DELHI PUBLIC SCHOOL RUBY PARK KOLKATA

REVISION WORKSHEET

CLASS XII (2018-19)

MATHEMATICS

ASSIGNMENT ON RELATION FUNCTION FOR CLASS XII 2018-19:-

1. For real numbers $'x'$ and $'y'$, define xRy, if and only if $x-y+\sqrt{2}$ is an irrational number. Is R transitive? Explain your answer.

2. Let $A=\{a,b,c\}$ and the relation R be defined on A as follows.
$R=\{(a,a),(b,c),(a,b)\}$.
Then, write minimum number of ordered pairs to be added in R to make reflexive and transitive.

3. Show that the relation S in set $A=\{x\in\mathbb{Z}:0\le x\le 12\}$ given by $S=\{(a,b):a,b\in A,|a-b|\,is\,divisible\,by\,4\}$ is an equivalence relation. Find the set of all elements related to 1.

4. Let a relation R on the set A of real numbers be defined as $(a,b)\mathbb{R}\Rightarrow 1+ab>0,\forall a,b\in A$. Show that R is reflexive and symmetric but not transitive.

5. Let $f:\mathbb{Z}\to\mathbb{Z}$ be defined by $f(n)=3n$ for all $n\in\mathbb{Z}$ and $g:\mathbb{Z}\to\mathbb{Z}$ be defined by
$g(n)=\begin{cases}\dfrac{n}{3} & \text{,if } n \text{ is a multiple of 3}\\ 0 & \text{,if } n \text{ is not a multiple of 3}\end{cases}$ for all $n\in\mathbb{Z}$. Find fog and gof.

6. Let $f:\mathbb{R}\to\mathbb{R}$ be a function given by $f(x)=ax+b,\ \forall x\in\mathbb{R}$. Find the constants a and b such that $fof=I_R$.

7. Let $f:A\to A$ be a function such that $fof=f$ show that f is into onto if f is one-one.
Describe f in this case.

8. Let $f,g:\mathbb{R}\to\mathbb{R}$ be a two function defined as $f(x)=|x|+x$ and $g(x)=|x|-x$ for all $x\in\mathbb{R}$. Then, find fog and gof.

9. Let f and g be real function defined by $f(x)=\dfrac{x}{x+1}$ and $g(x)=\dfrac{x}{x+3}$.Describe the functions gof and fog (if they exist).

10. Let $f:\mathbb{N}\cup\{0\}\to\mathbb{N}\cup\{0\}$ be defined by
$f(n)=\begin{cases}n+1 & \text{,if } n \text{ is even}\\ n-1 & \text{, if } n \text{ is odd}\end{cases}$ Show that f is invertible and $f=f^{-1}$.

ASSIGNMENT ON CONTINUITY AND DIFFERENTIABILITY FOR CLASS XII 2018-19:-

1. If the function $f(x)=\begin{cases} 3ax+b, & \text{if } x>1 \\ 11, & \text{if } x=1 \\ 5ax-2b, & \text{if } x<1 \end{cases}$ is continuous at $x=1$, then find the values of a and b

2. Test the continuity of the function $f(x)$ at the origin $f(x)=\begin{cases} \dfrac{x}{|x|}, & x\neq 0 \\ 1, & x=0 \end{cases}$

3. For what value of k is the function $f(x)$ continuous at $x=0$

$$f(x)=\begin{cases} \dfrac{\sin x+x\cos x}{x}, & \text{when } x\neq 0 \\ k, & \text{when } x=0 \end{cases}$$

4. Examine the continuity of the following function $f(x)=\begin{cases} \dfrac{x}{2|x|}, & x\neq 0 \\ \dfrac{1}{2}, & x=0 \end{cases}$

5. If $f(x)=\begin{cases} \dfrac{5x+|x|}{3x}, & \text{if } x\neq 0 \\ 2, & \text{if } x=0 \end{cases}$ then show that $f(x)$ is discontinuous at $x=0$.

