and (3)

Answer (4)

Sol. Diatomic gases have specific heats ratio of $\frac{7}{5}$. Polyatomic linear gas molecule has specific heat ratio $=\frac{7}{5}$.

10. **Assertion A:** Grease/Oil strains can not be removed by water wash

Reason R: The angle of contact between water and oil is obtuse

(1) Both A and R are true and R is correct explanation of A

(2) Both A and R are true but R is not correct explanation of A

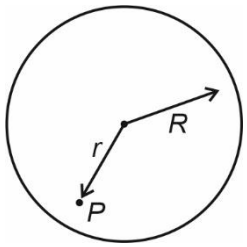
(3) A is true B is false

(4) A is false B is true

Answer (1)

Sol. Angle of contact between oil and water is obtuse therefore they do not stick and that's the reason grease/oil strain can not be removed by water wash

11. A solid sphere is charged such that charge density ρ varies with radial distance r as $\rho = \rho_0\left(1-\frac{r}{R}\right)$ for $r \le R$. The electric field at a radial distance r, at point P is



(1) $\frac{\rho_0 r}{\varepsilon_0}\left[\frac{1}{3}-\frac{r}{4R}\right]$ (2) $\frac{\rho_0 r}{\varepsilon_0}\left[\frac{1}{2}-\frac{r}{3R}\right]$

(3) $\frac{\rho_0 r}{\varepsilon_0}\left[1-\frac{r}{2R}\right]$ (4) $\frac{\rho_0 r}{\varepsilon_0}\left[\frac{1}{2}-\frac{r}{4R}\right]$

Answer (1)

Sol. Using gauss law (considering gaussian surface of radius x)

$$\oint \vec{E}\cdot\vec{dA} = \frac{\int_0^x 4\pi r^2 \rho(r)dr}{\varepsilon_0}$$

$$4\pi x^2 E = \frac{4\pi\rho_0}{\varepsilon_0}\int_0^x r^2\left(1-\frac{r}{R}\right)dr$$

$$x^2E = \frac{\rho_0}{\varepsilon_0}\left(\frac{x^3}{3}-\frac{x^4}{4R}\right)$$

So $E(x) = \frac{\rho_0 x}{\varepsilon_0}\left[\frac{1}{3}-\frac{x}{4R}\right]$

or $E(r) = \frac{\rho_0 r}{\varepsilon_0}\left[\frac{1}{3}-\frac{r}{4R}\right]$

12. **Statement:** Electric potential is constant inside and on the surface of a conductor.

Reason: Electric field just outside of the conductor is perpendicular to its surface.

(1) Only statement is correct

(2) Only reason is correct

(3) Both are correct but reason is not correct explanation

(4) Both are correct and reason in correct explanation for statement

Answer (3)

Sol. Reason is not completely explaining why potential is same inside the conductor.

13. Two objects of masses 'm' and '$3m$' are located at $\left(\hat{i}+2\hat{j}+\hat{k}\right)$ and $\left(-3\hat{i}-2\hat{j}+\hat{k}\right)$ respectively. Find position vector of COM of two objects is

(1) $+2\hat{i}+2\hat{j}-\hat{k}$

(2) $\hat{i}+\hat{j}-\hat{k}$

(3) $-2\hat{i}-\hat{j}+\hat{k}$

(4) $\hat{i}+2\hat{j}-2\hat{k}$

Answer (3)

Sol. $\vec{r}_{cm} = \frac{\Sigma m_i \vec{r}_i}{\Sigma m_i}$

$$= \frac{m\left(\hat{i}+2\hat{j}+\hat{k}\right)+3m\left(-3\hat{i}-2\hat{j}+\hat{k}\right)}{m+3m}$$

$$= \frac{-8m\hat{i}-4m\hat{j}+4m\hat{k}}{4m}$$

$$= -2\hat{i}-\hat{j}+\hat{k}$$

14. If L = 1 H, R = 100 Ω are connected to a battery 6 V DC in series. Find time when current is half of maximum and energy of inductor at t = 15 ms. $e^{-3/2} = 0.25$

(1) 7.5 ms, 1.01×10^{-3} J

(2) 7.5 μs, 2.53×10^{-4} J

(3) 2.5 ms, 7.62×10^{-3} J

(4) 25 ms, 4.98×10^{-3} J

Answer (1)

Sol. $\left(1-e^{-\frac{Rt}{L}}\right) = \frac{1}{2}$

$$\Rightarrow e^{-\frac{Rt}{L}} = e^{-3/4}$$

$$\Rightarrow \frac{100t}{1} = \frac{3}{4}$$

or t = 7.5 ms

$$I \text{ at 15 ms} = \frac{E}{R}\left(1-\frac{1}{4}\right)$$

$$= \frac{3E}{4R}$$

$$\text{Energy} = \frac{1}{2}\times L\times\left(\frac{3E}{4R}\right)^2 = \frac{1}{2}\times\frac{18\times18}{400\times400}$$

$$\cong 1\text{ mJ}$$

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. If one mole of monoatomic gas & 3 moles of diatomic gas are mixed, then the specific heat at constant volume is $\frac{\alpha^2 R}{4}$. Find value of 'α' is

Answer (3)