6. If $f(x)=\begin{cases} x\sin\dfrac{1}{x}, & \text{if } x\neq 0 \\ 0, & \text{if } x=0 \end{cases}$ then discuss the continuity of the $f(x)$ at $x=0$

7. The function $f(x)=\dfrac{\log_e(1+ax)-\log_e(1-bx)}{x}$ is not defined at $x=0$. Find the value of $f(0)$, so that $f(x)$ is continuous at $x=0$

8. Prove that $\lim\limits_{x\to\alpha}\dfrac{x^n g(x)+h(x)}{x^n+1}=\begin{cases} h(x), & \text{when } 0<x<1 \\ \dfrac{1}{2}\{h(x)+g(x)\}, & \text{when } x=1 \\ g(x), & \text{when } x>1 \end{cases}$

9. Evaluate $\lim\limits_{x\to 4}\dfrac{(\cos\alpha)^x-(\sin\alpha)^x-\cos 2\alpha}{x-4}$, $\left(0<\alpha<\dfrac{\pi}{4}\right)$

10. A function $f(x)$ is defined in the following way $f(x)=\begin{cases} -2\sin x, & \text{when } -\pi\leq x\leq -\dfrac{\pi}{2} \\ a\sin x+b, & \text{when } -\dfrac{\pi}{2}<x<\dfrac{\pi}{2} \\ \cos x, & \text{when } \dfrac{\pi}{2}\leq x\leq \pi \end{cases}$

If $f(x)$ is continuous in $-\pi \le x \le \pi$, then find a and b.

11. Find the values of a and b such that the function $f(x)=\begin{cases} x+a\sqrt{2}\sin x, & \text{when } 0\le x<\frac{\pi}{4} \\ 2x\cot x+b, & \text{when } \frac{\pi}{4}\le x\le\frac{\pi}{2} \\ a\cos 2x-b\sin x, & \text{when } \frac{\pi}{2}<x\le\pi \end{cases}$

is continuous for $0\le x\le\pi.$

12. Show that the function $\lim\limits_{n\to\infty}\frac{\cos\pi x-x^{2n}\sin(x-1)}{1+x^{2n+1}-x^{2n}}$ is continuous at $x=1$.

13. The value of $f(0)$, so that the function $f(x)=\sqrt{a^2-ax+x^2}-f(x)=\sqrt{a^2+ax+x^2}$ becomes continuous for all x is given by $f(0)=k,$ then find k.

14. Show that the function $f(x)=\begin{cases} |x-a|\sin\frac{1}{x-a}, & if\ \ x\ne a \\ 0, & if\ \ x=a \end{cases}$ is continuous at $x=a$.

15. For what value of 'k', the function $f(x)=\begin{cases} \frac{\sin 2x}{x}, & x\ne 0 \\ k, & x=0 \end{cases}$ is continuous at $x=0$?

16. The value of $f(0)$, so that the function $f(x)=\frac{(27-2x)^{\frac{1}{3}}-3}{9-3(243+5x)^{\frac{1}{5}}}\ (x\ne 0)$ becomes continuous for all x is given by $f(0)=k,$

then find k.

17. The value of $f(0)$, so that the function $f(x)=\frac{2-(256-7x)^{\frac{1}{8}}}{(5x+32)^{\frac{1}{5}}-2}\ (x\ne 0)$ becomes continuous for all x is given by $f(0)=k,$

then find k.

18. Find the value of the constants a,b and c for which the function $f(x)=\begin{cases} (1+ax)^{\frac{1}{x}}, & x<0 \\ b, & x=0 \\ \frac{(x+c)^{\frac{1}{3}}-1}{(x+1)^{\frac{1}{2}}-1}, & x>0 \end{cases}$

may be continuous at $x=0$

19. Test the differentiability of $f(x)=|\sin x-\cos x|$ at $x=\frac{\pi}{4}$.

20. If $f(x)=\begin{cases} ax^2+1, & \text{if } |x|<1 \\ \frac{1}{|x|}, & \text{if } |x\ge 1| \end{cases}$ is differentiable at $x=1$, then find a and b.

ASSIGNMENT ON INVERSE FOR CLASS XII (2018-19):-

1. Simplify : $\cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1+\sin x}}{\sqrt{1+\sin x}-\sqrt{1+\sin x}}\right);\ x\in\left(\dfrac{\pi}{2},\pi\right)$

2. Find the value of $\sin\left(\dfrac{1}{4}\sin^{-1}\dfrac{\sqrt{63}}{8}\right)$.

3. Solve for x: $\sin\left[2\cos^{-1}\left\{\cot\left(2\tan^{-1}x\right)\right\}\right]=0.$

4. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi,$ then prove that $x^4+y^4+z^4+4x^2y^2z^2=2\left(x^2y^2+y^2z^2+z^2x^2\right).$

5. Solve for x: $\cos^{-1}\dfrac{x^2-1}{x^2+1}+\tan^{-1}\dfrac{2x}{x^2-1}=\dfrac{2\pi}{3}.$

6. If $\left(\sin^{-1}x\right)^2+\left(\sin^{-1}x\right)^2+\left(\sin^{-1}x\right)^2=\dfrac{3\pi^2}{4}$, then find the minimum value of $x+y+z.$

7. Find the values of x for which $\sin^{-1}\left(\cos^{-1}x\right)<1$ and $\cos^{-1}\left(\cos^{-1}x\right)<1.$

8. Solve the equation $\sec^{-1}\dfrac{x}{a}-\sec^{-1}\dfrac{x}{b}=\sec^{-1}b-\sec^{-1}a, a\geq 1,\ b\geq 1,\ a\neq b.$

9. Solve for real values of x: $\dfrac{\left(\sin^{-1}x\right)^3+\left(\cos^{-1}x\right)^3}{\left(\tan^{-1}x+\cot^{-1}x\right)^3}=7$.

10. Let $f\left(x\right)=\sin x+\cos x+\tan x+\sin^{-1}x+\cos^{-1}x+\tan^{-1}x.$ Then find the minimum and maximum value of $f\left(x\right).$

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ASSIGNMENT FOR CLASS XII 2018-19:-

TOPIC : RELATION AND FUNCTION:-

1. Let $R = \{\left(a, a^3\right) : a$ is a prime number less than 5} be are relation. Find the range of $\mathbb{R}$.

2. State the reason for the relation R in the set $\{1, 2, 3\}$ given by $\mathbb{R} = \{(1,2),(2,1)\}$ is not to be transitive.

3. Let $A = \{0,1,2,3\}$ and defined a relation R on A as follows: $\mathbb{R} = \{(0,0),(0,1),(0,3),(1,1),(2,2),(3,0),(3,3)\}$
   Is $\mathbb{R}$ reflexive, symmetric and transitive?

4. For real numbers $'x'$ and $'y'$, define xRy , if and only if $x - y + \sqrt{2}$ is an irrational number. Is R transitive? Explain your answer.

5. Let $A = \{a, b, c\}$ and the relation R be defined on A as follows.
   $R = \{(a,a),(b,c),(a,b)\}$.
   Then, write minimum number of ordered pairs to be added in R to make reflexive and transitive.

6. Show that the relation S in set $A = \{x \in \mathbb{Z} : 0 \le x \le 12\}$ given by $S = \{(a,b) : a, b \in A, |a - b|\, is\, divisible\ by\, 4\}$ is an equivalence relation. Find the set of all elements related to 1.

7. Show that the relation R in the set A of real numbers defined as $R = \{(a,b)\, a \le b\}$ is reflexive and transitive but not symmetric.

8. Show that the relation S in the set $\mathbb{R}$ of real numbers defined as $S = \{(a,b) : a, b \in Rand\ a, b \in A\}$ is neither reflexive nor transitive.