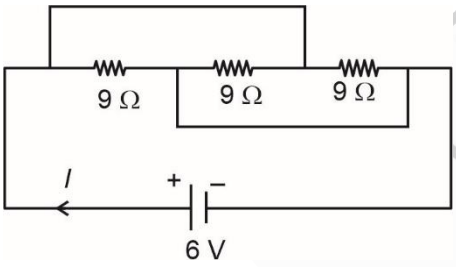
Sol. $C_{v_{mix}} = \frac{n_1 C_{v_1} + n_2 C_{v_2}}{n_1 + n_2}$

$= \frac{1 \times \frac{3}{2}R + 3 \times \frac{5}{2}R}{1+3}$

$= \frac{9}{4}R$

So, $\alpha = 3$

22. Find value of current in circuit, as shown in figure (in *A*)



Answer (2)

Sol. All the resistance are in parallel combination so R_{net} = 3 Ω

so $i = \frac{V}{R_{net}} = 2$ A

23. A wire of length 314 cm is made into a coil. Find its magnetic moment (in Am^2) if i = 14 A. (π = 3.14)

Answer (11)

Sol. $\mu = i\pi r^2$

$= i\pi\left(\frac{\ell}{2\pi}\right)^2$

$= 14\pi \times \left(\frac{3.14}{2 \times 3.14}\right)^2$

$= 11\ Am^2$

24. Find the value of electric field at depletion layer in p-n junction if its width is 6×10^{-6} m & potential difference is 0.6 V.

Is ____ $\times 10^5$ V/m

Answer (1)

Sol. $|E| = \left|\frac{\Delta V}{\Delta l}\right|$

$= \frac{0.6}{6 \times 10^{-6}} = 10^{+5}$ V/m

25. Value of acceleration due to gravity above the surface of earth at height h(h << R_e) is equal to acceleration due to gravity at depth of d below earth surface, then $d = \alpha h$. Value of α is equal to _____.

Answer (2)

Sol. $g_{above} = g_{below}$

$g_0\left(1 - \frac{2h}{Re}\right) = g_0\left(1 - \frac{\alpha h}{Re}\right)$

$\alpha = 2$

26. If two sounds of intensities $4I$ and $9I$ interfere at two points. At one, phase difference between them is 0 and $\frac{\pi}{2}$ at other. The difference between resultant intensity at the points is xI. Find the value of x.

Answer (12)

Sol. $I_1(\phi = 0) = 4I + 9I + 2\sqrt{4I \times 9I}\cos 0°$

$= 25\ I$

$I_2\left(\phi = \frac{\pi}{2}\right) = 4I + 9I + 2\sqrt{4I \times 9I}\cos\frac{\pi}{2}$

$= 13\ I$

so $I_1 - I_2 = 12\ I$

so $x = 12$

27.

28.

29.

30.

CHEMISTRY

SECTION – A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. $Zn + NaOH(aq) \longrightarrow$

Product for the reaction is

(1) ZnO (2) ZnO_2

(3) $[ZnO_3]^{4-}$ (4) $[Zn(OH)_4]^{2-}$

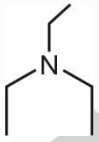
Answer (4)

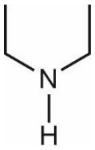
Sol. Zinc reacts with a base like NaOH to form sodium zincate and hydrogen gas is released.

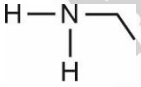
$$Zn + 2NaOH + 2H_2O \rightarrow Na_2[(Zn(OH)_4] + H_2(g)$$

2. Which of the following is the strongest bronsted base?

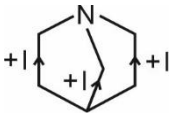
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol.  is the strongest base among the given compounds due to maximum +I effect over 'N' atom.

3. Which pair among the following is colourless?

(1) Sc^{3+}, Zn^{2+} (2) Cu^{2+}, Ti^{2+}

(3) Mn^{2+}, Fe^{3+} (4) Cu^{2+}, Fe^{3+}

Answer (1)

Sol. Species with d^0 and d^{10} electronic configuration are colourless as there is no d-d transition of electrons.

$Sc^{3+} – 3d^0$

$Zn^{2+} – 3d^{10}$

4. $N_2 + 3H_2 \longrightarrow 2NH_3$

10 g 5 g

Find limiting reagent and moles of NH_3 produced.

(1) N_2, 1.42

(2) N_2, 0.71

(3) H_2, 1.42

(4) H_2, 0.71

Answer (2)

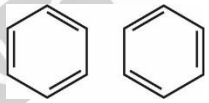
Sol. N_2 reacts with H_2 to forms NH_3 as per the reaction given below :

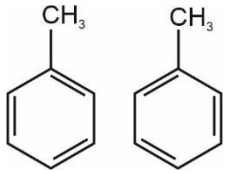
	N_2	+	$3H_2$	$\longrightarrow$	$2NH_3$
Initial moles	$\frac{10}{28}$		$\frac{5}{2}$		
Final moles	–		$\frac{5}{2}-\frac{30}{28}$		$\frac{2\times10}{28}$

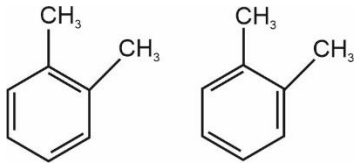
N_2 is the limiting reagent.