9. Let a relation R on the set A of real numbers be defined as $(a,b)\mathbb{R} \Rightarrow 1 + ab > 0, \forall a, b \in A$. Show that R is reflexive and symmetric but not transitive.

10. Let $\mathbb{N}$ be the set of all natural numbers and let $\mathbb{R}$ be a relation on $\mathbb{N}$, define by
    $R = \{(a,b) : a\ is\, a\, multiple\ of\ 'b'\}$. Show that R is reflexive and transitive but not symmetric.

11. Let A be the set of all points in a plane and R be a relation on A defined as
    $R = \{(P,Q) : distrance\, between\, P\ and\ Q\ is\ less\, than\, 2\ units\}$. Show that R is reflective and symmetric but not transitive.

12. Let $A = \{x \in \mathbb{R} : 0 \le x < 1\}$ .If $f : A \to A$ is defined by $f(x) = \begin{cases} x & ,if\ x \in \mathbb{Q} \\ 1 - x & ,if\ x \notin \mathbb{Q} \end{cases}$ then prove that
    $fof(x) = x$ for all $x \in A$.

13. Let $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ be two function such that $fog\,(x)=\sin x^2$ and $gof\,(x)=\sin^2 x$ .Then, find $f(x)$ and $g(x)$.

14. If $f:\mathbb{R}\to\mathbb{R}$ be given by for all $x\in\mathbb{R}$,and $g:\mathbb{R}\to\mathbb{R}$ be such that $g(5/4)=1$, then prove that $gof:\mathbb{R}\to\mathbb{R}$ its a constant function.

15. Let $f:\mathbb{Z}\to\mathbb{Z}$ be defined by $f(n)=3n$ for all $n\in\mathbb{Z}$ and $g:\mathbb{Z}\to\mathbb{Z}$ be defined by

$$g(n)=\begin{cases}\frac{n}{3} & \text{,if } n \text{ is a multiple of 3}\\ 0 & \text{,if } n \text{ is not a multiple of 3}\end{cases}$$ for all $n\in\mathbb{Z}$. Find fog and gof.

16. Let $f:\mathbb{R}\to\mathbb{R}$ be a function given by $f(x)=ax+b,\ \forall x\in\mathbb{R}$. Find the constants a and b such that $fof=I_R$.

17. Let $f:A\to A$ be a function such that $fof=f$ show that f is into onto if f is one-one. Describe f in this case.

18. Let $f,g:\mathbb{R}\to\mathbb{R}$ be a two function defined as $f(x)=|x|+x$ and $g(x)=|x|-x$ for all $x\in\mathbb{R}$. Then, find fog and gof .

19. Let f and g be real function defined by $f(x)=\frac{x}{x+1}$ and $g(x)=\frac{x}{x+3}$ .Describe the functions gof and fog (if they exist).

20. If $f(x)=\frac{3x-2}{2x-3}$, prove that $f(f(x))=x$ for all $x\in\mathbb{R}-\left\{\frac{3}{2}\right\}$.

21. Let $f:\mathbb{N}\cup\{0\}\to\mathbb{N}\cup\{0\}$ be defined by

$$f(n)=\begin{cases}n+1 & \text{,if } n \text{ is even}\\ n-1 & \text{, if } n \text{ is odd}\end{cases}$$ Show that f is invertible and $f=f^{-1}$.

22. Let $f(x)=x^3$ be a function with domain $\{0,1,2,3\}$. Then show that f invertible and then write the domain of $f^{-1}(x)$.

23. Consider $f:\mathbb{R}_+\to[3,\infty)$ given by $f(x)=x^2+3$. Show that f is invertible with the inverse 'f^{-1}' given by $f^{-1}(y)=\sqrt{y-3}$, where $\mathbb{R}_+$ is the set of all non-negative real numbers.

24. Find the inverse of the function $f:\mathbb{R}\to\{x\in\mathbb{R}:x<1\}$ given by $f(x)=\frac{e^x-e^{-x}}{e^x+e^{-x}}$.

25. If $f:\mathbb{R}\to(-1,1)$ is defined by $f(x)=\frac{-x|x|}{1+x^2}$, then show that $f^{-1}(x)$ equals to $-Sgn(x)\sqrt{\frac{|x|}{1-|x|}}$

TOPIC : MATRICES AND DETERMINANT:-

Q1. Find the matrix X such that $X.\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} = \begin{pmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{pmatrix}$

Q2.For what value of k, is the following matrix singular? $\begin{bmatrix} 3-2k & k+1 \\ 2 & 4 \end{bmatrix}$

Q3.Find x , if $\begin{pmatrix} x & 4 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & 2 \\ 1 & 0 & 2 \\ 0 & 2 & -4 \end{pmatrix} \begin{pmatrix} x \\ 4 \\ -1 \end{pmatrix} = 0$

Q4.Using the properties of determinant prove that

1. $\begin{vmatrix} x & x^2 & y+z \\ y & y^2 & z+x \\ z & z^2 & x+y \end{vmatrix}$=(x-y)(y-z)(z-x)(x+y+z).

2. $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)$

3. $\begin{vmatrix} a & b & c \\ a-b & b-c & c-a \\ b+c & c+a & a+b \end{vmatrix} = a^3+b^3+c^3 - 3abc$

4. $\begin{vmatrix} 3a & -a+b & -a+c \\ -b+a & 3b & -b+c \\ -c+a & -c+b & 3c \end{vmatrix}$=3(a+b+c)(ab+bc+ca)

5. $\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$, solve for x

6. If a, b, c are all positive and are pth , qth and rth elements of G. P. then

Show that $\begin{vmatrix} \log a & p & 1 \\ \log b & q & 1 \\ \log c & r & 1 \end{vmatrix} = 0.$

7. $\begin{vmatrix} a & b-c & c+b \\ a+c & b & c-a \\ a-b & b+a & c \end{vmatrix} = (a+b+c)(a^2+b^2+c^2)$

8. $\begin{vmatrix} x & x^2 & 1+cx^3 \\ y & y^2 & 1+cy^3 \\ z & z^2 & 1+cz^3 \end{vmatrix}$=(1+cxyz)(x-y)(y-z)(z-x)

9.Evaluate: $\begin{vmatrix} 10! & 11! & 12! \\ 11! & 12! & 13! \\ 12! & 13! & 14! \end{vmatrix}$

10. $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

11.Evaluate: $\begin{vmatrix} 1+p & 1 & 1+p+q \\ 3+2p & 2 & 4+3p+2q \\ 6+3p & 3 & 10+6p+3q \end{vmatrix}$

12. $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$

13. $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

Q14.Solve the following using the matrix method:

(i) $\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4, \quad \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1, \quad \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$

(ii) x – y+z = 4 , 2x+ y-3z = 0 , x+y+z = 2

Q15. If A= $\begin{pmatrix} 1 & 1 & -1 \\ 2 & 3 & 1 \\ 3 & -1 & -7 \end{pmatrix}$,find A^{-1}Using A^{-1} solve the system of equations:

x+y-z = 3

2x + 3y + z = 10

3x – y-7z = -1

TOPIC :CONTINUITY AND DIFFERENTIABILITY:-

1. If the function $f(x)=\begin{cases}3ax+b, & \text{if } x>1\\ 11, & \text{if } x=1\\ 5ax-2b, & \text{if } x<1\end{cases}$ is continuous at $x=1$, then find the values of a and b

2. Test the continuity of the function $f(x)$ at the origin $f(x)=\begin{cases}\dfrac{x}{|x|}, & x\neq 0\\ 1, & x=0\end{cases}$

3. For what value of k is the function $f(x)$ continuous at $x=0$

$$f(x)=\begin{cases}\dfrac{\sin x + x\cos x}{x}, & \text{when } x\neq 0\\ k, & \text{when } x=0\end{cases}$$

4. Examine the continuity of the following function $f(x)=\begin{cases}\dfrac{x}{2|x|}, & x\neq 0\\ \dfrac{1}{2}, & x=0\end{cases}$

5. If $f(x)=\begin{cases}\dfrac{5x+|x|}{3x}, & \text{if } x\neq 0\\ 2, & \text{if } x=0\end{cases}$ then show that $f(x)$ is discontinuous at $x=0$.