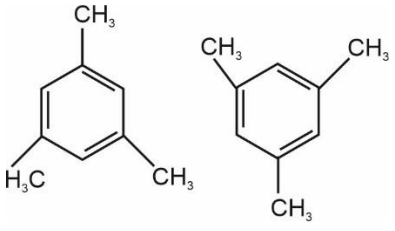
Moles of NH_3 gas formed = 0.71

5. Which of the following pair will give different products on ozonolysis?

(1) 

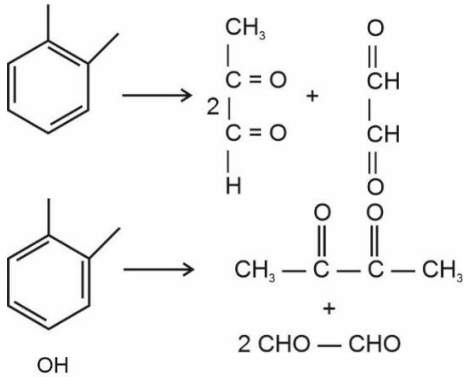
(2) 

(3) 

(4) 

Answer (3)

Sol.



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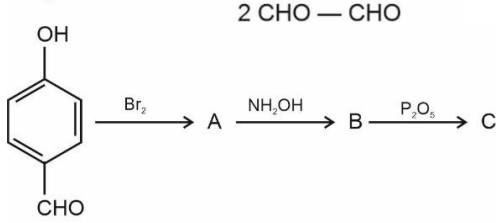
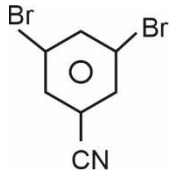
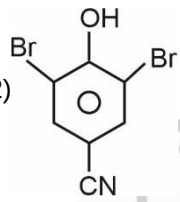
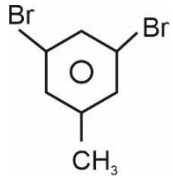
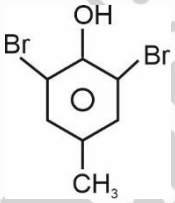


The Product 'C' is?

(1) 

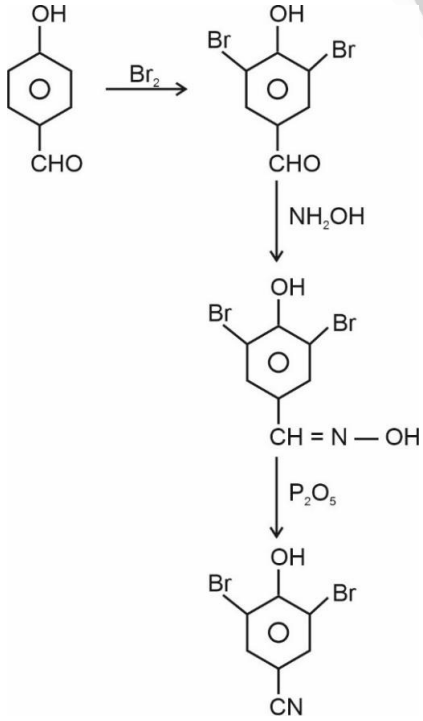
(2) 

(3) 

(4) 

Answer (2)

Sol.



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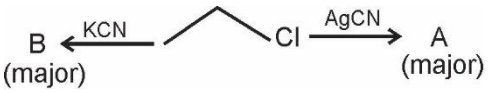
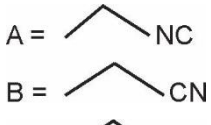
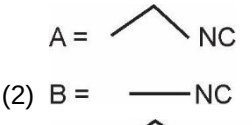
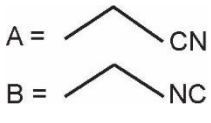
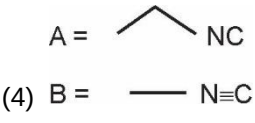


Find A and B respectively

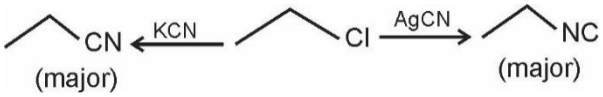
(1) 

(2) 

(3) 

(4) 

Answer (1)

Sol. 

Cyanide is ambident nucleophile it can attack through C as well as N

KCN is ionic

$KCN \rightarrow K^{+} + \bar{C}N$

Attack occurs through C giving cyanide as major product.

While AgCN has covalent character so attack starts through N

8. Which of the following is hypnotic drug?

(1) Seldane (2) Terpineol

(3) Amytal (4) Histamine

Answer (3)

Sol. Seldane → Antihistamine

Terpineol → Antiseptic

Amytal → Barbiturate (Hypnotic)

Histamine → Vasodilator

9. Which of the following pairs will have one of the compounds having odd number of electrons and will also contain a compound having expanded octet

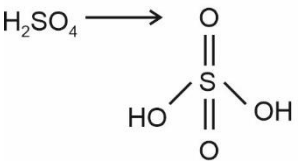
(1) BCl_3, H_2SO_4 (2) NO, H_2SO_4

(3) BCl_3, NO (4) NO, BCl_3

Answer (2)

Sol.