6. If $f(x)=\begin{cases}x\sin\dfrac{1}{x}, & \text{if } x\neq 0\\ 0, & \text{if } x=0\end{cases}$ then discuss the continuity of the $f(x)$ at $x=0$

7. Determine the constants a and b such that the function $f(x)=\begin{cases}ax^2+b, & \text{if } x>2\\ 2, & \text{if } x=2\\ 2ax-b, & \text{if } x<2\end{cases}$ is continuous.

8. Let $f(x)=\begin{cases}\dfrac{1-\sin^3 x}{3\cos^2 x}, & \text{if } x<\dfrac{\pi}{2}\\ a, & \text{if } x=\dfrac{\pi}{2}\\ \dfrac{b(1-\sin x)}{(\pi-2x)^2}, & \text{if } x>\dfrac{\pi}{2}\end{cases}$. If $f(x)$ be a continuous function at $x=\dfrac{\pi}{2}$. Find a and b.

9. Determine the values of a, b and c, for which the function $f(x)=\begin{cases}\dfrac{\sin(a+1)x+\sin x}{x}, & x<0\\ c, & x=0\\ \dfrac{\sqrt{x+bx^2}-\sqrt{x}}{b\sqrt{x^3}}, & x>0\end{cases}$

may be continuous at $x=0$

10. Let $f(x)=\begin{cases}\dfrac{1-\cos 4x}{x^2}, & x<0\\ a, & x=0\\ \dfrac{\sqrt{x}}{\sqrt{16+\sqrt{x}}-4}, & x>0\end{cases}$. Determine the value of a if possible so that the function is continuous at $x=0$.

11. Is the function $f(x)=\begin{cases}\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & x\neq 0\\ k, & x=0\end{cases}$ is continuous at $x=0$

12. The function $f(x)=\dfrac{\log_e(1+ax)-\log_e(1-bx)}{x}$ is not defined at $x=0$. Find the value of $f(0)$, so that $f(x)$ is continuous at $x=0$

13. Prove that $\lim\limits_{x\to\alpha}\dfrac{x^n g(x)+h(x)}{x^n+1}=\begin{cases}h(x) & \text{, when } 0<x<1\\ \dfrac{1}{2}\{h(x)+g(x)\}, & \text{when } x=1\\ g(x) & \text{, when } x>1\end{cases}$

14. Evaluate $\lim\limits_{x\to 4}\dfrac{(\cos\alpha)^x-(\sin\alpha)^x-\cos 2\alpha}{x-4}$, $\left(0<\alpha<\dfrac{\pi}{4}\right)$

15. Given that $f(x)=\dfrac{x^3+x^2-16x+20}{(x-2)^2}$, when $x\neq 2$

$= \quad k \quad$, when $x=2$ If $f(x)$ is continuous at the origin then find k.

16. If $f(x)=\dfrac{x}{1+e^{\frac{1}{x}}}$, when $x\neq 0$ and $f(0)=0$. Show that $f(x)$ is continuous at $x=0$

17. Discuss the continuity of the function $f(x)=x.\dfrac{e^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}$, when $x\neq 0$ and $f(0)=0$

18. A function $f(x)$ is defined in the following way $f(x)=\begin{cases}-2\sin x & \text{, when } -\pi\le x\le -\dfrac{\pi}{2}\\ a\sin x+b, & \text{when } -\dfrac{\pi}{2}<x<\dfrac{\pi}{2}\\ \cos x & \text{, when } \dfrac{\pi}{2}\le x\le\pi\end{cases}$

19. If $f(x)$ is continuous in $-\pi\le x\le\pi$, then find a and b.

20. Find the values of a and b such that the function $f(x)=\begin{cases}x+a\sqrt{2}\sin x, & \text{when } 0\le x<\dfrac{\pi}{4}\\ 2x\cot x+b & \text{, when } \dfrac{\pi}{4}\le x\le\dfrac{\pi}{2}\\ a\cos 2x-b\sin x, & \text{when } \dfrac{\pi}{2}<x\le\pi\end{cases}$

is continuous for $0\le x\le\pi$.

21. Show that the function $\lim\limits_{n\to\infty}\dfrac{\cos\pi x-x^{2n}\sin(x-1)}{1+x^{2n+1}-x^{2n}}$ is continuous at $x=1$.

22. The value of $f(0)$, so that the function $f(x)=\sqrt{a^2-ax+x^2}-f(x)=\sqrt{a^2+ax+x^2}$ becomes continuous for all x is given by $f(0)=k$, then find k.

23. Show that the function $f(x)=\begin{cases}|x-a|\sin\dfrac{1}{x-a}, & if\ \ x\neq a\\ 0, & if\ \ x=a\end{cases}$ is continuous at $x=a$.

24. For what value of 'k', the function $f(x)=\begin{cases}\dfrac{\sin 2x}{x}, & x\neq 0\\ k, & x=0\end{cases}$ is continuous at $x=0$?

25. Examine that $\sin|x|$ is a continuous function.

26. The value of $f(0)$, so that the function $f(x)=\dfrac{(27-2x)^{\frac{1}{3}}-3}{9-3(243+5x)^{\frac{1}{5}}}$ $(x\neq 0)$ becomes continuous for all x is given by $f(0)=k,$ then find $k.$

27. The value of $f(0)$, so that the function $f(x)=\dfrac{2-(256-7x)^{\frac{1}{8}}}{(5x+32)^{\frac{1}{5}}-2}$ $(x\neq 0)$ becomes continuous for all x is given by $f(0)=k,$ then find $k.$

28. Find the value of the constants a,b and c for which the function $f(x)=\begin{cases}(1+ax)^{\frac{1}{x}}, & x<0\\ b, & x=0\\ \dfrac{(x+c)^{\frac{1}{3}}-1}{(x+1)^{\frac{1}{2}}-1}, & x>0\end{cases}$ may be continuous at $x=0$

29. Find the values of a and b so that $f(x)=\begin{cases}x^2+3x+a, & \text{if } x\leq 1\\ bx+2, & \text{if } x>1\end{cases}$ is differentiable at each $x\in\mathbb{R}$.

30. For what value of a and b is the function $f(x)=\begin{cases}x^2, & \text{when } x\leq c\\ ax+b, & \text{when } x>c\end{cases}$ is differentiable at $x=c$.

31. Test the differentiability of $f(x)=|\sin x-\cos x|$ at $x=\dfrac{\pi}{4}$.

32. If $f(x)=\begin{cases}ax^2+1, & \text{if } |x|<1\\ \dfrac{1}{|x|}, & \text{if } |x\geq 1|\end{cases}$ is differentiable at $x=1$, then find a and $b.$

33. If $f(x)=\sqrt{x^2+9}$, evaluate $\lim\limits_{x\to 4}\dfrac{f(x)-f(4)}{x-4}$

34. Discuss the differentiability of the following functions in their domain
   (i) $f(x)=|\cos x|$,
   (ii) $f(x)=\sin|x|$

35. Discuss the continuity and differentiability of the function $f(x)=[x]$ at $x=1,2.5$

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TOPIC : DIFFERENTIATION:-

1. If $x\sin y = 3\sin y + 4\cos y$, then find $\frac{dy}{dx}$.

2. Find the derivative of $\sec^{-1}\left(\frac{1}{2x^2-1}\right)$ with respect to $\sqrt{1+3x}$ at $x = -\frac{1}{3}$

3. Let f be twice differential such that $f''(x) = -f(x)$ and $f'(x) = g(x)$. If $h(x) = (f(x))^2 + (g(x))^2$ and $h(5) = 7$, find $h(10)$.