NO ⟶ 15 electrons (odd number)



('S' has expanded octet as represented by the structure)

10. Identify the products formed in the following reaction.

Lithium nitrate + $NaNO_3 \xrightarrow{\Delta}$ product.

(1) $Li_2O + NaNO_2$

(2) $LiNO_2 + NaNO_2$

(3) $Li_2O + Na_2O$

(4) $LiNO_2 + Na_2O$

Answer (1)

Sol. Both $LiNO_3$ and $NaNO_3$ undergo thermal decomposition according to the following reaction

$$2LiNO_3(s) \xrightarrow{\Delta} Li_2O(s) + 2NO_2(g) + \frac{1}{2}O_2(g)$$

$$NaNO_3(s) \xrightarrow{\Delta} NaNO_2(s) + \frac{1}{2}O_2(g)$$

$$2LiNO_3(s) + NaNO_3(s) \xrightarrow{\Delta} Li_2O(s) + NaNO_2(s) + 2NO_2(g) + O_2(g)$$

11. The number of lone pairs present on the following :

SCl_2, ClF_3, O_3. SF_6 respectively are

(1) 4, 2, 2, 1

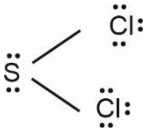
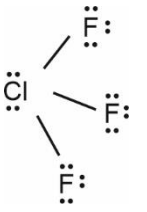
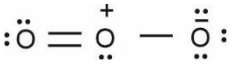
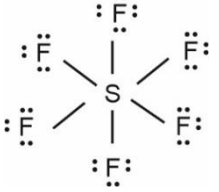
(2) 6, 4, 4, 9

(3) 6, 2, 1, 0

(4) 8, 11, 6, 18

Answer (4)

Sol.

Species	Lone pairs
	8
	11
	6
	18

12. Which of the following are herbicides

(1) Sodium chlorate and Sodium Arsenite

(2) Aldrin and Dieldrin

(3) Aldrin and Sodium Chlorate

(4) Dieldrin and Sodium Arsenite

Answer (1)

Sol. Sodium chlorate ($NaClO_3$) and sodium Arsenite (Na_3AsO_3) are examples of Herbicides

Aldrin and dieldrin are examples of pesticides

13. In a 5% w/v NaCl solution, we add albumin of egg and stir well. The resultant solution is:

(1) Lyophobic

(2) Lyophilic

(3) Emulsion

(4) Precipitate

Answer (2)

Sol. The resultant colloidal solution will have lyophilic colloid as albumin of egg contains, proteins which mix with water on stirring.

14. Calculate the ratio of energy emitted by H atom when electron jumps from infinity to ground state and 1st excited state to ground state.

(1) $\frac{3}{2}$

(2) $\frac{4}{3}$

(3) $\frac{4}{5}$

(4) $\frac{7}{9}$

Answer (2)

Sol. Electron of H-atom jumps from infinity to ground state E_1, magnitude of energy emitted = +13·6 eV.

Electron of H-atom jumps from 1st excited state to ground state E_2, magnitude of energy emitted =

$$=13\cdot6\left(1-\frac{1}{4}\right)=13\cdot6\times\frac{3}{4}\text{eV}$$

$$\frac{E_1}{E_2}=\frac{13\cdot6\times4}{13\cdot6\times3}=\frac{4}{3}.$$

15. The first ionisation enthalpy of Na, Mg and Si are 496, 737 and 786 KJ/mol. Then the value of first ionisation enthalpy of Al is (in kJ/mol)

(1) 788

(2) 747

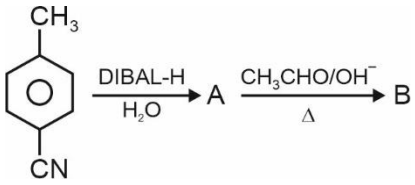
(3) 577

(4) 840

Answer (3)

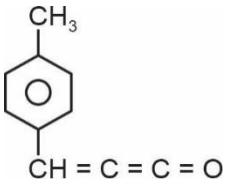
Sol. The value of first ionisation enthalpy of Al will be less than Mg, greater than Na and less than the corresponding value of Si.

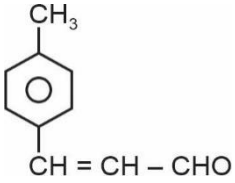
Thus, the value of first ionisation enthalpy of Al is 577 KJ/mol.

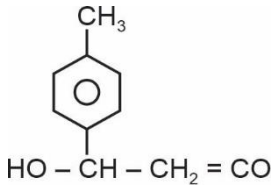
16. 