4. If $x = e^t \sin t$ and $y = e^t \cos t$, then show that $(x+y)^2 \frac{d^2y}{dx^2} = 2\left(x\frac{dy}{dx} - y\right)$

5. If $y = \sin^{-1}\left(\frac{x^2-1}{x^2+1}\right) + \sec^{-1}\left(\frac{x^2+1}{x^2-1}\right)$, $|x|>1$, then find $\frac{dy}{dx}$.

6. Find the derivative of $\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$ *w.r.t.* $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $(x>0)$

7. If $y = 1 + \frac{p}{x-p} + \frac{qx}{(x-p)(x-q)} + \frac{rx^2}{(x-p)(x-q)(x-r)}$, then show that $\frac{dy}{dx} = \frac{y}{x}\left(\frac{p}{p-x} + \frac{q}{q-x} + \frac{r}{r-x}\right)$.

8. If $y = \cos^{-1}\left[x^{4/3} - \sqrt{(1-x^2)(1-x^{2/3})}\right]$, $0 \le x \le 1$, show that $\frac{dy}{dx} = -\frac{1}{\sqrt{1+x^2}} - \frac{1}{3x^{2/3}\sqrt{1-x^{2/3}}}$.

9. If $y = \sin^{-1}\frac{a+b\cos x}{b+a\cos x}$, prove that $\frac{dy}{dx} = \mp\frac{\sqrt{b^2-a^2}}{b+a\cos x}$.

10. If $y = \frac{x^2+x-1}{x^3+x^2-6x}$, show that $y_1 = -\frac{1}{6x^2} - \frac{1}{2(x-2)^2} - \frac{1}{3(x+3)^2}$.

TOPIC : INTEGRATION:-

1. Evaluate : $\int \sqrt{\frac{x-1}{x^5}}\ dx$

2. Evaluate: $\int \sin\left(\log_e x\right) dx$

3. Show that $\int_0^{\alpha} \frac{dx}{1-\cos\alpha\cos x} = \frac{\pi}{2\sin\alpha}$.

4. Using definition of definite integral (limit of sum): show that $\int_0^1 x\sqrt{x}dx = \frac{2}{5}$.

5. Evaluate: $\int \frac{\left(x^4 - x\right)^{\frac{1}{4}}}{x^5}\ dx$.

6. Integrate $\int \left(\frac{\tan^{-1} x}{x} + \frac{\ln x}{1+x^2} \right) dx$

7. Integrate $\int \frac{2x^3\left(x^2-1\right)}{x^{10}+1} dx$.

8. Integrate $\int \frac{x^2-1}{x} \cdot \frac{dx}{\sqrt{\left(x^2+ax+1\right)\left(x^2+bx+1\right)}}$

9. Integrate $\int \sqrt{\frac{3-x}{3+x}\sin^{-1}\sqrt{\frac{3-x}{6}}}\ dx$

10. Integrate $\int \frac{\cos^2 x}{\sin^2 x\left(\sin^2 x - \sin^2 \alpha\right)} dx$

11. Integrate $\int \sqrt{2+\tan^2 x}\ dx$.

12. Evaluate $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}\ dx$

13. Evaluate the following $\int_0^{\pi/2} \frac{x\sin x\cos x}{\sin^4 x + \cos^4 x}$

14. Evaluate $\int_0^4 \left(x+e^{2x}\right) dx$ as the limit of a sum.

15. Evaluate: $\int \frac{\log|x|}{(x+1)^2}\ dx$.