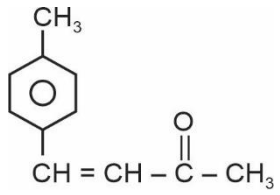
(Major)

The product B is

(1) 

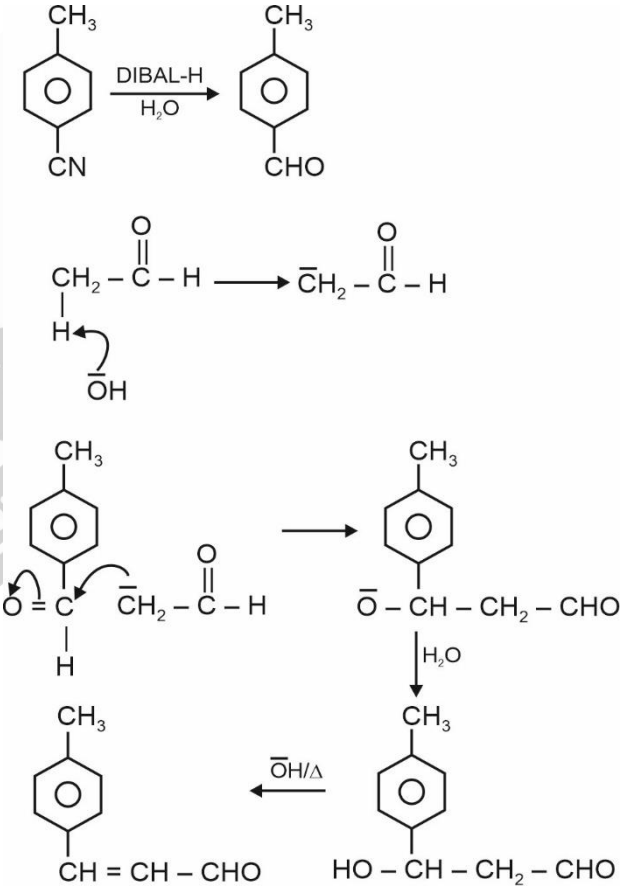
(2) 

(3) 

(4) 

Answer (2)

Sol.



17. What is gangue?

(1) Impurity in ore

(2) High quality ore

(3) Mineral

(4) Flux

Answer (1)

Sol. Gangue is impurity in the one

Eg:→ Silica (SiO_2) is present as impurity in haematite ore is gangue

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. Ionic radii for A^+ and B^- are 281 and 108 pm respectively forming a ccp structure. If B^- forms a ccp lattice and A^+(in pm) fills the octahedral voids them what is the value of edge length?

Answer (778)

Sol. $\underset{\left(\text{Radius of cation}\right)}{r^{\oplus}} + \underset{\left(\text{Radius of anion}\right)}{r^{\ominus}} = \frac{a}{2}$

$281+108 = \frac{a}{2}$

$2(389) = a$

$a = 778$ pm

22. Consider a complex $\left[Fe(CN)_6\right]^{-3}$ which acts as an inner orbital complex.

If the CFSE value after ignoring pairing energy is represented as $-x\Delta_0$, then x is

[Δ_0 is splitting energy in octahedral field]

Answer (02.00)

Sol. In $\left[Fe(CN)_6\right]^{-3}$, iron is present in (+3) oxidation state and has a d^5 configuration.

As the complex formed is an inner orbital complex,

CFSE value

$= -0.4\ \Delta_0(5)$

$= -2\Delta_0$

23. The magnitude of change in oxidation state of manganese in $KMnO_4$ in faintly alkaline or neutral medium is:

Answer (03.00)

Sol. In faintly alkaline or neutral medium, $MnO_4^{\ominus}$ change to MnO_2.

So, change in oxidation state = 7 – 4 = 3

24. K_{sp} of PbS is given as 9×10^{-30} at a given temperature. Its solubility is 'x' $\times 10^{-15}$ M.

Find the value of ''x''.

Answer (03.00)

Sol. $PbS(s) \rightleftharpoons \underset{s}{Pb^{+2}}(aq) + \underset{s}{S^{2-}}(aq)$

$K_{sp(PbS)} = S^2$

$\sqrt{K_{sp}} = S$

$\Rightarrow \quad S = \sqrt{9\times10^{-30}}$

$= 3 \times 10^{-15}$ M

$\therefore \quad x = 3$

25.

26.

27.

28.

29.

30.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. $\int_0^{\pi/2} \frac{dx}{3+2\sin x+\cos x}$ is equal to

(1) $\tan^{-1}(2)$ (2) $\tan^{-1}2-\frac{\pi}{4}$

(3) $\frac{1}{2}\tan^{-1}(2)-\frac{\pi}{8}$ (4) $\frac{\pi}{3}-\tan^{-1}(2)$

Answer (2)

Sol. $\int_0^{\pi/2} \frac{dx}{3+2\sin x+\cos x} = \int_0^{\pi/2} \frac{\left(1+\tan^2\frac{x}{2}\right)dx}{3+3\tan^2\frac{x}{2}+4\tan\frac{x}{2}+1-\tan^2\frac{x}{2}}$

Let $\tan\frac{x}{2}=t \Rightarrow \sec^2\left(\frac{x}{2}\right)dx = 2dt$

$= 2\int_0^1 \frac{dt}{2t^2+4t+4} = \int_0^1 \frac{dt}{(t+1)^2+1}$

$= \tan^{-1}(t+1)\Big|_0^1 = \tan^{-1}2-\tan^{-1}1 = \tan^{-1}2-\frac{\pi}{4}$

2. Let $Z = 2 + 3i$, then value of $(Z)^5+\left(\overline{Z}\right)^5$ is

(1) 246

(2) 244

(3) 248

(4) 234

Answer (2)

Sol. $Z^5+\overline{Z}^5 = (2+3i)^5+(2-3i)^5$

$= 2\left[{}^5C_0\cdot 2^5+{}^5C_2\cdot 2^3(3i)^2+{}^5C_4\cdot 2^1(3i)^4\right]$

$= 2[32-720+810]$

$= 244$

3. Let $\vec{a}=3\hat{i}+\hat{j}, \vec{b}=\hat{i}+2\hat{j}+\hat{k}$ and $\vec{a}\times(\vec{b}\times\vec{c})=\vec{b}\times\lambda\vec{c}$, $\vec{b}$ is non-parallel to $\vec{c}$, then value of λ is

(1) 5 (2) -5

(3) 1 (4) -1

Answer (2)

Sol. Given $\vec{a}=3\hat{i}+\hat{j}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$

also $\vec{a}\times(\vec{b}\times\vec{c})=\vec{b}+\lambda\vec{c}$

$\Rightarrow (\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c}=\vec{b}+\lambda\vec{c}$

$\therefore \lambda = -(\vec{a}\cdot\vec{b}) = -(2+3) = -5$

4. If $\lim_{x\to0}\frac{\alpha e^x+\beta e^{-x}+\gamma\sin x}{x\sin^2 x}=\frac{2}{3}$, then which of the following option is correct?

(1) $\alpha^2+\beta^2+\gamma^2=1$

(2) $\alpha\beta+\beta\gamma+\gamma\alpha+1=0$

(3) $\alpha\beta^2+\beta\gamma^2+\gamma\alpha^2+3=0$

(4) $\alpha^2-\beta^2+\gamma^2+4=0$

Answer (2)

Sol. $\lim_{x\to0}\frac{\alpha e^x+\beta e^{-x}+\gamma\sin x}{x\sin^2 x}=\frac{2}{3}$

for indeterminacy $\alpha+\beta=0$ ...(1)

$\Rightarrow \lim_{x\to0}\frac{\alpha e^x+\beta e^{-x}+\gamma\sin x}{x^3}\ \frac{x^2}{\sin^2 x}$

Applying L'Hospital rule

$\Rightarrow \lim_{x\to0}\frac{\alpha e^x-\beta e^{-x}+\gamma\cos x}{3x^2}=\frac{2}{3}$

$\therefore \alpha-\beta+\gamma=0$ ...(2)

$\Rightarrow \lim_{x\to0}\frac{\alpha e^x+\beta e^{-x}-\gamma\sin x}{bx}=\frac{2}{3}$

$\Rightarrow \lim_{x\to0}\frac{\alpha e^x-\beta e^{-x}-\gamma\cos x}{b}=\frac{2}{3}$

$\Rightarrow \alpha-\beta-\gamma=4$ ...(3)

$\Rightarrow \beta=-1, \alpha=1, \gamma=-2$

5. If $A = \{1, 2, \ldots\ldots 60\}$ and B is relation on A defined as $B = \{(x, y) : y = pq$ where p and q are primes $\geq 3\}$ then number of elements in B is

(1) 720 (2) 660

(3) 540 (4) 600

Answer (2)

Sol. Given $y = pq$ {p, q are prime number ≥ 3}

$\therefore$ y can be generated from

$3 \times 3, 3 \times 5, 3 \times 7, 3 \times 11, 3 \times 13, 3 \times 19, 5 \times 5, 5 \times 7, 5 \times 11$

7×7 $\Rightarrow$ Total 11 possibilities

x can be {1, 2, …… 60}

No. of relations = $60 \times 11 = 660$

6. If $f(x) = 3^{(x^2-2)^3} + 4$ and

P : $f(x)$ attains maximum value at $x = 0$

Q : $f(x)$ have point of inflection at $x = \sqrt{2}$

R : $f(x)$ is increasing for $x > \sqrt{2}$, then

Which of the following statement are correct

(1) P and R

(2) Q and R

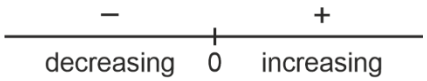
(3) P and Q

(4) All three statement P, Q, R

Answer (2)

Sol. $f(x) = 3^{(x^2-2)^3} + 4$

$$f'(x) = 6x(x^2-2)^2 \cdot 3^{(x^2-2)^3}$$



So, $x = 0$ is point of local minima.

Also $x = \sqrt{2}$ and $x = -\sqrt{2}$ are twice repeated roots of $f'(x) = 0$.

So, $f''(x)$ will change it's sign at $x = \pm\sqrt{2}$

Hence, $x = \pm\sqrt{2}$ are points of inflection.

7. Let $f(x) = |(x - 1)| \cos|x - 2| \sin|x - 1| + |x - 3| |(x^2 - 5x + 4)|$. The number of points where the function is not differentiable is

(1) 3 (2) 4

(3) 5 (4) 6

Answer (1)

Sol. $f(x) = \dfrac{|(x-1)|(\cos|x-2|)(\sin|x-1|)}{g(x)} + \dfrac{|x-3|\left|(x^2-5x+4)\right|}{h(x)}$

As polynomial is always differentiable, the points only except critical points of modulus

$\therefore$ $h(x)$ is non-diff. at $x = 3$, 1 & 4

$x = 1$, 2 are required to check for differentiability of g(x)

Examining differentiability of $g(x)$ at $x = 1$ & 2

$$g'(1^+) = \lim_{h\to 0} \frac{g(1+h) - g((1)}{h} = \lim_{h\to 0} \frac{|h|\cos(|1-h|)\sin h}{h} = 0$$

$$g'(1^-) = \lim_{h\to 0} \frac{g(1-h) - g((1)}{-h} = \lim_{h\to 0} \frac{|-h|\cos(|1+h|)\sin(-h)}{-h} = 0$$

$\therefore$ $g(x)$ is differentiable at $x = 1$

Similarly,

$$g'(2^+) = \lim_{h\to 0} \frac{|1+h|\cdot\cos|h|\cdot\sin|1+h| - \sin 1}{h} = \frac{\cos 1}{2}$$

$$g'(2^-) = \lim_{h\to 0} \frac{|1-h|\cdot\cos|-h|\cdot\sin|1-h| - \sin 1}{-h} = -\frac{\cos 1}{2}$$

$\therefore$ $f(x)$ is non-differentiable at $x = 1$, 3 & 4

8. Let A and B are two 3×3 non zero real matrix and $AB = 0$, then which of the following option is correct

(1) $AX = B$ has unique solution

(2) $AX = B$ has infinite solution

(3) B is invertible

(4) (adj(A))B is invertible

Answer (2)

Sol. $\because$ $AB = 0 \Rightarrow |A| = 0 = |B|$

So, B is not invertible as $|B| = 0$

(adj$A$)$\cdot B$ is not invertible as

$$\left|(\text{adj}A)B\right| = \left|\text{adj}A\right|\left|B\right| = 0$$

$AX = B$ has either no solution or infinitely many solution.

So only possible correct option is (2)

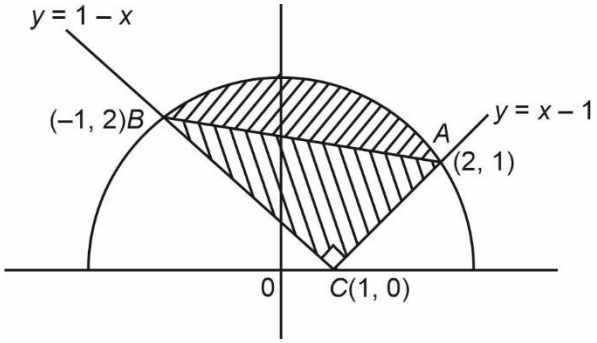
9. If $|x - 1| \leq y \leq \sqrt{5 - x^2}$, then the area of region bounded by the curves, is

(1) $\dfrac{5\pi}{4} - \dfrac{1}{2}$ (2) $\dfrac{5\pi}{4} - \dfrac{3}{2}$

(3) $\dfrac{3\pi}{4} - \dfrac{1}{2}$ (4) $\cos^{-1}\dfrac{1}{3} - \dfrac{1}{2}$

Answer (1)

Sol.



Clearly chord AB subtends a right angle at centre.

Required area = Area of ΔABC

+ Area of segment of circle on chord AB

$= \frac{1}{2}AC.BC$ [Area of quarter circle – Area of ΔAOB]

$= \frac{1}{2}\sqrt{2}.2\sqrt{2} + \left(\frac{5\pi}{4} - \frac{1}{2}\sqrt{5}.\sqrt{5}\right)$

$= \frac{5\pi}{4} - \frac{1}{2}$

10. The straight line $y = mx + c$ focal chord of parabola $y^2 = 4x$, which also touches the hyperbola $x^2 - y^2 = 4$, then the value of 'm' is

(1) $m = \pm\frac{2}{\sqrt{3}}$ (2) $m = \pm\frac{\sqrt{3}}{2}$

(3) $m = \pm\frac{2}{3}$ (4) $m = \pm\frac{3}{2}$

Answer (1)

Sol. Focal of parabola is (1, 0)

$\therefore \quad m + c = 0 \qquad \ldots(i)$

Also, if touches $x^2 - y^2 = 4$

$\therefore \quad c^2 = 4m^2 - 4$

$\Rightarrow \quad m^2 = \frac{4}{3}$

$m = \pm\frac{2}{\sqrt{3}}$

11. Let $S = \{1, 2, 3, \ldots\ldots, 2022\}$ and $A = \{(a, b) : a, b \in S$ and HCF of b and 2022 is 1$\}$. If an ordered pair (α, β) such that $\alpha, \beta \in S$ is selected then the probability that $(\alpha, \beta) \in A$ is

(1) $\frac{112}{337}$ (2) $\frac{674}{1011}$

(3) $\frac{526}{1011}$ (4) $\frac{112}{1011}$

Answer (1)

Sol. Total number of ordered pairs of $(a, b) = (2022)^2$

For favourable cases,

No. of ways to select a = 2022

No. of ways to select b as b is coprime to 2022

$= 2022\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{337}\right)$

$= 2 \times 336$

No. of favourable ways = $2022 \times 2 \times 336$

Required probability $= \frac{2022 \times 2 \times 336}{(2022)^2}$

$= \frac{112}{337}$

12. The straight line 'L' passing through point of intersection of line $bx + 10y - 8 = 0$ and $2x = 3y$. The line also passes through the point (1, 1) and is tangent to circle $17(x^2 + y^2) = 16$, then the value of $9b^2 + 16$ is

(1) 22 (2) 36

(3) 20 (4) 34

Answer (4)

Sol. Equation of required line

$(bx + 10y - 8) + \lambda(2x - 3y) = 0$

$\downarrow (1,1)$

$(b + 2) - \lambda = 0 \qquad \ldots(i)$

$\therefore \quad (b + 2b + 4)x + (10 - 3b - 6)y - 8 = 0$

also this line is tangent to $x^2 + y^2 = \frac{16}{17}$

$\therefore$ dist. from centre = radius

i.e., $\left|\frac{8}{\sqrt{(3b+4)^2 + (4-3b)^2}}\right| = \sqrt{\frac{16}{17}}$

$\Rightarrow \quad 9b^2 + 24b + 16 + 16 + 9b^2 - 24b = 68$

$\Rightarrow \quad 9b^2 + 16 = 34$

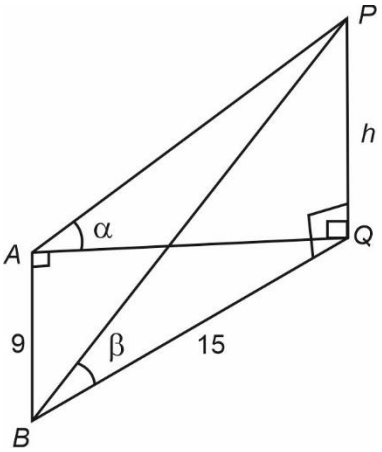
13. Tower of height h, angle of elevation from point A is α. Let B be a point 9 m to the west of A from where angle of elevation is $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$. Distance of tower from B is 15 m. The value of $\cot\alpha$ is

(1) $\frac{4}{3}$ (2) $\frac{6}{5}$

(3) $\frac{5}{7}$ (4) $\frac{7}{6}$

Answer (2)

Sol. Given $\tan\beta = \frac{2}{3}$ as $\beta = \cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$



$\because \tan\beta = \frac{h}{15} = \frac{2}{3} \Rightarrow h = 10$

Also, $AQ = \sqrt{15^2 - 9^2}$

$= 12$

So, $\cot\alpha = \frac{AQ}{h}$

$= \frac{12}{10}$

$= \frac{6}{5}$

14.

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 10 questions. In Section B, attempt any five questions out of 10. The answer to each question is a **NUMERICAL VALUE.** For each question, enter the correct numerical value (in decimal notation, truncated/rounded-off to the second decimal place; e.g. 06.25, 07.00, –00.33, –00.30, 30.27, –27.30) using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer.

21. A matrix of 3 × 3 order, should be filled either by 0 or 1 and sum of all elements should be prime number then the number of such matrix is equal to

Answer (282)

Sol. Sum of elements can be 2, 3, 5, 7

If sum = 2 No. of ways = ${}^9C_2$

If sum = 3 No. of ways = ${}^9C_3$

If sum = 5 No. of ways = ${}^9C_5$

If sum = 7 No. of ways ${}^9C_7$

$\therefore$ Total ways = ${}^9C_2 + {}^9C_3 + {}^9C_5 + {}^9C_7$

$=$ 36 + 84 + 126 + 36 = 282

22. Let $a_1, a_2, a_3, \ldots\ldots a_n$ are in A.P. and $\sum_{r=1}^{\infty}\frac{a_r}{2^r} = 4$, then $4a_2$ is equal to

Answer (16)

Sol. Let $S = \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \ldots$

$$\frac{S}{2} = \frac{a_1}{2^2} + \frac{a_2}{2^3} + \ldots$$

$$\frac{S}{2} = \frac{a_1}{2} + \frac{d}{2^2} + \frac{d}{2^3} + \frac{d}{2^4} + \ldots\infty$$

Given $S = 4$, $2 = \frac{a_1}{2} + \frac{\frac{d}{4}}{1-\frac{1}{2}}$

$\Rightarrow a_1 + d = 4 = a_2$

$\Rightarrow 4a_2 = 16$

23. If $\frac{1}{2.3.4} + \frac{1}{3.4.5} + \ldots. + \frac{1}{100.101.102} = \frac{k}{101}$ then $34k$ is equal to

Answer (286)

Sol. $\frac{1}{2.3.4} + \frac{1}{3.4.5} + \ldots. + \frac{1}{100.101.102} = \frac{k}{101}$

$$\Rightarrow \frac{1}{2}\left(\frac{4-2}{2.3.4} + \frac{5-3}{3.4.5} + \ldots + \frac{102-100}{100.101.102}\right) = \frac{k}{101}$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{2.3} - \frac{1}{3.4} + \frac{1}{3.4} - \frac{1}{4.5} + \ldots + \frac{1}{100.101} - \frac{1}{101.102}\right) = \frac{k}{101}$$

$$\Rightarrow \frac{1}{2}\left(\frac{1}{2.3}-\frac{1}{101.102}\right)=\frac{k}{101}$$

$$\Rightarrow k=\frac{1}{2}\left(\frac{101}{2.3}-\frac{1}{102}\right)$$

$$\Rightarrow \frac{1}{2}\left(\frac{10296}{2.3.102}\right)=\frac{858}{102}$$

$$\therefore \quad 34k=\frac{34.858}{102}=286$$

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